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SEMI-SYMMETRIC MIRON CONNECTIONS IN DUAL FINSLER SPACES

Dedicated to Professor Dr. Radu Miron on the occasion of his sixtieth birthday

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Abstract

The purpose of the present paper is to give a dual theory, corresponding to the theory of semi-symmetric metrical Finsler connections in Finsler spaces given in R. Miron-M. Hashiguchi [6].

Introduction

In his recent paper [5], R. Miron studied a dual space (M, H) of a Finsler space (M, L) , and established a beautiful dual theory of Finsler geometry. All properties of such a space (M, H) are based on the existence of one canonical d -connection $M^*\Gamma$. Since these concepts were obviously made explicit by him for the first time, we hope to call (M, H) a *Miron space* and $M^*\Gamma$ a *Miron connection*.

In the present paper we shall define semi-symmetric metrical d -connections called *semi-symmetric Miron connections* in a Miron space, and study the group of transformations of these connections and its invariants. As the results of these considerations, we have two important d -tensor fields $L_j^i{}_{kl}$ and M_j^{ikl} which are invariants of semi-symmetric Miron connections (Theorem 5.2), and consider some of their properties.

The terminology and notations follow those in Miron [5], with some modifications (e. g., *Cartan space* \rightarrow *Miron space*, $N_{i\tau} \rightarrow N_{\tau i}$, $H\Gamma \rightarrow F^*\Gamma$, $H_{jk}^i \rightarrow F_j^i{}_{k\tau}$, $R_{jk\tau} \rightarrow R_{\tau jk}$, etc.). For convenience' sake, in the first two preliminary sections we shall sketch the materials necessary for our discussions from the theory of Miron [5].

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1. Distinguished geometrical objects on the cotangent bundle

Let M be an n -dimensional C^∞ -manifold and (T^*M, π^*, M) its cotangent bundle.

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Since a point of T^*M is a covector (x, p) at a point x of the base manifold M , a coordinate system $x=(x^i)$ in M induces a canonical coordinate system $(x, p)=(x^i, p_i)$ in T^*M by $p=p_i(dx^i)$.

Let V be the vertical distribution on T^*M given by $(x, p) \in T^*M \rightarrow V_{(x,p)} = \{X \in T_{(x,p)}T^*M \mid \pi^*X=0\}$. A *non-linear connection* N on T^*M is a distribution of class C^∞ given by $(x, p) \in T^*M \rightarrow N_{(x,p)} \subset T_{(x,p)}T^*M$ such that $T_{(x,p)}T^*M = N_{(x,p)} \oplus V_{(x,p)}$. N is characterized by

$$(1.1) \quad \delta_i = \partial_i + N_{ri} \partial^r,$$

where $\partial_i = \partial/\partial x^i$, $\partial^i = \partial/\partial p_i$. $\{\delta_i, \partial^i\}$ is a local basis adapted to the distributions N and V , and its dual basis is $\{dx^i, \delta p_i\}$, where

$$(1.2) \quad \delta p_i = dp_i - N_{ir} dx^r.$$

A *distinguished tensor field*, a *d-tensor field* for short, of type (r, s) on M is defined by its components, satisfying the classical law of transformations with respect to a transformation of canonical coordinate systems in T^*M :

$$(1.3) \quad \tilde{x}^i = \tilde{x}^i(x^1, \dots, x^n), \quad \tilde{p}_i = (\partial x^r / \partial \tilde{x}^i) p_r, \quad \det(\partial \tilde{x}^i / \partial x^r) \neq 0.$$

