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著者	Morita Kazuo, Shiga Thoru, Taharazako Shoji
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Optical Grading of Satsuma Mandarin Using Linear Discriminant Function

Kazuo MORITA, Thoru SHIGA* and Shoji TAHARAZAKO
(*Laboratory of Agricultural Systems Engineering*)
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Introduction

In order to improve the grading process of agricultural products in the packinghouse line, a lot of attempts have been carried out to develop a simple, rapid and non-destructive method for quality evaluation[2,5,6]. This study was executed to obtain some basic data for quality evaluation of Satsuma mandarin using light reflectance technique and to establish automatic grading system in the packinghouse.

Authors reported[1] previously that the surface color of Satsuma mandarin was found to be related with the peel chlorophyll contents, and that spectral reflectance at the wavelength of 680 nm was highly correlated with it. In previous paper[3], the spectral reflectance properties of Satsuma mandarin with typical kind of defects were measured, and the spectral reflectance at the wavelengths of 740 and 520 nm was shown to be most suitable for detecting of the defects. Moreover, in order to apply the spectral reflectance method for the automatic grading system, an experimental equipment for easy practical application was designed and the grading accuracy was determined[4].

The purpose of this study is to analyse statistically the light reflectance data related to surface color and defect of Satsuma mandarin, and to apply the theory of discriminant function[7–9] for automatic grading system.

Theory of discriminant function

1. Linear discriminant function

Let's assume that there are two populations with the same kind of variable, P. In fixing a judgement to which population an unknown individual with variable, P belongs, a linear discriminant function is one of the most suitable methods in the statistical analysis. When an idividual is classified in the population A_1 or A_2, there may occur two kinds of error. That is to mis-classify the individual in each population, and these mis-classifications are equally important.

There is a P dimensional space and it is assumed that an individual in a specimen exists as one point in the space. It is considered that the population is a gathering point around central point. When f_1 dx and f_2 dx are elements of probability density in each population A_1 and A_2, it is realized in following equation (1):

$$\int_R f_2dx = \int_{1-R} f_1dx = 1 - \int_R f_1dx \qquad \cdots\cdots(1)$$

* Faculty of Agriculture, Utsunomiya University, Utsunomiya 311

Assuming that there is a sphere, R in the space, the boundary condition of equation (1) is equal to the following equation (2):

$$\int_R (f_1+f_2)dx=1 \qquad \cdots\cdots(2)$$

The final goal is to minimize $\int_R f_2 dx$ under the above condition. It is equal to obtain minimum value of $\int_R \{f_2-\rho(f_1+f_2)\}dx$. It can also be re-written $\int_R \psi(f_2-f_1)dx$. The constant value is calculated by equation (2) and it is assumed that any point that $\psi(f_2-f_1)$ is minus value belongs to sphere, R. Then, the boundary of sphere, R is given by the following equation (3):

$$f_1/f_2=\psi \qquad \cdots\cdots(3)$$

According to the concept, the discriminant boundary is determined by a ratio of probability. An individual with the boundary condition of $f_1/f_2>\psi$, belongs to f_1 and when the boundary condition is $f_1/f_2<\psi$, the individual belongs to f_2. Then, the probability of mis-classification comes to be minimized and be equal in the both cases. The minimum error is shown by the following equation (4):

$$\int_{f1/f2>\psi} f_2 dx=\int_{f2/f1>\psi} f_1 dx \qquad \cdots\cdots(4)$$

There are two populations $u_1(i)$, $u_2(i)$ with variable of normal distribution. The averages of each element are not equal but the variances are equal. Then, the element X(j) is a discriminant factor and is combined to the linear function (5):

$$X=\sum_{j=1}^{P} S(j)\cdot x(j) \qquad \cdots\cdots(5)$$

Two populations are to be classified by the linear discriminant function, X. To decrease the mis-classification, it is to be desirable that central line of distribution in one population is as far away from the other center line in other population as possible and the variance had better be small. Therefore, the difference of expected value in the two populations is squared and is divided by the variance of the linear discriminant function, X. When its value is maximized, S(j) is determined. This relation is shown by the following equation (6):

$$F=\frac{\{\sum_j S(j)\cdot(u_1(j)-u_2(j))\}^2}{\sum_{i,j} S(i)\cdot S(j)\cdot a(i,j)} \qquad \cdots\cdots(6)$$

