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ON DETERMINATIONS OF FINSLER CONNECTIONS
BY DEFLECTION TENSOR FIELDS

By

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The author [2] discussed parallel displacements in Finsler spaces and showed that the connection $\mathcal{F}$ defined by E. Cartan [1] is the shortest and fittest from a natural standpoint. In that case we imposed as a natural condition the torsion tensor field to vanish, but in its definition the supporting elements are confined to be parallel. And, M. Matsumoto [4] has proposed, from the standpoint of his modern Finsler theory, the following elegant axioms that determine uniquely that connection $\mathcal{F}$ and the associated non-linear connection $\mathcal{N}$:

(C1) the connection $\mathcal{F}$ be metrical,
(C2) the deflection tensor field $D=0$,
(C3) the $(h)h$-torsion tensor field $T=0$,
(C4) the $(v)v$-torsion tensor field $S^1=0$,

where the axiom C2 expresses the geometrical meaning as above stated.

So, from the standpoint that the supporting elements may be displaced with respect to any non-linear connection $\mathcal{N}$ in the tangent bundle, we shall replace the condition C2 by some weaker conditions and find the conditions to be imposed thereon in order that the connection $\mathcal{F}$ defined by E. Cartan be obtained (Theorem A).

As a result of this consideration we shall notice that Finsler connections with the deflection tensor field $D=-\delta$ are somewhat canonical. We shall give an example of such a Finsler connection (Theorem B).

Throughout the present paper we shall use the terminology and notations described in M. Matsumoto [5]. In §1, we shall briefly sketch the materials in need of our discussions.

The author wishes to express his sincere gratitude to Prof. M. Matsumoto for the invaluable suggestions and encouragements.

§ 1. Preliminaries

1°. Given a differentiable manifold $M$ of dimension $n$, we denote by $L(M)(M, \pi, GL(n, R))$ the bundle of linear frames and by $T(M) (M, \tau, F, GL(n, R))$ the tangent bundle, where the standard fiber $F$ is a vector space of dimension $n$ with a fixed base $\{e_a\}$. 
The induced bundle $\tau^{-1}L(M) = \{(y, z) \in T(M) \times L(M) | \tau(y) = \tau(z)\}$ is called the Finsler bundle of $M$ and denoted by $F(M)$ $(T(M), \pi_1, GL(n, R))$. The projection $\pi_1$ is the mapping

$$\pi_1: F(M) \rightarrow T(M) \mid (y, z) \rightarrow y,$$

and we shall denote by $\pi_2$ the mapping

$$\pi_2: F(M) \rightarrow L(M) \mid (y, z) \rightarrow z.$$

The Lie algebra of the structural group $GL(n, R)$ of $L(M)$ or $F(M)$ is denoted by $L(n, R)$ and the canonical base by $\{L_i^j\}$.

2°. A Finsler connection $(\Gamma, N)$ is by definition a pair of a connection $\Gamma$ in the Finsler bundle $F(M)$ and a non-linear connection $N$ in the tangent bundle $T(M)$.

Given a Finsler connection $(\Gamma, N)$, let $l_u(u \in F(M))$ and $l_y(y \in T(M))$ be the respective lifts with respect to $\Gamma$ and $N$. In terms of a canonical coordinate system $(x^i, y^j, z^k)$ of $F(M)$, they are expressed by

\begin{align}
(1) \quad l_u\left(\frac{\partial}{\partial x^k}\right)_y &= \left(\frac{\partial}{\partial x^k}\right)_y - z^i\Gamma^j_{ik}\left(\frac{\partial}{\partial z^j}\right)_u, \\
(2) \quad l_u\left(\frac{\partial}{\partial y^k}\right)_y &= \left(\frac{\partial}{\partial y^k}\right)_y - z^iC^j_{ik}\left(\frac{\partial}{\partial z^j}\right)_u, \\
\text{and} \\
(3) \quad l_y\left(\frac{\partial}{\partial x^k}\right)_x &= \left(\frac{\partial}{\partial x^k}\right)_x - F^i_j\left(\frac{\partial}{\partial y^i}\right)_y,
\end{align}

where $\Gamma^j_{ik}$, $C^j_{ik}$ are called the components of $\Gamma$ and the $F^i_j$ the components of $N$. $C^j_{ik}$ are also the components of the $(h)\nu$-torsion tensor field $\mathcal{C}$.

