

Diameters of Planar Nets of the Regular Dodecahedron

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Diameters of Planar Nets of the Regular Dodecahedron

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Abstract

Incongruent planar nets of the regular dodecahedron are classied according to their diameters, and their frequency distribution is shown.

1 Introduction

Planar nets of a polyhedron have attracted acute interest from ancient times (for example, see Pedoe (1976)). It has been well-known that the cube (the regular hexahedron) has the number 11 kinds of incongruent planar nets. It is also known that, by the duality between the cube and the regular octahedron, the regular octahedron has the same number of incongruent planar nets.

How about the regular dodecahedron and its dual, i.e., the regular icosahedron? Hippenmeyer(1979) solved the problem: both regular solids have the number 43,380 kinds of incongruent planar net. He obtained the number by use of the Matrix-Tree theorem and the Burnside formula.

Although the number 43,380 has been determined, there is little knowledge about shapes of these nets themselves. Only one exception is a research by Horiyama and Shoji (2011). They shows that these nets never overlap.

In this paper we classify incongruent planar nets of the regular dodecahedron according to their diameters. The diameter of a finite point sets, say $V = \{P_1, P_2, \dots, P_n\}$, is defined as the maximum of Euclidean distances of all pairs of points, i.e.,

$$\text{diamater of } V = \max_{1 \leq i < j \leq n} d(P_i, P_j),$$

where d denotes the Euclidean distance of two points. In other words, the diameter of a finite point sets is the minimum of diamters of circles that contain all the points.

2 Diameters

For every planar net of the regular dodecahedron, we construct a graph if we regard faces of the net as vertices of a graph and draw edges of a graph between adjacent faces of the net. Obviously the graph is a

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tree, and its number of vertices is given by $12 \times 5 - (12 - 1) \times 2 = 38$. We compute the diameter of these 38 vertices of the net.

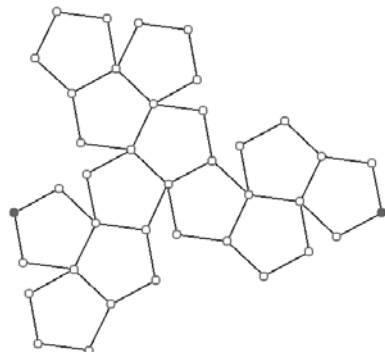
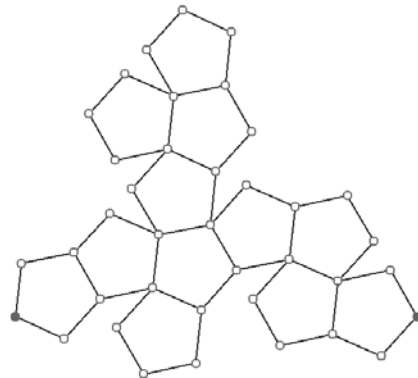
The following table gives the distribution of diameters for all the incongruent planar nets of the regular dodecahedron of unit edge length: the leftmost column shows exact values of diameters, where τ denotes the golden ratio ($\sqrt{5} + 1$)/2; the middle column shows their approximate values; and the rightmost column shows frequencies of nets. Of course the sum of frequencies is equal to 43,380.