Differentiate the equation (6) by S(j), and the derivative is equal to zero. Then, equations (7) and (8) are obtained.

$$u_1(j)-u_2(j)=\frac{\{\sum_j S(j)\cdot(u_1(j)-u_2(j))\}\cdot\sum_i S(i)\cdot a(i,j)}{2\cdot\sum_{i,j} a(i,j)\cdot S(i)\cdot S(j)} \qquad \cdots\cdots(7)$$

$$S(j)=\sum_j a(i,j)^{-1}\cdot(u_1(j)-u_2(j)) \qquad \cdots\cdots(8)$$

Where, a(i, j) is co-variance matrix and $a(i,j)^{-1}$ is inversive matrix of a(i, j). When the average specimen in each population are $x_1(j)$ and $x_2(j)$, the assumptive value is shown by the following equation (9):

$$S(j)=\sum_{j} a(i,j)^{-1}\cdot\{x_1(j)-x_2(j)\} \qquad \cdots\cdots(9)$$

The aim of the linear discriminant function, X is to classify the population and the statistical distance between two populations is not measured. Equation (9) divided by S(1) is re-written by the equation (10):

$$X=\frac{1}{S(1)}\cdot\sum_{j=1}^{P} S(j)\cdot x(j) \qquad \cdots\cdots(10)$$

2. Standard value of discrimination

Standard values of discrimination of the two populations is calculated by the linear discriminant function. Fig. 1 shows the basic concept of statistical classification by the linear discriminant function. The linear discriminant functions X_1 and X_2 in each population A_1 and A_2 are obtained by substituting each average value of elements for the equation (10). The standard value of discrimination is shown by the euqation (11):

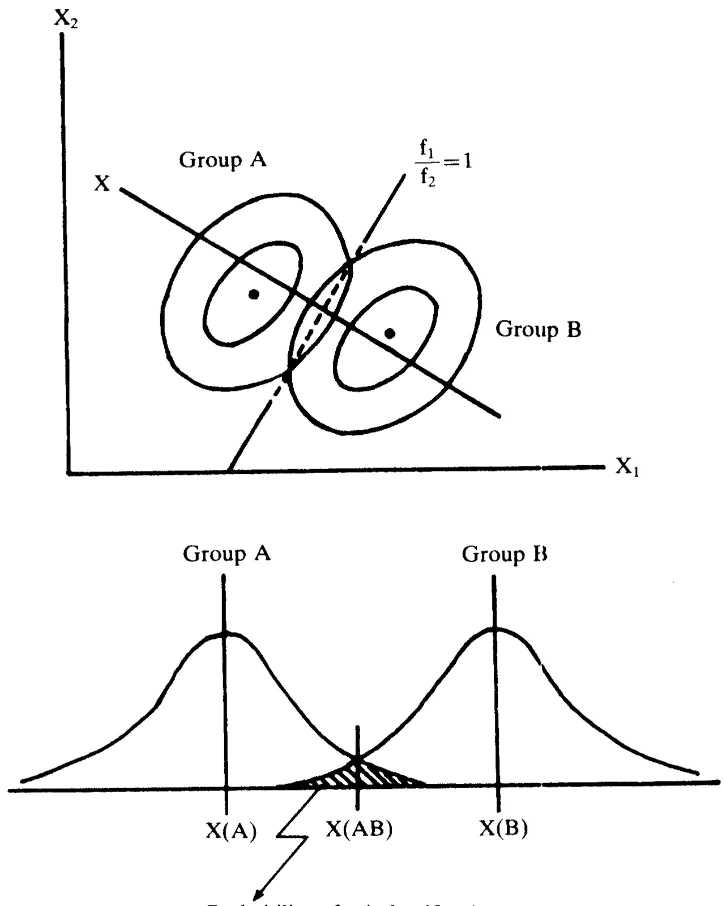

Fig. 1. Statistical classification by linear discriminant function.