A *d-connection* on M is a triad $F^*\Gamma = (N, F, C)$, where $N = (N_{jk})$ is a non-linear connection, $F = (F_j^i{}^k)$ is a special *d-object* and $C = (C_i^{jk})$ is a *d-tensor field* of type $(2, 1)$. The transformation formulas of N_{jk} , $F_j^i{}^k$, C_i^{jk} with respect to (1.3) are as follows:

$$(1.4) \quad \begin{aligned} \tilde{N}_{jk} &= (\partial x^s / \partial \tilde{x}^j)(\partial x^t / \partial \tilde{x}^k) N_{st} + p_r (\partial^2 x^r / \partial \tilde{x}^j \partial \tilde{x}^k), \\ \tilde{F}_j^i{}^k &= (\partial \tilde{x}^i / \partial x^r)(\partial x^s / \partial \tilde{x}^j)(\partial x^t / \partial \tilde{x}^k) F_s^r{}^t + (\partial \tilde{x}^i / \partial x^r)(\partial^2 x^r / \partial \tilde{x}^j \partial \tilde{x}^k), \\ \tilde{C}_i^{jk} &= (\partial x^r / \partial \tilde{x}^i)(\partial \tilde{x}^j / \partial x^s)(\partial \tilde{x}^k / \partial x^t) C_r^{st}. \end{aligned}$$

Given a *d-connection*, the *h-* and *v-covariant derivatives* are defined for *d-tensor field*, e. g., K_j^i , by

$$(1.5) \quad K_{j|k}^i = \delta_k K_j^i + K_j^r F_{rk}^i - K_r^i F_j^r{}^k, \quad K_j^i|{}^k = \partial^k K_j^i + K_j^r C_r^{jk} - K_r^i C_j^{rk}.$$

The five *torsion* and three *curvature tensor fields* of a *d-connection* are given by

$$(1.6) \quad T_j^i{}^k = \mathfrak{A}_{jk} \{F_j^i{}^k\}, \quad C_i^{jk},$$

$$(1.7) \quad R_{ijk} = -\mathfrak{A}_{jk} \{\delta_k N_{ij}\}, \quad P_{ij}{}^k = -(\partial^k N_{ij} - F_i^k{}^j), \quad S_i^{jk} = -\mathfrak{A}_{jk} \{C_i^{jk}\},$$

$$(1.8) \quad R_j^i{}^kl = \mathfrak{A}_{kl} \{\delta_l F_j^i{}^k + F_j^r{}^k F_r^i{}^l\} + C_j^{ir} R_{rkl},$$

$$(1.9) \quad P_j^i{}^kl = \partial^l F_j^i{}^k - C_j^{il}{}^k + C_j^{ir} P_{rk}{}^l,$$

$$(1.10) \quad S_j^{ikl} = \mathfrak{A}_{kl} \{\partial^l C_j^{ik} + C_j^{rk} C_r^{il}\},$$

where $\mathfrak{A}_{jk} \{\dots\}$ denotes the alternate summation.

2. Miron spaces and metrical connections

We shall consider a C^∞ -function $H(x, p)$ defined in $T^*_\circ M = \{(x, p) \in T^*M \mid p \neq 0\}$,

and we put

$$(2.1) \quad g^{ij} = (\partial^i \partial^j H) / 2.$$

The pair (M, H) is called a *Miron space*, if H satisfies the following conditions:

- (i) H is (2) p -homogeneous: $H(x, \lambda p) = \lambda^2 H(x, p)$ for $\lambda > 0$,
- (ii) g^{ij} is non-degenerate: $\det(g^{ij}) \neq 0$.

The H and g^{ij} are called the *fundamental function* and the *fundamental metric tensor field* of (M, H) respectively. We put $p^i = g^{ir} p_r$ and $(g_{ij}) = (g^{ij})^{-1}$.

It is noted that a Miron space differs with a so-called Cartan space [2], where p_i is not a covector but a covector density.

In a Miron space (M, H) there exists a canonical d -connection determined by H only. A d -connection is called *metrical* if $g^{ij}_{|k} = 0$, $g^{ij}|^k = 0$. Let $\gamma_j^i{}^k$ be the Christoffel symbols formed with respect to g_{ij} .