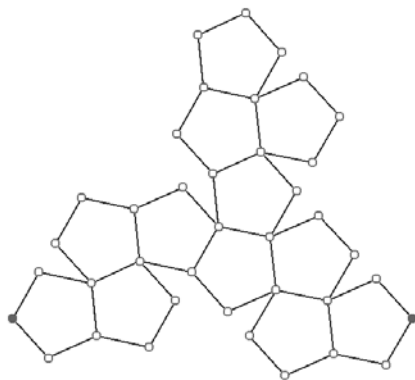
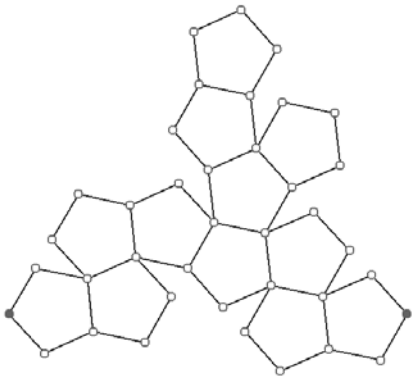
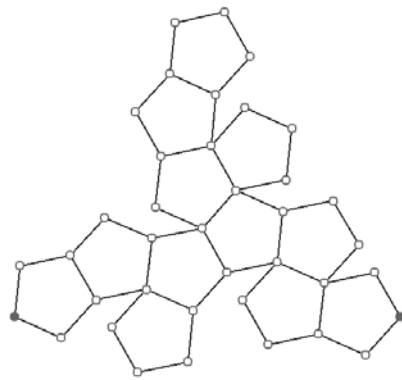
exact	appr.	freq.	exact	appr.	freq.	exact	appr.	freq.
$\sqrt{25\tau + 16}$	7.513	20	$\sqrt{36\tau + 28}$	9.287	294	$\sqrt{49\tau + 33}$	10.596	127
$\sqrt{26\tau + 17}$	7.686	415	$\sqrt{38\tau + 25}$	9.300	2804	$\sqrt{48\tau + 35}$	10.614	40
$\sqrt{25\tau + 19}$	7.710	176	$\sqrt{37\tau + 28}$	9.374	336	$\sqrt{51\tau + 32}$	10.701	699
$\sqrt{25\tau + 20}$	7.775	81	$\sqrt{36\tau + 31}$	9.447	45	$\sqrt{48\tau + 37}$	10.708	2
$\sqrt{25\tau + 21}$	7.839	67	$\sqrt{40\tau + 25}$	9.472	463	$\sqrt{47\tau + 40}$	10.773	1
$\sqrt{27\tau + 20}$	7.980	95	$\sqrt{39\tau + 28}$	9.545	11	$\sqrt{52\tau + 33}$	10.823	83
$\sqrt{29\tau + 18}$	8.057	1404	$\sqrt{39\tau + 29}$	9.597	208	$\sqrt{52\tau + 35}$	10.915	99
$\sqrt{25\tau + 25}$	8.090	3	$\sqrt{41\tau + 26}$	9.609	2003	$\sqrt{53\tau + 35}$	10.989	11
$\sqrt{29\tau + 19}$	8.119	2105	$\sqrt{41\tau + 27}$	9.661	238	$\sqrt{50\tau + 41}$	11.041	1
$\sqrt{30\tau + 19}$	8.218	4077	$\sqrt{41\tau + 28}$	9.713	1098	$\sqrt{55\tau + 34}$	11.090	44
$\sqrt{27\tau + 25}$	8.288	16	$\sqrt{40\tau + 31}$	9.784	26	$\sqrt{55\tau + 35}$	11.135	50
$\sqrt{29\tau + 22}$	8.302	469	$\sqrt{38\tau + 35}$	9.823	7	$\sqrt{54\tau + 37}$	11.152	21
$\sqrt{26\tau + 27}$	8.311	2	$\sqrt{43\tau + 27}$	9.827	2192	$\sqrt{55\tau + 36}$	11.180	36
$\sqrt{18\tau + 25}$	8.385	32	$\sqrt{39\tau + 34}$	9.854	14	$\sqrt{56\tau + 36}$	11.252	58
$\sqrt{31\tau + 21}$	8.436	2	$\sqrt{41\tau + 32}$	9.917	72	$\sqrt{56\tau + 37}$	11.296	22
$\sqrt{32\tau + 20}$	8.472	3396	$\sqrt{38\tau + 37}$	9.924	2	$\sqrt{55\tau + 39}$	11.313	1
$\sqrt{31\tau + 23}$	8.553	39	$\sqrt{44\tau + 28}$	9.960	15	$\sqrt{58\tau + 37}$	11.439	52
$\sqrt{29\tau + 27}$	8.598	12	$\sqrt{40\tau + 35}$	9.986	10	$\sqrt{59\tau + 37}$	11.509	10
$\sqrt{32\tau + 23}$	8.647	837	$\sqrt{44\tau + 29}$	10.010	470	$\sqrt{59\tau + 38}$	11.553	4
$\sqrt{33\tau + 22}$	8.683	3843	$\sqrt{45\tau + 29}$	10.090	929	$\sqrt{59\tau + 39}$	11.596	11
$\sqrt{31\tau + 26}$	8.727	191	$\sqrt{41\tau + 36}$	10.116	2	$\sqrt{61\tau + 38}$	11.692	5
$\sqrt{31\tau + 27}$	8.784	10	$\sqrt{44\tau + 32}$	10.158	159	$\sqrt{59\tau + 42}$	11.725	1
$\sqrt{34\tau + 23}$	8.833	3367	$\sqrt{45\tau + 31}$	10.189	115	$\sqrt{60\tau + 41}$	11.751	3
$\sqrt{30\tau + 31}$	8.919	10	$\sqrt{44\tau + 33}$	10.208	59	$\sqrt{65\tau + 41}$	12.090	2
$\sqrt{35\tau + 24}$	8.979	1006	$\sqrt{41\tau + 38}$	10.215	1	$\sqrt{64\tau + 43}$	12.106	1
$\sqrt{36\tau + 23}$	9.014	477	$\sqrt{46\tau + 31}$	10.268	404	$\sqrt{66\tau + 41}$	12.157	6
$\sqrt{35\tau + 25}$	9.035	437	$\sqrt{47\tau + 30}$	10.298	459	$\sqrt{65\tau + 44}$	12.214	1
$\sqrt{35\tau + 26}$	9.090	329	$\sqrt{46\tau + 33}$	10.365	67	$\sqrt{66\tau + 47}$	12.401	1
$\sqrt{37\tau + 23}$	9.103	5418	$\sqrt{44\tau + 37}$	10.402	5	$\sqrt{69\tau + 43}$	12.436	1
$\sqrt{34\tau + 29}$	9.166	48	$\sqrt{47\tau + 33}$	10.443	133	$\sqrt{72\tau + 45}$	12.708	1
$\sqrt{36\tau + 27}$	9.233	246	$\sqrt{49\tau + 32}$	10.549	40	$\sqrt{74\tau + 47}$	12.913	1
$\sqrt{35\tau + 29}$	9.254	83	$\sqrt{50\tau + 31}$	10.578	591			

