Critical Note on the Surface Resistance Coefficient in the Atmospheric Flow over the Ocean

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1. Introduction

It is a well known fact that the wind over the sea is comparatively stronger than over land, as explained by the smallness of the frictional drag over the sea. Moreover this fact has a familiar synoptic significance that the inclination angle (α) of the wind to the isobars over the sea is smaller than that over land.

Assuming $F = -kV$ for the frictional force (k, coefficient of friction; V, wind vector), Guldberg and Mohn[1] obtained a simple relation of $\tan\alpha = k/f$ ($f = 2\omega\sin\varphi$: Coriolis parameter) under the balanced three forces, i.e. pressure gradient, deflecting, and frictional. Taking the inferred value of $k = 0.00002$ over the sea and that of $k = 0.00012$ over land, they derived the synoptically reasonable value of $\alpha = 10°58'$ and $\alpha = 49°19'$ respectively (at $\varphi = 45°$). However this simple assumption was besides the mark and a new treatment by eddy theory had to be introduced.

The internal friction due to turbulence is expressed by $K\rho\frac{\partial^2 V}{\partial z^2}$, where K is eddy diffusivity assumed to be constant in the frictional layer, and ρ is density of air. Slipping velocity (V_s) upon the ground or sea surface is connected with the surface stress (τ_0) by the equation $\tau_0 = \kappa\rho V_s^2$, where κ is the coefficient of surface resistance or skin friction. Using $\kappa = \frac{2KBG \sin\alpha}{V_s^2}$ ($B = \frac{\omega \sin\varphi}{K}$), derived from the bottom condition of $K\frac{\partial V}{\partial z}\Big|_{z=0} = \kappa \left| V_s \right| V_s$, G. I. Taylor[2] calculated the value of κ from Dobson's pilot balloon observations over Salisbury Plane, and obtained $\kappa = 0.0023$, 0.0032, 0.0022 for light, moderate, and strong wind respectively.

D. Brunt[3] proposed the most probable value of κ to be 0.004 from Stanton's experiment of pipe flow. He noticed that Taylor's value of κ did not show systematic variation with wind speed, and that the values of κ were comparable in the above-mentioned two cases in spite of the ratio of the Reynold's numbers which was more than 10^5. And he stated that the same law appeared to hold over very wide range. Since then, the idea that the coefficient of skin friction is independent of surface states has been prevailed implicitely among meteorologists.

However, this assumption of independency is very doubtful. We can show this point by an example. Shaw's working rule,[4] i. e. "the ratio of surface wind to the corresponding geostrophic wind (V_s/G) is about one-third over land and is about two-third over the sea", is generally acceptable. And we have

$\alpha \fallingdotseq 32°$ for land and $\alpha \fallingdotseq 17°$ for the sea, by using the well known Taylor's relation of $V_s/G = \cos\alpha - \sin\alpha$. By the aid of the equation of $\kappa = \frac{2KBG \sin\alpha}{V_s^2}$, then, we get $K = 1.7 \times 10^5$ for the sea and $K = 3.4 \times 10^3$ for land (at $\varphi = 45°$ and when $G = 10$ m/sec). This conclusion, which shows that the value of eddy diffusivity over the sea is higher than over land, is obviously contradictory to the fact.

2. Determination of coefficient of surface resistance from pilot balloon observations

A method of the estimation of κ from pilot balloon observations devised by Sutcliffe[5] is employed here. When the pressure gradient, Coriolis force, and internal friction are in equilibrium, x-component equation of motion in cartesian coordinates is $fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} = 0$, where v is y-component of wind velocity, τ_x is x-component of shearing stress, and p pressure. By substituting the y-component of the geostrophic wind (G_y) for the barometric gradient term, the above equation becomes $\frac{\partial \tau_x}{\partial z} = \rho f(G_y - v)$, and by integration

$$\tau_x(z) - \tau_x(0) = f\int_0^z \rho(G_y - v)\,dz.$$

Choosing x-axes in the direction of the surface wind, we have a working formula of $-\tau_0 = f\int_0^h \rho(G_y - v)\,dz$, where $\tau_0 = \tau_x(0)$ and h is a certain level at which $G_y = v$ or $\tau_x(h) = 0$ is to be satisfied.