Theorem 2.1. *In a Miron space there exists a unique metrical d -connection $F^* \Gamma$ satisfying $D_{jk} = -(p_r F_j^r{}^k - N_{jk})$, $T_j^i{}^k = 0$, $S_i^{jk} = 0$. If we denote this $F^* \Gamma$ by $M^* \Gamma = (N, F, C)$, then it is given by*

$$(2.2) \quad N_{jk} = p_r \gamma_j^r{}^k - p_r p^i \gamma_s^r{}^i (\partial^s g_{jk}) / 2,$$

$$(2.3) \quad F_j^i{}^k = g^{ir} (\delta_k g_{jk} + \delta_j g_{kr} - \delta_r g_{jk}) / 2,$$

$$(2.4) \quad C_i^{jk} = -g_{ir} (\partial^k g^{jr}) / 2.$$

The above d -connection $M^* \Gamma$ was first obtained by Miron [5] as a dual notion of the well-known Cartan connection of a Finsler space (cf. [3, 4]). So we shall call $M^* \Gamma$ the *Miron connection* and also N the *Miron non-linear connection*.

When we discuss d -connections with a fixed non-linear connection N , a d -connection is denoted by $F^* \Gamma(N) = (F, C)$. The set of all metrical d -connections $F^* \Gamma(N)$ was given in the following form by Miron [5], and was first used in [1], together with $M^* \Gamma$.

Theorem 2.2. *In a Miron space there exists a unique metrical d -connection $F^* \Gamma(N) = (F, C)$ having the torsion tensor fields $T_j^i{}^k$, S_i^{jk} given a priori. It is given by*

$$(2.5) \quad F_j^i{}^k = \overset{m}{F}_j^i{}^k + g^{ir} (g_{rs} T_j^s{}^k - g_{js} T_r^s{}^k + g_{ks} T_j^s{}^r) / 2,$$

$$(2.6) \quad C_i^{jk} = \overset{m}{C}_i^{jk} - g_{ir} (g^{rs} S_s^{jk} - g^{js} S_s^{rk} + g^{ks} S_s^{jr}) / 2.$$

3. Semi-symmetric Miron connections and their group of transformations

We denote the set of all d -covector fields and the set of all d -vector fields in M by $X^*(M)$ and $X(M)$ respectively.

A d -connection on M is called *semi-symmetric* if the torsion tensor fields $T_j^i{}^k$, S_i^{jk} have the form

$$(3.1) \quad T_j^i{}^k = \sigma_j \delta_k^i - \sigma_k \delta_j^i, \quad S_i^{jk} = -(\tau^j \delta_i^k - \tau^k \delta_i^j),$$

where $\sigma_j \in X^*(M)$, $\tau^j \in X(M)$.

The Miron connection $M^*\Gamma$ on a Miron space is considered as a special semi-symmetric metrical d -connection $F^*\Gamma(N)$. We call a semi-symmetric metrical d -connection $F^*\Gamma(N)$ a *semi-symmetric Miron connection*. From Theorem 2.2 and (3.1) we have

Theorem 3.1. *In a Miron space the set of all semi-symmetric Miron connections (N, F, C) is given by*

$$(3.2) \quad N_{jk} = \overset{m}{N}_{jk}, \quad F_{j^i k} = \overset{m}{F}_{j^i k} + \sigma_j \delta_k^i - g_{jk} \sigma^i, \quad C_i^{jk} = \overset{m}{C}_i^{jk} + \tau^j \delta_i^k - g^{jk} \tau_i,$$

where $\sigma_j \in X^*(M)$, $\tau^j \in X(M)$ are arbitrary, and $\sigma^i = g^{ir} \sigma_r$, $\tau_i = g_{ir} \tau^r$.

Hence two semi-symmetric Miron connections (N, F, C) , $(\bar{N}, \bar{F}, \bar{C})$ are related in the form

$$(3.3) \quad \bar{N}_{jk} = N_{jk}, \quad \bar{F}_{j^i k} = F_{j^i k} + s_j \delta_k^i - g_{jk} s^i, \quad \bar{C}_i^{jk} = C_i^{jk} + t^j \delta_i^k - g^{jk} t_i,$$

where $s_j = \bar{\sigma}_j - \sigma_j$, $t^j = \bar{\tau}^j - \tau^j$, and $s^i = g^{ir} s_r$, $t_i = g_{ir} t^r$.