$$X_s=1/2\cdot(X_1+X_2) \qquad \cdots\cdots(11)$$

When the value of an individual which is calculated by the linear discriminant function, X is bigger than the standard value of discrimination X_s, the individual is classified the population A_1

and when it is smaller than X_s, the individual is classified the population A_2. The variance value calculated by the linear discriminant function X is shwn by the equation (12) and (13):

$$\begin{aligned} Var(X) &= \sum_{i,j} S(i)\cdot S(j)\cdot cov(x(i), x(j)) \\ &= \sum_{i,j} S(i)\cdot S(j)\cdot a(i, j) \\ &= \sum_{i,j} S(i)\cdot a(i, j)\cdot \sum_{k} a(j, k)\cdot \{x_1(k)-x_2(k)\} \\ &= \sum_{i} S(i)\cdot \{x_1(i)-x_2(i)\} \\ &= X_1 - X_2 \qquad \cdots\cdots(12) \end{aligned}$$

$$Var(X) = (X_1 - X_2)/(\rho\cdot S(1)) \qquad \cdots\cdots(13)$$

The variance could be assumed from remainder among average values in the equation (13). The ρ in equation (13) is an estimated value and the probability of mis-classification by the standard value of discrimination is shown by the area of diagonal line in Fig. 1. By comparing Y value in the equation (14) with a table of normal distribution, it comes to be easy to find the probability of mis-classification.

$$Y = \frac{X_1 - X_2}{SQR\{Var(X)\}} \qquad \cdots\cdots(14)$$

The standard value of discrimination and probability of mis-classification are discussed in case of the two populations, and the statistical classification by the linear discriminant function is established.

In agricultural products, there are several grades for the quality and Satsuma mandarin are sorted out into four grades for the quality. As the theory of linear discriminant function is applied for more than two populations, the statistical classification of four populations such as the grade of Satsuma mandarin could be carried out by the same method.

Materials and Methods

1. Materials

Satsuma mandarin (variety of citrus Unshu : Miyagawa) were used in this experiment and fruits were visually sorted out four grades by trained people in the packinghouse. The four grades of Satsuma mandarin are classified into 1st, 2nd, 3rd and spoiled grade, and one hundred samples were randomly selected in each grade.

2. Measuring of light relfectance properties

The experimental apparatus and measuring of light reflectance properties were shown in the previous paper[4]. The interference filters (KL-520, 540, 720 and 740) were adopted for detecting surface defect of Satsuma mandarin and the interference filter (KL-680) was adopted for detecting surface color. The detecting directions and combination of interference filters were shown in Fig. 2.

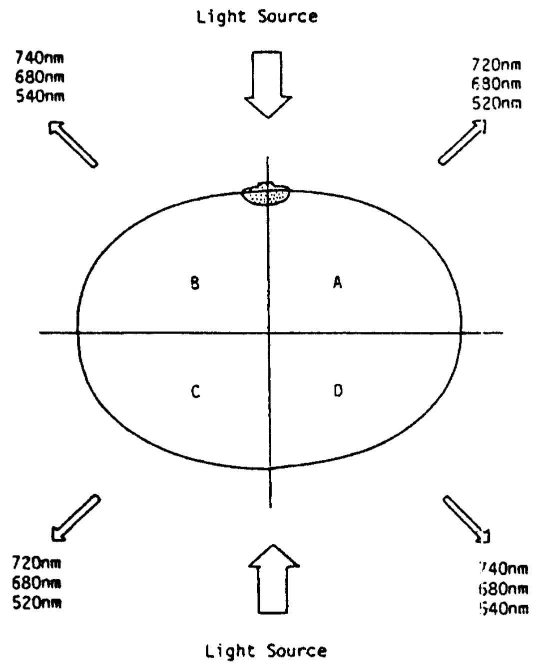

Fig. 2. Detecting direction of each side on the fruit surface of Satsuma mandarin.