The prominent merit of this method is that it involves no assumption as to the wind distribution along the vertical, while Taylor's method involves the assumption of the constancy of K in the frictional layer. Adopting this method to kite observations made on board the German research vessel "Meteor" over the North and South Atlantic, Sutcliffe found $\kappa = 4 \times 10^{-4}$ (mean value). But this low value of κ over the ocean was not accepted by some meteorologists who anticipated the independency of κ from surface states.

For a preliminary research of the general circulation of the atmosphere some trials to determine a reasonable value of the coefficient of skin friction have been made by P. A. Sheppard and others[6] recently. For this purpose, they performed detailed pilot balloon observations over the sea in the westerlies. But the determination of κ by Sutcliffe's method failed because of their missing a level where $\tau_x(z)$ vanished.

Though this trial of the determination failed in the westerly wind zone, P.A.Sheppard and M. H. Omar[7] succeeded in the trade wind belt. The results got from the observational data at very small islands in this belt are listed in Table 1. With the exception of the values for San Juan Island, the order of the obtained values of κ indicates 10×10^{-4}, and without having any systematic variation with the wind speed. (The inducement of the exceptional higher values for San Juan Island is attributed by the authors to the orographic effects, while in C. H. B. Priestly's suggestion it is imputed to those of the vertical velocity.)

So far, the value of $\kappa = 10 \times 10^{-4}$ is plausible, that is the smallness of κ over the ocean is assured to be a fact.

Table 1

Island	Range of V_s (knots)	κ
Wake	3…… 8	8×10^{-4}
	9……10	10 〃
	11……12	10 〃
	13……19	10 〃
Eniwetok	5……12	11 〃
	13……16	10 〃
	17……19	11 〃
	21……26	6 〃
Johnson	4……10	15 〃
	11……13	14 〃
	14……16	16 〃
	17……21	15 〃
San Juan	2…… 7	93 〃
	8……11	40 〃
	12……15	30 〃
	16……21	21 〃

3. Determination of coefficient of surface resistance from the velocity profiles in the skin layer

C.G.Rossby and R.B.Montgomery's method[8] for this subject is briefly reviewed here. Assuming the velocity profile in the skin layer obeys Prandtl's formula of $V = \frac{1}{k_o}\sqrt{\frac{\tau}{\rho}} \ln \frac{z+z_o}{z_o}$, where k_o is von Kármán's constant of 0.38, z_o roughness parameter, and τ is shearing stress which is constant throughout the skin layer, and eliminating τ from the relation $\tau = \kappa \rho V^2$, they get the working formula of $\kappa = \frac{k_o^2}{\left(\ln \frac{z_a + z_o}{z_o}\right)^2}$, where z_a indicates anemometer level at which synoptical surface wind is measured. They obtained $\kappa = 19 \times 10^{-4}$ with $z_o = 0.5$cm deduced by them from Hellman's data over the lawn at Nauen, and $\kappa = 31 \times 10^{-4}$ with $z_o = 3.2$cm from Shaw's data over open grass land.

Similarly they analysed the Wüst's observations of the wind over the Baltic Sea with the result that Prandtl's formula holds good for the mean wind profile getting $z_o = 4$cm. Rossby[9] re-analysed these materials, dividing them into two groups, i. e. light wind group (wind speed at 6 meters level ≤ 5.5m/sec) and strong wind group (wind speed at 6 meters level ≥ 5.8m/sec). As he got unreasonably high value of $z_o = 10$cm for the light wind group, he stopped the further study, regarding the observational data were not available for steady state. For the strong wind group, he found that the wind profile above 100cm obeys Prandtl's formula with $z_0=0.6$cm but below this level it obeys von Kármán's formula for air-flow over smooth surface $\frac{V}{\sqrt{\frac{\tau_0}{\rho}}}=5.5+\frac{1}{k_0}\ln\frac{z}{\nu}\sqrt{\frac{\tau_0}{\rho}}$, where ν is kinematic viscosity of air.

This fact was also identified by him from the analysis of Shoulejkin's data over the Black Sea and Montgomery's over Buzzards Bay.