For each $f \in F$ the $h$- and the $\nu$-basic vector fields $B^h(f)$ and $B^\nu(f)$ are defined by

\begin{align}
(4) \quad B^h(f)_u &= l_u l_y(zf), \\
(5) \quad B^\nu(f)_u &= l_u l_y(zf)
\end{align}

at $u = (y, z)$ respectively, where $l_y$ is the vertical lift expressed by

\begin{align}
(6) \quad l_y\left(\frac{\partial}{\partial x^i}\right)_x &= \left(\frac{\partial}{\partial y^i}\right)_y.
\end{align}

The $h$- and the $\nu$-basic forms $\theta^h$ and $\theta^\nu$ constitute, with the connection form $\omega$ of $\Gamma$, the dual system of $(B^h(f), B^\nu(f), Z(A))$, where $Z(A)$ is the fundamental vector field corresponding to $A \in L(n, R)$. They are expressed by

\begin{align}
(7) \quad \theta^h &= z^{-1}d x^i e_a,
\end{align}
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\[ \theta^e = z^{-1} \gamma (d y^i + F^i_k dx^k) e_a \]

and

\[ \omega = z^{-1} \alpha (d z^i + z^i \Gamma^i_j_k dx^k + z^i C^i_j_k dy^k) L^b_a. \]

If we denote by \( \theta \) the basic form in \( L(M) \) then

\[ \theta^b = \pi_2 \theta. \]

3. Given a Finsler connection \((\Gamma, N)\), we get the associated non-linear connection \( \overline{N} \) with the subordinate \( F \)-connection \( \Gamma_F \) to \((\Gamma, N)\). The pair \((\Gamma, \overline{N})\) is a Finsler connection and is called the associated connection with the given one. We shall denote by putting - the quantities with respect to \((\Gamma, \overline{N})\).

If we put

\[ F^i_j_k = \Gamma^i_j_k - C^i_j_m F^m_k, \]

the components \( F^i_j \) of \( N \) are

\[ F^i_k = y^i F^i_k, \]

and differ by \( y^i F^i_j_k - F^i_k \) from \( F^i_k \). The quantities

\[ D^i_k = y^i F^i_k - F^i_k \]

are the components of the deflection tensor field \( D \) defined by

\[ D(f) = B^h(f) \gamma, \]

where \( \gamma \) is the characteristic field defined by

\[ \gamma : F(M) \rightarrow F(y, z) \rightarrow z^{-1} y = z^{-1} y^i y^i e_a. \]

Between the h-basic vector fields \( B^h(f) \) and \( \overline{B^h(f)} \) there exists the relation

\[ B^h(f) = \overline{B^h(f)} + B^v(D(f)), \]

therefore, as the dual relation, we have

\[ \theta^e = \overline{\theta^e} - D(\theta^b). \]

4. Given a Finsler metric function \( L \), the usual metric tensor field \( G \) is defined, its components \( g_{ij} \) being given by

\[ g_{ij} = \frac{1}{2} \frac{\partial^2 L^2}{\partial y^i \partial y^j}. \]

A Finsler space means here a differentiable manifold \( M \) endowed with such a metric tensor field \( G \).
We put

\[(18) \quad \tau_{ijk} = -\frac{1}{2} \left( \frac{\partial g_{ij}}{\partial x^k} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} \right), \]

\[(19) \quad g^i = \frac{1}{2} \tau^i_{jk} y^j y^k, \]

and

\[(20) \quad g^i_k = \frac{\partial g^i}{\partial y^k}, \]

where \(\tau^i_{jk} = g^{ih} \tau_{ijk} \).

And we shall sometimes use the notations

\[(21) \quad l^i = \frac{y^i}{L}, \quad l_j = g_{ij} l^i. \]

5. Let a Finsler connection \((\Gamma, N)\) be given in a Finsler space \((M, G)\). The conditions C1–C4 are expressed as follows:

\[(22) \quad \Gamma_{jkh} + \Gamma_{hjk} = \frac{\partial g_{jh}}{\partial x^k}, \]

\[(23) \quad C_{jkh} + C_{hjk} = \frac{\partial g_{jh}}{\partial y^k}, \]

\[(24) \quad F^i_k = y^i F^i_{jk}, \]

\[(25) \quad F_{jkh} = F_{khj}, \]

\[(26) \quad C_{jkh} = C_{khj}, \]

where \(\Gamma_{jkh} = g_{ih} \Gamma^i_{jk}, \ C_{jkh} = g_{ih} C^i_{jk}\) and \(F_{jkh} = g_{ih} F^i_{jk}\). We shall here explain some geometrical meanings of these conditions.