3. Evaluation of the surface color

The surface color of Satsuma mandarin was observed at four directions by differential colorimenter (Nippon Denshoku, ND-544AA). The surface color was measured at UCS value consisting of Hunter (L, a, b) and represented at Lightness (L), Chroma($\sqrt{a^2+b^2}$) and Hue ($\tan^{-1}a/b$).

4. Optical grading using linear discriminant function

The light reflectance of Satsuma mandarin was measured with three filters at four directions and collected data was analyzed using linear discriminant function. Fig. 3a shows the flow-chart of a computer analyzing system. Twelve elements for the discrimination were represnted in x(i, j), and the average, the co-variance matrix and the inversive matrix were calculated. The linear discriminant function was determined by the equations (9) and (10). The standard value of discrimination between 1st and 2nd grades of Satsuma mandarin was computed using the equation (11). The standard values of discrimination between 2nd and 3rd grades, and between 3rd and sopiled grades were computed by the same method. By comparing each standard values of discrimination with value of each sample computed by the linear discriminant function, all fruits were re-graded into four grades. Fig. 3b shows the flow-chart of the discriminant program for optical grading of Satsuma mandarin.

Results and Discussion

1. Visual grading vs. surface color

The Lightness, Chroma and Hue values for 1st, 2nd, 3rd and spoiled grades of Satsuma mandarin were investigated. Means and standard deviations of the measured size of fruits were shown in Table 1. All data except fruit weight are not significant at 5% level among each grade.