Conversely, given $s_j \in X^*(M)$, $t^j \in X(M)$, the above (3.3) is thought to be a transformation of d -connections preserving the non-linear connection. Then it transforms a semi-symmetric Miron connection to a semi-symmetric Miron one. We shall denote this transformation by $t(s_j, t^j)$. Let T be the set of all such transformations. In T a product is defined by the mapping product $t(\bar{s}_j, \bar{t}^j) \circ t(s_j, t^j) = t(s_j + \bar{s}_j, t^j + \bar{t}^j)$, and we have

Theorem 3.2. *The set $\overset{s}{T}$ of all transformations $t(s_j, t^j)$ given by (3.3), together with the mapping product, is an Abelian group. This group acts on the set of all $\overset{d}{m}$ -connections effectively, and it acts on the set of all semi-symmetric Miron connections $F^*\Gamma(N)$ transitively. The group $\overset{s}{T}$ is the direct product of its subgroups $T_c = \{t(s_j, 0) \in T\}$ and $T_F = \{t(0, t^j) \in T\}$: $T = T_c \times T_F$.*

4. Various transformation formulas

In order to find invariants of the group $\overset{s}{T}$, we shall first treat a transformation of general d -connections, preserving the non-linear connection:

$$(4.1) \quad \bar{N}_{jk} = N_{jk}, \quad \bar{F}_{j^i k} = F_{j^i k} - B_{j^i k}, \quad \bar{C}_i^{jk} = C_i^{jk} - D_i^{jk}.$$

Proposition 4.1. *By a transformation (4.1) of d -connections the torsion and curvature tensor fields are transformed as follows:*

$$(4.2) \quad \bar{T}_{j^i k} = T_{j^i k} - \mathfrak{A}_{jk} \{B_{j^i k}\}, \quad \bar{C}_i^{jk} = C_i^{jk} - D_i^{jk},$$

$$(4.3) \quad \bar{R}_{ij k} = R_{ij k}, \quad \bar{P}_{ij}{}^k = P_{ij}{}^k - B_{ij}{}^k, \quad \bar{S}_i{}^{jk} = S_i{}^{jk} + \mathfrak{A}_{jk} \{D_i{}^{jk}\},$$

$$(4.4) \quad \bar{R}_{j^i kl} = R_{j^i kl} - B_{j^i r} T_{k^r l} - D_j{}^{ir} R_{rkl} - \mathfrak{A}_{kl} \{B_{j^i kl} - B_{j^i k} B_{r^l}{}^i\},$$

$$(4.5) \quad \bar{P}_{j^i k}{}^l = P_{j^i k}{}^l - B_{j^i r} C_k{}^{rl} - D_j{}^{ir} P_{rkl} - (B_{j^i k}{}^l - B_{j^i k}{}^r D_r{}^{il}) + (D_j{}^{il} - D_j{}^{rl} B_r{}^i{}^k),$$

$$(4.6) \quad \bar{S}_j^{ikl} = S_j^{ikl} - D_j^{ir} S_r^{kl} - \mathfrak{A}_{kl} \{D_j^{ik}\}^l - D_j^{rk} D_r^{il}.$$

If we eliminate R_{rkl} from (4.4), we have

Proposition 4.2. *The d-tensor field defined by*

$$(4.7) \quad K_j^i{}_{kl} = R_j^i{}_{kl} - C_j^{ir} R_{rkl}$$

is transformed by the transformation (4.1) as follows:

$$(4.8) \quad \bar{K}_j^i{}_{kl} = K_j^i{}_{kl} - B_j^i{}_{r} T_k^r{}_{l} - \mathfrak{A}_{kl} \{B_j^i{}_{kl} - B_j^r{}_{k} B_r^i{}_{l}\}.$$

In a Miron space, let us consider the d-tensor fields

$$(4.9) \quad \Omega_{jk}^{hi} = (\delta_j^h \delta_k^i - g^{hi} g_{jk})/2, \quad \Omega_{jk}^{*hi} = (\delta_j^h \delta_k^i + g^{hi} g_{jk})/2,$$

which have the well-known properties and play the same role as the operators Λ_1, Λ_2 given by M. Obata [7] (cf. [6]).