Let \(C\) be a differentiable curve in \(M\) and \(\bar{C}\) be a differentiable curve in \(T(M)\) mapped on the \(C\) by the projection \(\tau\). Tangent vectors \(X(t)\) along \(C\) are said to be parallel along \(C\) with respect to \(\bar{C}\), if the equations

\[(27) \quad \frac{dX^i}{dt} + \Gamma^i_{jkh}(x, y) \frac{dx^j}{dt} + C^i_{jkh}(x, y) \frac{dy^k}{dt} = 0 \]

are satisfied, where \(C\) is expressed by \(x'(t)\) and \(\bar{C}\) by \(x'(t), \ y'(t)\).

Under the parallel displacement along a curve \(C\), if we take in particular \(\bar{C}\) to be a lift \(\bar{C}_N\) with respect to the non-linear connection \(N\), i.e.

\[(28) \quad \frac{dy^i}{dt} + F^i_k(x, y) \frac{dx^k}{dt} = 0, \]
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the equations (27) may be written in the form

\[ \frac{dX^i}{dt} + F_j^i(x, y)X^j \frac{dx^k}{dt} = 0. \]

(29)

The supporting elements \( y^i \) (the points of the lift \( \tilde{C}_N \)) are parallel with respect to \( \tilde{C}_N \), i.e.

\[ \frac{dy^i}{dt} + F_j^i(x, y)y^j \frac{dx^k}{dt} = 0, \]

(30)

if and only if the equations (24) are satisfied, which is a geometrical meaning of the condition C2.

The connection \( \Gamma \) is called to be metrical if the length of a vector remains unchanged under the parallel displacement along any curve \( C \) with respect to any \( \tilde{C} \), which is a geometrical meaning of the condition C1. On the other hand, the non-linear connection \( N \) is called to be metrical if the supporting elements as the points of a lift \( \tilde{C}_N \) of any curve \( C \) have a constant length, that is, the (28) yields

\[ \frac{d}{dt}(g_{ij}(x, y)y^iy^j) = 0. \]

(31)

In the case that the \( \Gamma \) is metrical, the non-linear connection \( N \) is metrical if and only if

\[ g_{jk}y^jD^k_h = 0, \quad \text{or} \quad l_iD^i_k = 0. \]

(32)

This is easily verified by (22), (23), (28) and (13). Hence, if the condition C2 is satisfied, the non-linear connection \( N \) is metrical.

Let \( T(x) \) be the fibre \( \pi^{-1}x \) over a point \( x \in \mathcal{M} \) and \( F(x) \) be the Finsler subbundle \( \pi^{-1}T(x) \). If we denote by \( \Gamma^v \) the restriction of the distribution \( \Gamma \) to \( F(x) \), the \( \Gamma^v \) is regarded as a linear connection on the differentiable manifold \( T(x) \), whose components are \( C^v_{jk} \). Since the (v)\( v \)-torsion tensor field \( S^v \) is expressed by \( S^v_{jk} = C^v_{jh} - C^v_{hj} \), the condition C4 requires this connection \( \Gamma^v \) to be without-torsion.

If we restrict the metric tensor field \( G \) to \( T(x) \), then the \( T(x) \) becomes a Riemannian space. Thus, the connection satisfying (23) and (26) is the Riemannian connection, which is uniquely determined by the \( G \) as follows:

\[ C_{jkh} = \frac{1}{2} \frac{\partial g_{ih}}{\partial y^k}. \]

(33)

Therefore, \( C_{jkh} \) are symmetric and the relations

\[ C_{jkh} y^k = 0, \quad \text{or} \quad C_{jkh} l^k = 0 \]

(34)

hold good.

Now, since \( F_j^k = \Gamma_j^k - C^m_{jm}F^k_m \), the (h)\( h \)-torsion tensor field \( T \), which is expressed by \( T^i_{jk} = F^i_j - F^i_k \), depends not only on the \( \Gamma \) but on the \( N \). However, the conditions C1 and C4 do not depend on the \( N \). So, the condition C2 gives an influence upon the
§ 2. Determinations of Finsler connections by deflection tensor fields

6°. First, we shall consider the case that any non-linear connection is given in the tangent bundle of a Finsler space.