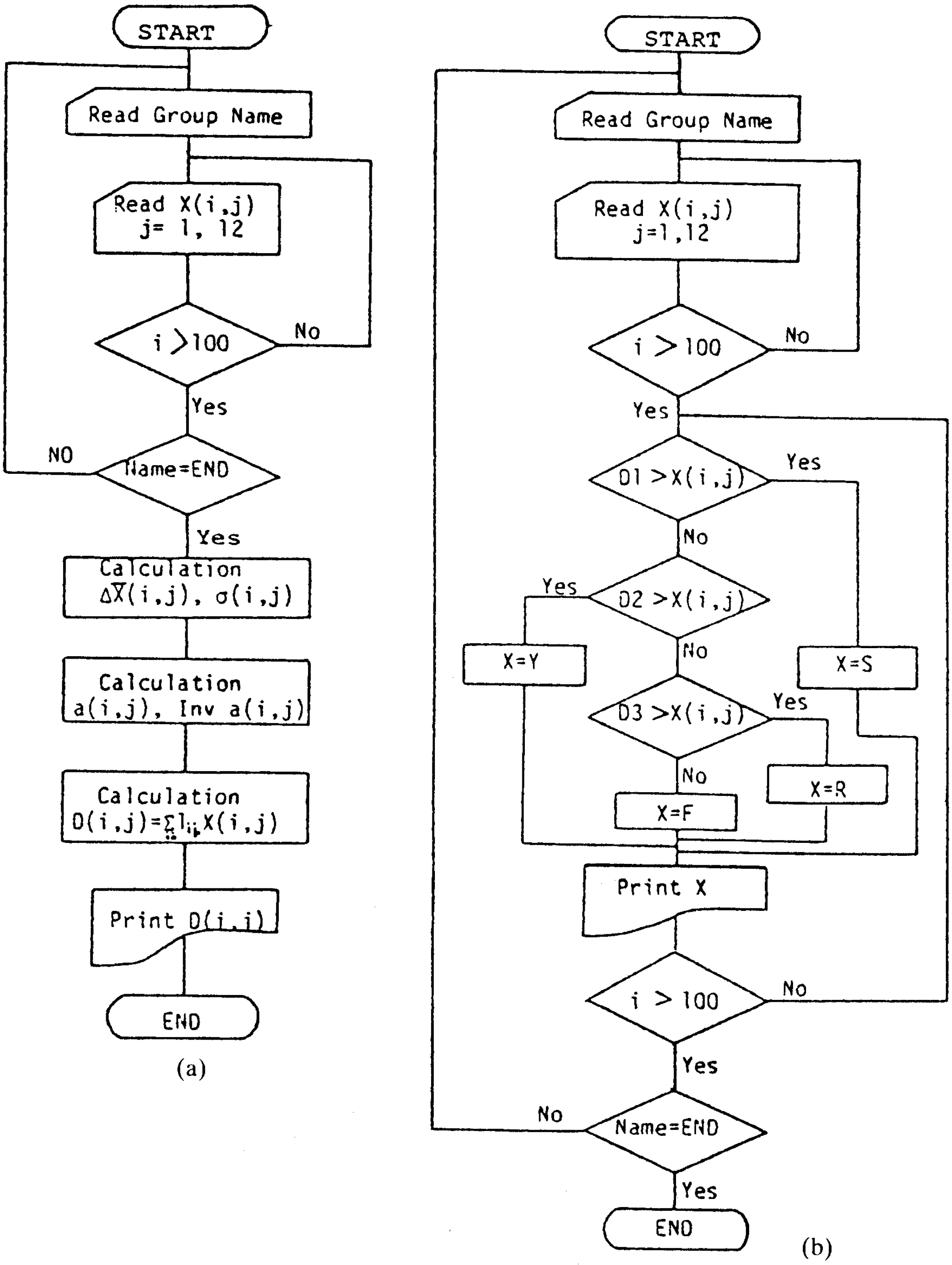

Fig. 3. Flowchart of the discrimination program for optical grading of Satsuma mandarin.

As the light reflectance is not affected by fruit weight, fruits among each grade were considered to be same in size and shape. Fig. 4, 5 and 6 show the changes of Lightness, Chroma and Hue values for one hundred fruits in each grade with average values and standard deviations. Lightness at stem end was not different among each grade but Lightness of the spoiled grade was higher than the other grades at blossom end (Fig. 4). This was the reason why the surface of fruits in spoiled grade was covered with scabs, especially at blossom end. Chroma value decreased with the grade of fruits except the spoiled grade. Chroma value in the spoiled grade was higher than that in 3rd grade but its variance was larger than that. Hue value tends to be different between blossom end and stem end (Fig. 6). In case of Satsuma mandarin, as surface color at the blossom end changes from green to yellow faster than that at stem end, Hue value at the blossom end was higher than

Table 1. Means and standard deviations of measured size of Satsuma mandarin

		1st grade	2nd grade	3rd grade	Spoiled
M	DIA. (mm)	63.34	63.38	64.35	64.45
E	HEIGHT (mm)	49.70	50.44	50.75	51.78
A	H/D	0.789	0.796	0.789	0.805
N	WAIGHT (g)	103.5	106.1	105.7	112.9
S	DIA. (mm)	2.06	2.09	2.06	2.50
.	HEIGHT (mm)	2.69	3.11	3.31	3.21
D	H/D	0.060	0.049	0.051	0.047
	WAIGHT (g)	8.32	9.66	9.00	12.82

1st grade means visually classified "Shu".
2nd grade means visually classified "Yu".
3rd grade means visually classified "Ryo".
Spoiled means visually classified "Kakugai".

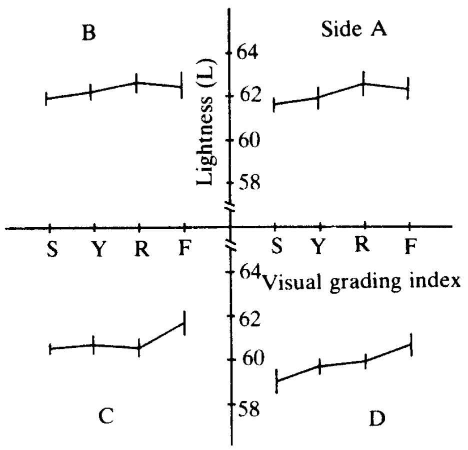

Fig. 4. Relationship between visual grading index and Lightness value of Satsuma mandarin.
(S: 1st, Y: 2nd, R: 3rd, F: Spoiled grade)