Now, we shall treat the transformation (3.3) of semi-symmetric Miron connections. Then $B_j^i{}_{k}, D_i^{jk}$ in (4.1) are expressed as $B_j^i{}_{k} = -2\Omega_{jk}^{ri} S_r, D_i^{jk} = -2\Omega_{ri}^{jk} t^r$. From Proposition 4.1 we have

Proposition 4.3. *The d-tensor fields $T_j^i{}_{k}, C_i^{jk}, S_{ij}{}^k, P_i{}^{jk}, K_j^i{}_{kl}, S_j^i{}_{kl}$ of a semi-symmetric Miron connection are transformed by the transformation (3.3) as follows:*

$$(4.10) \quad \bar{T}_j^i{}_{k} = T_j^i{}_{k} + 2\mathfrak{A}_{jk} \{\Omega_{jk}^{ri} S_r\}, \quad \bar{C}_i^{jk} = C_i^{jk} + 2\Omega_{ri}^{jk} t^r,$$

$$(4.11) \quad \bar{S}_i^{jk} = S_i^{jk} - 2\mathfrak{A}_{jk} \{\Omega_{ri}^{jk} t^r\}, \quad \bar{P}_{ij}{}^k = P_{ij}{}^k + 2\Omega_{ij}^{rk} S_r,$$

$$(4.12) \quad \bar{K}_j^i{}_{kl} = K_j^i{}_{kl} + 2\mathfrak{A}_{kl} \{\Omega_{jk}^{ri} S_{rl}\},$$

$$(4.13) \quad \bar{S}_j^{ikl} = S_j^{ikl} + 2\mathfrak{A}_{kl} \{\Omega_{rj}^{ik} t^{rl}\},$$

where

$$(4.14) \quad s_{ri} = s_{ri} - s_r s_i - s_r \sigma_i + (s/2) g_{ri}; \quad s = g^{rs} s_r s_s,$$

$$(4.15) \quad \bar{t}^{rl} = \bar{t}^r{}^l - \bar{t}^r \bar{t}^l + \bar{t}^r \tau^l + (t/2) g^{rl}; \quad \bar{t} = g_{rs} \bar{t}^r \bar{t}^s.$$

5. Invariants of semi-symmetric Miron connections

We have the following important invariants from Proposition 4.3 by the well-known elimination method [4], [6].

Theorem 5.1. *The following d-tensor fields of semi-symmetric Miron connections are invariants of the group T :*

$$(5.1) \quad T_j^i{}_{k} - \mathfrak{A}_{jk} \{T_j \delta_k^i\} / (n-1), \quad C_i^{jk} - 2\Omega_{ri}^{jk} C^r / (n-1), \quad \tilde{C}^k,$$

$$(5.2) \quad R_{ij}{}^k, \quad S_i^{jk} - \mathfrak{A}_{jk} \{S^j \delta_i^k\} / (n-1), \quad P_{ij}{}^k - 2\Omega_{ij}^{rk} P_r / (n-1), \quad \tilde{P}_j,$$

where $T_j = T_j^i{}_{i}, C^j = C_i^{ji}, S^j = S_i^{ji}, P_i = P_{ij}{}^j$, and $\tilde{C}^k = C_i^{ik}, \tilde{P}_j = P_{ij}{}^i$.