If we allow the assumption that the state of the upper part of skin layer which obeys Prandtl's formula indicates real conditions over the sea, we shall be able to find the value of κ from the character of the upper part only without paying any attention to that of the lowest layer. Thus we have $\kappa=33\times10^{-4}$ with $z_0=4$cm from the mean of Wüst's data and $\kappa=20\times10^{-4}$ with $z_0=0.6$cm from the strong wind group. In the calculation, we choose $z_a=30$ meters in order to compare these with Taylor's results of Salisbury Plane. These values keep the same order of κ as in the case over land, accordingly we are obliged to conclude that κ is independent of surface states.

But we must remember that these observations were made over the shore waters, not over the mid-ocean. The assumption, that the state of the upper part over the shore waters is equivalent to the condition over the open sea, is doubtful. In other words, we must recognize that the upper part is much modified by the land condition, while the lowest layer is free from the land disturbance. Thus, it may be inferred that when the observations are made over mid-ocean the wind profile which obeys von Kármán's formula, existing in the lowest layer over the shore waters, may be extended to the upper part of the skin layer.

A similar situation over land was found in Mildner's pilot balloon observations in Leipzig. From these observations Lettau[10] deduced τ_0-value of 5.31 dynes/cm^2. By using this value we get $\kappa=69\times10^{-4}$, and connecting this value with the skin layer, we obtain $z_0=31.4$cm. However, the observations were made at the field with the z_0-value of about 0.5cm. This contradiction may be explained by the fact that the air reaching the field had just before passed over the city.

From this reason, it may be supposed that von Kármán's formula over the smooth surface will hold not only within the lowest layer but throughout the skin layer over the actual sea. If we allow this supposition, we shall have the following equation: $\frac{1}{\sqrt{\kappa}}+5.75\log\frac{1}{\sqrt{\kappa}}=5.5+5.75\log\frac{zV}{\nu}$. Unfortunately, the value of κ derived from this equation is not a pure constant but depends upon the wind speed. Therefore the assumption that the skin friction is $\kappa\rho V^2$

is not exempt from any modification. At present, we have too poor observational materials to examine these theoretical inconsistency. Leaving this question

Table 2

V (m/sec)	1	5	10	15	20	30	40
10^4 κ	9.2	7.4	6.8	6.5	6.3	6.0	5.8

unsettled, we calculate the values of κ for different V-values from this equation with the results listed in Table 2. In addition to these, the values thus obtained from the actual data treated by Rossby are fixed in Table 3. The order of these values coincides with the value 10×10^{-4} indicated in the preceding section. In consequence, it is proved that the value of κ is considerably lower over the sea than over land.

Table 3

Materials		κ
Wüst's data (Baltic Sea)		7.6×10^{-4}
Shoulejkin's data (Black Sea)	Series 1	8.0 〃
	Series 2	7.7 〃
Montgomery's data (Buzzards Bay)	Sept. 20th	7.1 〃
	22nd	8.3 〃
	23rd	7.7 〃
	24th	8.2 〃
	mean	7.7 〃
Mean		7.8×10^{-4}

4. Summary

A determination of the coefficient of the skin friction (κ) over the sea is attempted in this critical note. The assumption that the coefficient κ is independent of surface states is discussed, and it is illustrated by an example that this assumption leads to a result contradictory with the observed fact. P.A. Sheppard and others' determinations of κ from pilot balloon observations are reviewed, and it is noticed that the order of the obtained value of $\kappa \fallingdotseq 10\times10^{-4}$ is reliable; and by the way the value is about one-forth of that over land.

Rossby's treatment for the determination of κ from the velocity profile in the skin layer is discussed in the latter part of this paper. Noticing Rossby's results that in the upper portion of the skin layer the wind profile obeys Prandtl's formula while in the lower portion it obeys von Kármán's formula for the smooth surface, the writer of this paper supposes that the condition of the upper portion of the skin layer, shown by the available data observed at near coast, may be much modified by the neighbouring land condition without representing the true condition over the open sea, while the condition of the lower portion indicates the true condition over the open sea. In this view point, rejecting the too high values of κ ($20 - 33\times10^{-4}$) obtained from the upper profiles of the skin layer, and assuming that von Kármán's formula is available up to the top of the skin layer, the writer fixes the value $\kappa \fallingdotseq 8\times10^{-4}$, which is comparable with the value obtained from the pilot balloon observations over the open sea.

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