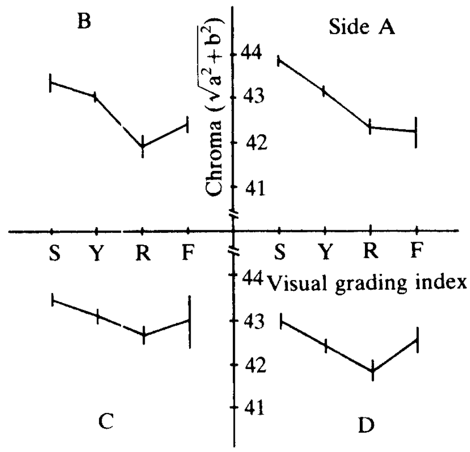

Fig. 5. Relationship between visual grading index and Choroma value of satsuma mandarin.

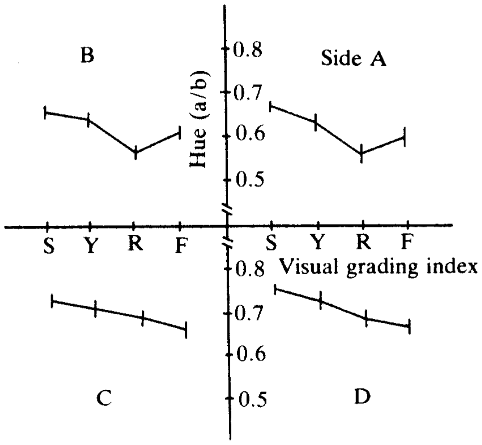

Fig. 6. Relationship between visual grading index and Hue value of Satsuma mandarin.

that at stem end. Also, Hue value decreased with the grade of fruits except the spoiled grade. This is the reason why fruits in spoiled grade have some defects on the surface.

Consequently, the results indicated that fruits among 1st, 2nd, 3rd grades were mainly judged by the change of Hue and Chroma values. In the spoiled grade, although Lightness, Chroma and Hue changed with the surface color, the change of those was irregular under the influence of defects on the surface.

2. Visual grading vs. light reflectance

Fig. 7 shows relationship between visual grading index of Satsuma mandarin and light reflectance (680 nm) for detecting surface color. The light reflectance (680 nm) decreased with visual grade as similar as the change of Hue value. Although it was difficult to make a separation between 1st and 2nd grades in the change of Hue value, the change of light reflectance (680 nm) between 1st and 2nd grades had significant at 1% level.

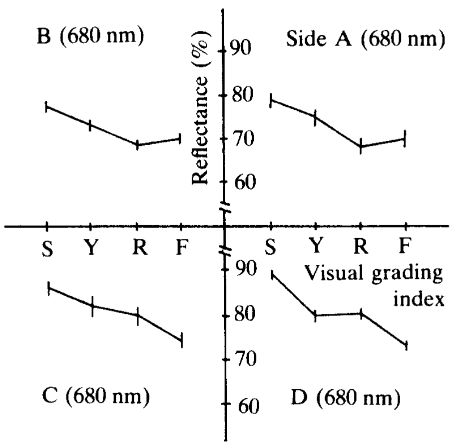

Fig. 7. Relationship between visual grading index and light reflectance (680 nm) of Satsuma mandrin.

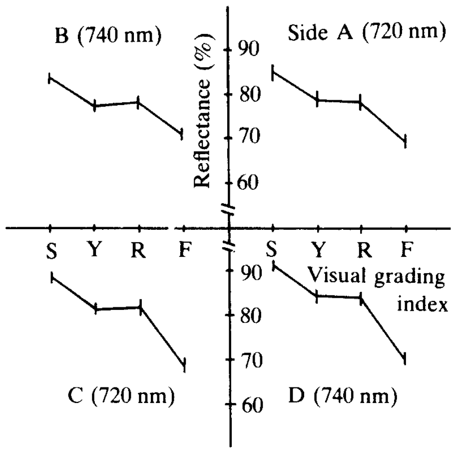

Fig. 8. Relationship between visual grading index and light reflectance (720, 740 nm) of Satsuma mandarin.