Proposition 1. Given a non-linear connection \( N \) in the tangent bundle of a Finsler space, there exists a unique Finsler connection \((\Gamma, N)\) satisfying the following four conditions:

(C1) the connection \( \Gamma \) be metrical,
(C2') the non-linear connection be the given \( N \),
(C3) the \((h)h\)-torsion tensor field \( T_0 = 0 \),
(C4) the \((v)v\)-torsion tensor field \( S_1 = 0 \).

The components \( \Gamma_{jkh} \) and \( C_{jkh} \) of the \( \Gamma \) are

\[
\Gamma_{jkh} = \gamma_{jkh} + \frac{1}{2} \left( \frac{\partial g_{ih}^{m}}{\partial y^{m} h} - \frac{\partial g_{hh}^{m}}{\partial y^{m} i} F_{j}^{m} \right),
\]

\[
C_{jkh} = \frac{1}{2} \frac{\partial g_{ij}^{h}}{\partial y^{h}},
\]

where \( F_{j}^{i} \) are the components of the given non-linear connection \( N \).

In this case \( F_{jkh} \) are

\[
F_{jkh} = \gamma_{jkh} - \frac{1}{2} \left( \frac{\partial g_{ij}^{m}}{\partial y^{m} h} - \frac{\partial g_{hh}^{m}}{\partial y^{m} i} F_{j}^{m} + \frac{\partial g_{ih}^{m}}{\partial y^{m} h} - \frac{\partial g_{hh}^{m}}{\partial y^{m} i} F_{j}^{m} \right),
\]

and if we put

\[
\frac{\delta}{\partial x^{k}} = \frac{\partial}{\partial x^{k}} - F_{j}^{m} \frac{\partial}{\partial y^{m}},
\]

then they are expressed by

\[
F_{jkh} = \frac{1}{2} \left( \frac{\partial g_{ij}^{k}}{\partial x^{k}} + \frac{\partial g_{hh}^{k}}{\partial x^{j}} - \frac{\partial g_{ih}^{k}}{\partial x^{k}} \right).
\]

Proof. (33) follows from (23) and (26) as remarked in 5°. If we put

\[
\Gamma_{jkh} = \gamma_{jkh} + \frac{1}{2} \left( \frac{\partial g_{ij}^{m}}{\partial y^{m} h} - \frac{\partial g_{hh}^{m}}{\partial y^{m} i} F_{j}^{m} \right) + A_{jkh},
\]

then we obtain by (22) and (18)

\[
A_{jkh} + A_{kjh} = 0,
\]
and by (11), (33) and (25)

\[ A_{jhk} = A_{khj}. \]

From these equations it follows that \( A_{jhk} = 0 \). Hence, (39) becomes (35), and (36) follows.

And the \( \Gamma \) defined by (35) and (33) satisfies with the \( N \) our conditions.

From (36) and (34), we have

\[ y^j F^i_j = y^j r^i_j - \frac{1}{2} g^{ih} \frac{\partial g_{hk}}{\partial y^m} F^m_i y^j. \]

We may solve \( F^i_j \) from (13) and (42), and obtain

\[ F^i_j = G^i_j + C^i_j D^i_k y^k - D^i_k. \]

Substituting (43) into (35), we have

**Proposition 2.** Given a Finsler tensor field \( D \) of type \((1, 1)\) in a Finsler space, there exists a unique Finsler connection \((\Gamma, N)\) satisfying the following four conditions:

\[(C1)\] the connection \( \Gamma \) be metrical,

\[(C2')\] the deflection tensor field be the given \( D \),

\[(C3)\] the \((h)h\)-torsion tensor field \( T = 0 \),

\[(C4)\] the \((v)v\)-torsion tensor field \( S^1 = 0 \).