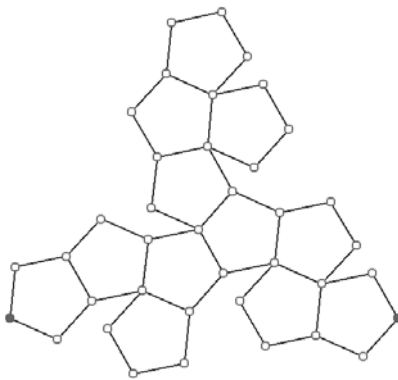
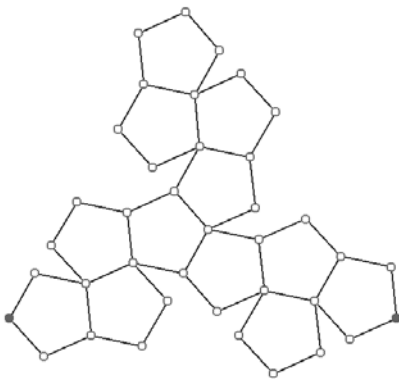
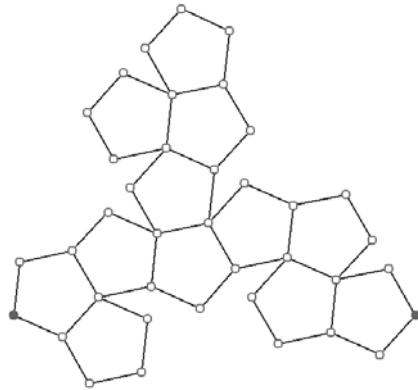
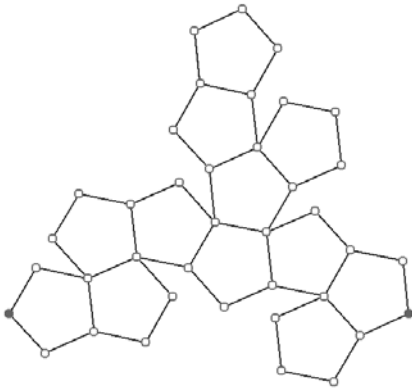
3 Incongruent planar nets of the minimal diameter

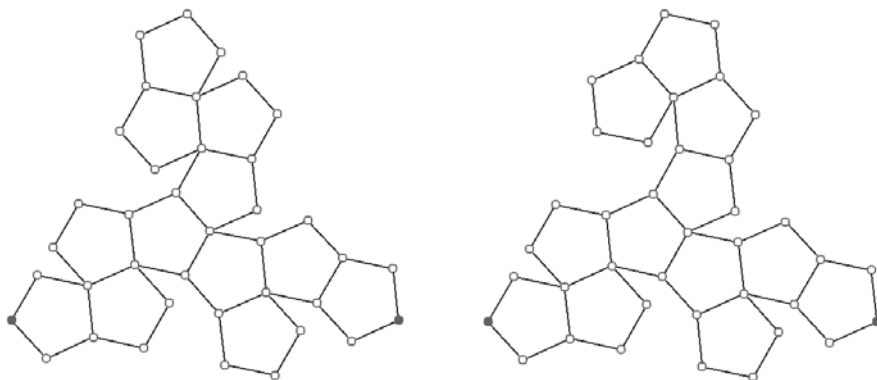
The following figures show all the incongruent planar nets of the minimal diameter $\sqrt{25\tau + 16}$.

They are the most 'compact' planar nets. In each figure the distance between two filled points corresponds to the minimal diameter.



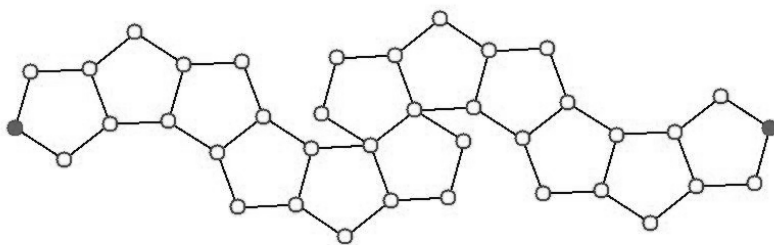






4 Unique planar net of the maximal diameter

The following figure shows the unique incongruent planar net of the maximal diameter $\sqrt{74\tau + 47}$. This is the most 'long' planar net. Again the distance between two filled points corresponds to the maximal diameter.



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