Fig. 8 shows relationship between visual grading index and light reflectance (720 and 740 nm) for detecting defect on the surface. The light reflectance in the spoiled grade was remarkably lower than light relfectance in the other grades. It was considered to be valid for making a separation between the spoiled grade and the other grades. Essentially, although the light reflectance (720 and 740 nm) is mainly influenced by defect on the surface, there were a little different among the light reflectances in the other grades with defect. However, as the light reflectance was slightly related to Chroma value of the surface color, it is assumed that the reflectance decreases in accordance with visual grading index.

Fig. 9 shows relationship between visual grading index and light reflectance (520 and 540 nm) for detecting defect on the surface. The light reflectance in the spoiled grade was slightly higher than that in the other grades. By comparing reflectance (520 nm) with reflectance (540 nm), it was clarified that the wavelength of 520 nm was more sensitive than the wavelength of 540nm.

Consequetly, it became possible to trace the color change on the surface using the light reflectance (680 nm) and to detect the defect on the surface using the light reflectance (740 and 520 nm). Three kinds of wavelengths are considered to be available for the classficiation of Satsuma mandarin.

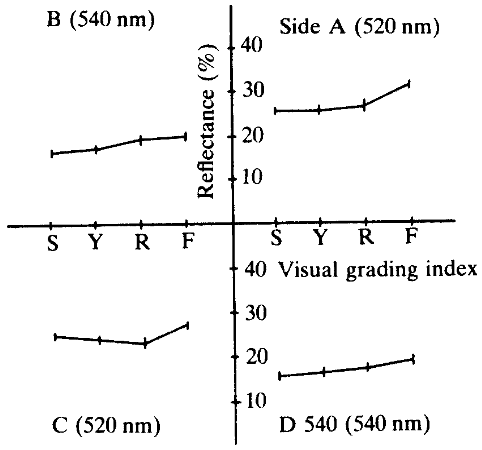

Fig. 9. Relationship between visual grading index and light reflectance (520, 540 nm) of Satsuma mandrin.

3. Classification by linear discriminant function

The light reflectances at the three wavelengths were measured at the four directions and these twelve grading values from a fruit were applied to calculate the linear discriminant function for each visual grade. After that all samples were re-graded by standard value of discrimination. In linear discriminant function, the varience of each element has to be euqal in each grade. As the variences were significant at 5% level in F-test, the variences were calculated from the gathered variables in two groups. The linear discriminant function using twelve elements from R_1 to R_{12} was calculated by a micro-computer (TEAC PS-80). Coefficients of linear discriminant function, X in each grade were shown in Table 2. Then, the probability of mis-classification was calculated using the equation (14) and was also shown in Table 2. All fruits were re-graded using this function and these results were shown in Table 3. Although only nine fruits were re-graded in the grade which skipped the next grade, fifty-six fruits were re-graded in the neighbor grade. However, visual grading standard is questionable and the judgement is not always consistent and

Table 2. Coefficients of linear discriminant function

S (i)	Wavelength (nm)	Direction	X (1st-2nd)	X (2nd-3rd)	X (3rd-spolled)
1	680	A	1	1	1
2	680	B	−5.349	2.191	1.310
3	680	C	0.096	−0.622	−2.218
4	680	D	−18.209	−1.070	−3.149
5	720	A	1.972	−1.645	−1.100
6	740	B	15.106	−3.315	−0.398
7	720	C	2.011	0.927	2.051
8	740	D	16.041	0.048	5.773
9	520	A	−1.021	0.223	−2.183
10	540	B	−21.018	2.396	2.303
11	520	C	8.944	−0.954	−4.419
12	540	D	−10.330	1.320	0.658
Xs			2538.6	−233.9	379.6
P.M.			(17%)	(21%)	(14%)

P.M.: Probability of misclassification.