The components \( \Gamma_{jhk}, C_{jhk} \) and \( F^i_j \) of the \((\Gamma, N)\) are

\[
\Gamma_{jhk} = \gamma_{jhk} + \frac{1}{2} \left( \frac{\partial g_{ij}}{\partial y^m} G^m_j - \frac{\partial g_{jk}}{\partial y^m} G^m_i \right) \\
+ C_{jkm} C^i_j D^i_k y^k - C_{hkm} C^m_j D^m_i y^i - C_{jkm} D^m_i + C_{hkm} D^m_i,
\]

\[(33)\] 

\[ C_{jhk} = \frac{1}{2} \frac{\partial g_{ij}}{\partial y^k}, \]

and

\[(43)\] 

\[ F^i_j = G^i_j + C^i_j D^i_k y^k - D^i_k, \]

where \( D^i_k \) are the components of the given Finsler tensor field \( D \).

7°. Proposition 2 shows that the connection \( \Gamma \) determined in Proposition 1 or 2 is the one defined by E. Cartan if and only if

\[(45)\] 

\[ C_{jkm} C^m_i D^i_j y^j - C_{hkm} C^m_i D^m_j y^j - C_{jkm} D^m_i + C_{hkm} D^m_i = 0. \]

It is easily verified by (34) that (45) is equivalent to
Thus we have

Theorem A. Given a Finsler tensor field $D$ of type $(1, 1)$ in the Finsler bundle of a Finsler space, there exists a unique Finsler connection $(\Gamma, N)$ satisfying the following four conditions:

(C1) the connection $\Gamma$ be metrical,
(C2') the deflection tensor field be the given $D$,
(C3) the $(h)h$-torsion tensor field $T = 0$,
(C4) the $(v)v$-torsion tensor field $S^v = 0$.

And, a necessary and sufficient condition that the $\Gamma$ thus determined be the one defined by E. Cartan is that the deflection tensor field $D$ satisfies the condition

\[ C(f_1, D(f_2)) = C(f_2, D(f_1)), \]

where $C$ is the $(h)v$-torsion tensor field of the $(\Gamma, N)$, or equivalently that the components $D_k^j$ of the deflection tensor field $D$ satisfy the conditions

\[ \frac{\partial g_{i h}}{\partial y^m} D_k^m = \frac{\partial g_{h k}}{\partial y^m} D_j^m. \]

In this case the conditions

\[ \frac{\partial g_{i h}}{\partial y^m} D_k^m y^v = 0 \]

hold good, and the components $\Gamma_{jkh}, C_{jkh}$ and $F^i_k$ of the $(\Gamma, N)$ are

\[ \Gamma_{jkh} = \gamma_{jkh} + \frac{1}{2} \left( \frac{\partial g_{i h}}{\partial y^m} G_m^k - \frac{\partial g_{h k}}{\partial y^m} G_j^m \right), \]

\[ C_{jkh} = \frac{1}{2} \frac{\partial g_{i h}}{\partial y^k}, \]

and

\[ F^i_k = G^i_k - D_k^i. \]

8. As a special example of the $D$ satisfying the condition (48), we have

Proposition 3. In a Finsler space there exists a unique Finsler connection $(\Gamma, N)$ satisfying the following four conditions:

(C1) the connection $\Gamma$ be metrical,
(C2''') the deflection tensor field $D$ be given by

\[ D^i_k = \lambda l^i l_k + \mu \delta^i_k, \]
where $\lambda$ and $\mu$ are scalar functions on the tangent bundle,

(C3) the $(h)$-torsion tensor field $T=0$,

(C4) the $(v)$-torsion tensor field $S^1=0$.

The connection $\Gamma$ is the one defined by E. Cartan. And, the non-linear connection $N$ is metrical if and only if $\lambda + \mu = 0$.

This is easily proved by (34) and (32). Thus, we have noticed that, in order to determine the connection $\Gamma$ defined by E. Cartan, the condition (C2) may be replaced by the weaker condition (C2'). If we take $D$ in (C2'') such that

\[ D = \lambda (l^i l_k - \delta^i_k), \]

then the non-linear connection $N$ is metrical, and so we have a generalization of the $(\Gamma, N)$ defined by E. Cartan.

However, in order to obtain the $\Gamma$ only, it does not need the non-linear connection to be metrical. In particular, if $\lambda = 0$, $\mu = -1$ (i.e. $D = -\delta$) then the components $F^i_1$ of the non-linear connection $N$ become $F^i_1 = G^i_1 + \delta^i_1$, which are somewhat canonical in features. So, it seems to be interesting that, apart from Finsler metrics, we treat Finsler connections with the deflection tensor field $D = -\delta$. Next, we shall give an example of such a Finsler connection.

