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Yôichi KOSHIBA *

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Abstract

We report some results on the calculations of the n-th cyclotomic polynomial $\Phi_n(x)$ for $n = 111546435 = 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$. In the coefficients of this polynomial,the maximum value and the minimum value are $\geqq 4071770387$ and $\leqq -4248451085$, respectively from the point of view of our calculations.

Key words:cyclotomic polynomial,coefficients,supercomputing.

1 Grytczuk-Tropak's Formula

Let $\Phi_n(x) = \sum_{j=0}^{\phi(n)} a_j(n)x^j$ be the n-th cyclotomic polynomial,where $\phi(n)$ is the *Euler* function. The coefficients of the polynomial $\Phi_n(x)$ are given as follows (A.Grytczuk & B.Tropak[4]):

$$a_j(n) = -\frac{1}{j} \cdot \mu(n) \cdot \sum_{m=0}^{j-1} a_m(n)\mu((n, j-m))\phi((n, j-m))$$

where $\mu(n)$ is the *Möbius* function and $(n, j-m)$ is the greatest common divisor of integers n and $j-m$.

This formula is deducible both from the recurrence expression representing of the roots of the algebraic equations by use of the elementary expressions (van der Waerden[5, 102page,Aufgaben.1]) and from the *Hölder* equation for the *Ramanujan* sums (*Hölder*[6, Theorem 272]).

Thanks to Professor B.Tropak's suggestions,the running program for the calculations of the cyclotomic polynomial is rewritten to this formula. To obtain the

*Department of Mathematics and Computer Science, Faculty of Science, Kagoshima University, Kagoshima,890-0065, Japan.

complete list of the coefficients of polynomial $\Phi_n(x)$,it is sufficient for degree term $[\phi(n)/2]+1$,since $\Phi_n(x)$ is a symmetric polynomial. We put $n = 111546435 = 3\times5\times7\times11\times13\times17\times19\times23$. In this case ,$\phi(n) = 36495360$. Due to the time limit of our supercomputing, we could not obtain the complete list of $a_j(n)$'s.But the first term with degree $\leqq \phi(n)/10 = 3649536$ was outputed.

Main Output Results

The maximum and minimum values of the coefficients in the first term with degree $\leqq \phi(n)/10 = 3649536$ are 4071770387 and -4248451085 respectively. That is,

$$\Phi_{3\times5\times7\times11\times13\times17\times19\times23}(x) = \underbrace{x^{36495360}+\cdots+\bigcirc x^{32845824}}_{\text{maximum=4071770387, minimum=-4248451085}}+\cdots+1.$$

2 The program

The followings are the Fortran90 program by the algorithm of the above Grytczuk-Tropak's Formula. The greater part of the consumptions of the time in our program runnings is due to the label 400 of do loop.

```
!A.Grytczuk and B.Tropak
      parameter (isize=36495360)
      integer*8 T(isize)
      integer*8 a(0:isize)
      integer*8 prime(20)
      integer*8 euler,r,s,n,sum,myu,const
!         read(5,*)n
           n=3*5*7*11*13*17*19*23
!          n=3*5*7
         call check(n,prime,s)
          myu=(-1)**s
!
          euler=1
         do 100 j=1,s
```

```
          euler=euler*(prime(j)-1)
 100    continue
!
         do 500 r=1,euler
         T(r)=1
 500   continue
!
      do 300 j=1,s
      do 300 r=1,euler/prime(j)
      T(prime(j)*r)=T(prime(j)*r)*(1-prime(j))
 300   continue
!
          a(0)=1
           const=euler/10
!         do 400 j=1,euler
         do 400 j=1,const
!        do 400 j=1,10000000
          sum=0
            do 600 m=0,j-1
               sum=sum+a(m)*T(j-m)
 600         continue
           a(j)=-myu*(sum/j)
 400     continue
       write(6,*)'max=',MAXVAL(a),'min=',MINVAL(a)
        end
```

The file containing subroutine check is omitted as it is a simple and short program.

3 Running Informations

Our calculations have been done by the NEC supercomputer SX-4.The followings are compiler and system messages.This lists show that User Time(sec) is 27018.020478 sec=7 hours 30 minutes.The integers 4071770387 and -4248451085 must have overflow as 32-bits integer data.

```
cyc_Grytczuk_Tropak_64.f90:

f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 14:
                                        Vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 18:
                                        Vectorized loop.
f90: vec(2): cyc_Grytczuk_Tropak_64.f90, line 22:
                                        Partially vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 23:
                                        Vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 32:
                                        Vectorized loop.
f90: vec(4): cyc_Grytczuk_Tropak_64.f90, line 37:
                                        Vectorized array expression.
f90: vec(4): cyc_Grytczuk_Tropak_64.f90, line 37:
                                        Vectorized array expression.
f90: cyc_Grytczuk_Tropak_64.f90, _MAIN: There are 7 diagnoses.
check_64.f90:

f90: vec(1): check_64.f90, line 14: Vectorized loop.
f90: check_64.f90, check: There is 1 diagnosis.
moebius_64.f90:

f90: vec(3): moebius_64.f90, line 8: Unvectorized loop.
f90: moebius_64.f90, moebius: There is 1 diagnosis.
start time=Tue Nov 23 18:00:11 JST 1999
 max=  4071770387  min=  -4248451085


      ******    Program Information    ******
  Real Time(sec)                :      28949.717421
  User Time(sec)                :      27018.020478
  Sys Time(sec)                 :         64.466699
  Vector Time(sec)              :      27015.028925
```

```
  Inst.Count                    :      1561253529530.
  V.Inst.Count                  :       416268499499.
  V.Element Count               :    106561013327958.
  FLOP Count                    :         7300322.
  MOPS                          :       3986.450393
  MFLOPS                        :          0.000270
  VLEN                          :        255.991057
  V.Op.Ratio (%)                :         98.936935
  Memory Size(MB)               :        561.031250
  MIPS                          :         57.785637
  I-Cache(sec)                  :          2.754349
  O-Cache(sec)                  :          1.222460
  Bank(sec)                     :          0.692612
  end time=Wed Nov 24 02:02:40 JST 1999
logout
```

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