Table 3. Classification of Satsuma mandarin by light reflectance method

Optical grade		Visual grade: Marketable 1st	Marketable 2nd	Marketable 3rd	Spoiled	TOTAL
Marketable	1st	87	10	2	0	99
	2nd	12	79	12	3	106
	3rd	1	8	78	6	93
Spoiled		0	3	8	91	102
TOTAL		100	100	100	100	400

accurate. The light reflectance (680 nm) had a high correlation with chlorophyll content of peel and the chlorophyll content was correlated with internal quality of fruits. The light reflectance (740 and 520 nm) was valid for detecting defect on the surface. It was concluded that the re-grading of Satsuma mandarin using the linear discriminant function was more advantageous in judging errors or scatterings in a grade than visual grading. Moreover, these probabilities of mis-classification were not absolute ones. With the consideration of the internal quality of fruits, the classification by the statistical method might be better than the visual grading. As the standard value of discrimination was calculated by the computer, it was rather easy to make improvement fit for various kinds of fruit. The optical grading with the linear discriminant function was considered to be accomplishable in the automatic grading system.

Summary

Surface color and defect are the major factors in the present visual grading of fruits and those are to be estimated on the status of appearance. The theory of discriminant function is applied to calculate the value of statistical grading index. According to the properties of spectral reflectance at the wavelengths of 680, 520 and 740 nm for detecting surface color and defect, Satsuma mandarin are attempted to be re-grade into four groups that have already been graded by human beings in the packinghouse.

The UCS values (Lightness, Chroma and Hue) for the fruit surface are measured on both the blossom and stem ends, and visual gradng of acceptable group is significant at the values of Chroma and Hue for each grade, but the spoiled grade is not significant at all the UCS values. Therefore, these three grades are noted to be capable of being graded by Hue and Chroma values but it is impossible for the spoiled grade to be sorted out from the other grades by the UCS values.

The light reflectance at the wavelength of 680 nm is measured for detecting surface color and the light reflectance at 520 and 740 nm is measured for detecting surface defects at the both ends. These twelve grading values from a fruit are applied to calculate the linear discriminant function for mechanical grading. After re-grading with this method, the measuring and grading errors of all factors (color and defect indices) of each grade become smaller as compared with the results that have been obtained from the visual grading at the packinghouse. This shows that this method can be used for autmoatic grading.

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References

1) Chuma, Y., Shiga, T. and Morita, K.: Evaluation of surface of Japanese persimmon fruits by light reflectance. J. Soc. Agr. Machinery Japan, **42**(1), 115–120 (1980) (in Japanese)
2) Meyers J. B. Jr., Prussia S. E., Thai C. N., Sadosky T. L. and Campbell D. T.: Visual inspection of agricultural products moving along sorting conveyors. Trans. ASAE 33(2), 367–372 (1990)
3) Morita, K., Shiga, T. and Taharazako, S.: Light reflectance properties of defects of Satsuma mandarin. Mem. Fac. Agr. Kagoshima Univ., **26**, 77–85 (1990)
4) Morita, K., Shiga, T. and Taharazako, S.: Development of automatic sorting system of Satsuma mandarin by Light reflectance. Mem. Fac. Agr. Kagoshima Univ., **27**, 63–77 (1991)
5) Morita, K., Shiga, T. and Taharazako, S.: Evaluation of change in quality of ripening banana using light relfectance technique. Mem. Fac. Agr. Kagoshima Univ., **28**, 125–134 (1992)
6) O'Brien M., Prussia S. E. and Smith N. E.: Man-machine factors when computerinzing tomato gradng. Trans. ASAE, **26**(2), 997–1002 (1983)
7) Sugiyama K.: Tahenryo-data-kaiseki nyumon. Asakura, Tokyo, 57-97 (1983) (in Japanese)
8) Suzuki Y., Miyake A. : Hanbetsu Bunseki. Gendai sugakusha, Tokyo, 35–48 (1979) (in Japanese)
9) Ura S., Takenami T.: Tahenryo-kaiseki no kiso. Saiensusha, Tokyo, 121–142 (1972) (in Japanese)