§ 3. Finsler connections derived from affine connections

9°. Let $F(M)$ be the affine bundle over $M$, where $\tilde{G} = GL(n, \mathbb{R}) \times F$ is the affine group with the multiplication

\[ (g_1, v_1) (g_2, v_2) = (g_1 g_2, g_1 v_2 + v_1). \]

Each $(g, v) \in \tilde{G}$ acts on $F(M)$ by

\[ T_{(g, v)} : F(M) \to F(M) \mid (y, z) \mapsto (y + vz, zg), \]

so we have the restrictions

\[ T_g : F(M) \to F(M) \mid (y, z) \mapsto (y, zg) \]

and

\[ S_v : F(M) \to F(M) \mid (y, z) \mapsto (y + vz, z). \]

Therefore, a connection in the affine bundle is invariant not only by $T_g$ but by $S_v$.

The Lie algebra of the structural group $\tilde{G}$ is $L(n, \mathbb{R}) + F$, if we identify the Lie algebra of the additive group $F$ with $F$ itself. If we denote by $Z(A)$ and $Y(f)$ the respective fundamental vector fields corresponding to $A \in L(n, \mathbb{R}) + 0$ and $f \in 0 + F$, then $Z(A)$ is also the fundamental vector field in the Finsler bundle $F(M)$, and $Y(f)$ is the induced fundamental vector field.
The induced vertical distribution $F^i$ defined by

$$F(M) \ni u \to \{X \in F(M)_u | \pi_2 X = 0\}$$

is spanned by $Y(f)$, where $F(M)_u$ is the tangent space at $u \in F(M)$.

10°. Let $\tilde{F}$ be a connection in the affine bundle $F(M)$. Then, a Finsler connection $(\Gamma, N)$ is obtained by pairing $\tilde{F}$ with the induced vertical distribution $F^i$. In this case the $v$-basic vector field $B^v(f)$ is $Y(f)$.

Since the $\tilde{F}$ is $S^r$-invariant, the $h$-basic vector field $B^h(f)$ is $S^r$-invariant. Therefore, the subordinate $F$-connection to $(\Gamma, N)$ is a linear connection and the deflection tensor field $D$ of $(\Gamma, N)$ is $S^r$-invariant.

Now, we shall treat the connection forms.

**Proposition 4.** Let $\omega$ and $\omega'$ be the connection forms of $\tilde{F}$ and $F$ respectively. If we consider the form $\omega + \theta^v$ to take values in the Lie algebra $L(n, R) + F$, then

$$\tilde{\omega} = \omega + \theta^v.$$  

**Proof.** Since $(\theta^h, \theta^v, \omega)$ constitutes the dual system of $(B^h(f), Y(f), Z(A))$, we have

$$\tilde{\omega} = \omega + \theta^v.$$  

These relations show that $\omega + \theta^v$ is just the connection form $\tilde{\omega}$ of the $\tilde{F}$. Because, with respect to the connection in the affine bundle $F(M)$ over $M$, the horizontal subspace is spanned by $B^h(f)$ and the vertical subspace by the fundamental vector fields $Z(A)$ and $Y(f)$.

**Proposition 5.** Let $\omega$ be the connection form of $\tilde{F}$, and $\omega'$ be the connection form of the subordinate linear connection to $(\Gamma, N)$. If $\varepsilon$ is the injection

$$\varepsilon: L(M) \to F(M) | z \to (0, z),$$

then

$$\varepsilon^* \tilde{\omega} = \omega - D(\theta).$$

The proof will be obtained from (55), (16), (12) and (10). A connection $F$ in the affine bundle is canonical, if the $\varepsilon^* \tilde{\omega}$ has the form

$$\varepsilon^* \tilde{\omega} = \omega + \theta,$$

and is called the affine connection $[3]$. The formula (58) shows that the connection $\tilde{F}$ is affine if and only if

$$D = -\theta.$$

Thus we have
Theorem B. Let $\tilde{F}$ be a connection in the affine bundle $F(M)$ over $M$. Then, a Finsler connection $(\tilde{T}, N)$ of $M$ may be defined by the Finsler pair $(\tilde{T}, F')$, where $F'$ is the induced vertical distribution. Its subordinate F-connection becomes a linear connection and its deflection tensor field $D$ is $S_v$-invariant. In particular, the connection $\tilde{F}$ is an affine connection of $M$ if and only if $D=-\delta$.

References