Theorem 5.2. For $n > 2$ the following d -tensor fields of semi-symmetric Miron connections are invariants of the group T :

$$(5.3) \quad L_j^i{}_{kl} = K_j^i{}_{kl} + 2\mathfrak{A}_{kl}\{\Omega_{jk}^{ri}(K_{rl} - g_{rl}K/2(n-1))\}/(n-2),$$

$$(5.4) \quad M_j^{ikl} = S_j^{ikl} + 2\mathfrak{A}_{kl}\{\Omega_{rj}^{ik}(S^{rl} - g^{rl}S/2(n-1))\}/(n-2),$$

where $K_{jk} = K_j^i{}_{ki}$, $S^{ik} = S_j^{ikj}$, $K = g^{jk}K_{jk}$ and $S = g_{ik}S^{ik}$.

We shall show some properties of the invariants $L_j^i{}_{kl}$, M_j^{ikl} .

Theorem 5.3. In a Miron space the invariants $L_j^i{}_{kl}$, M_j^{ikl} of semi-symmetric Miron connections $F^*\Gamma(N)$ have the following properties:

$$(5.5) \quad \Omega_{jr}^{*si} L_s^r{}_{kl} = -C_j^{is} R_{skl}, \quad \Omega_{jr}^{*si} M_s^{rkl} = 0,$$

$$(5.6) \quad L_i^i{}_{kl} = -\tilde{C}^s R_{skl}, \quad L_j^i{}_{ki} = 0; \quad M_i^{ikl} = 0, \quad M_j^{ikj} = 0,$$

$$(5.7) \quad \mathfrak{S}_{jkl}\{L_j^i{}_{kl}\} = -\tilde{C}^s \mathfrak{S}_{jkl}\{\delta_j^i R_{skl}\}/(n-2); \quad \mathfrak{S}_{ikl}\{M_j^{ikl}\} = 0,$$

where $\tilde{C}^s = S^s + C^s$, and $\mathfrak{S}_{jkl}\{\dots\}$ denotes the cyclic summation.

For the proof it is sufficient to study the properties of $L_j^i{}_{kl}$, M_j^{ikl} formed with respect to the Miron connection $M^*\Gamma$.

As is shown in [5], from the metrical property of $M^*\Gamma$ we have $\Omega_{jr}^{*si} R_s^r{}_{kl} = 0$, $\Omega_{jr}^{*si} S_s^{rkl} = 0$, from which we have

$$(5.8) \quad \Omega_{jr}^{*si} K_s^r{}_{kl} = -C_j^{is} R_{skl}, \quad K_i^i{}_{kl} = -C_i^{is} R_{skl}, \quad S_i^{ikl} = 0.$$

On the other hand, from the Bianchi identities $\mathfrak{S}_{jkl}\{K_j^i{}_{kl}\} = 0$, $\mathfrak{S}_{ikl}\{S_j^{ikl}\} = 0$, we have

$$(5.9) \quad \mathfrak{A}_{kl}\{K_{kl}\} = C_i^{is} R_{skl}, \quad \mathfrak{A}_{kl}\{S^{kl}\} = 0.$$

Thus the proof follows in the same way as in [6, p. 34].

We shall finally give an example of a Miron space satisfying $L_j^i{}_{kl} = 0$, $M_j^{ikl} = 0$.

Theorem 5.4. If the Miron connection $M^*\Gamma$ has the properties of h - and v -isotropy:

$$(5.10) \quad K_j^i{}_{kl} = h(\delta_j^i g_{jk} - \delta_k^i g_{jl}), \quad S_j^{ikl} = v(\delta_j^i g^{ik} - \delta_j^k g^{il}),$$

where $h(x, p)$, $v(x, p)$, are arbitrary functions in T_0^*M , then for any semi-symmetric Miron connection $F^*\Gamma(N)$ we have

$$(5.11) \quad L_j^i{}_{kl} = 0, \quad M_j^{ikl} = 0.$$

Indeed, $L_j^i{}_{kl} = 0$, $M_j^{ikl} = 0$ follow from the respective conditions of (5.10).

It would be an interesting problem to characterize the Miron spaces satisfying $L_j^i{}_{kl} = 0$, $M_j^{ikl} = 0$. On the other hand, we can also treat semi-symmetric metrical d -connection such that deflection tensor fields D_{jk} vanish, and consider some special Miron spaces.

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