Design of H-infinity Extended Recursive Wiener Estimators in Discrete-Time Stochastic Systems

Kagoshima University Faculty of Education

NAKAMORI Seiichi*

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Abstract

This paper designs (1) the H-infinity RLS Wiener fixed-point smoother and filter for the observation equation with the linear modulation and (2) the extended H-infinity recursive Wiener fixed-point smoother and filter in discrete-time wide-sense stationary stochastic systems. In the extended estimators, it is assumed that the signal is observed with the nonlinear modulation and with additional white observation noise. In the estimators, the system matrix Φ for the state vector $x(k)$, the observation vector C for the state vector, the variance $K(k,k)=K(0)$ of the state vector, the nonlinear observation function and the variance of the white observation noise are used. Φ, C and $K(0)$ are calculated from the auto-covariance data of the signal.

A simulation example, on the estimation of a speech signal in the phase demodulation problem, is demonstrated to show the estimation characteristics of the proposed extended H-infinity recursive Wiener estimators.

Keyword：H-infinity estimation, Discrete-time stochastic systems, Extended recursive Wiener estimators, Covariance information, Nonlinear modulation

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● Professor of Kagoshima University, Faculty of Education

1. Introduction

The extended Kalman filter [1],[2],[3] is useful in the wide area of engineering such as signal demodulation problems etc. for the signal observed with nonlinear observation mechanism and with additional white observation noise. Also, the extended recursive Wiener fixed-point smoother and filter are designed in discrete-time wide-sense stationary stochastic systems [4]. The extended recursive Wiener estimators use the information of the system matrix Φ, the observation vector C for the state vector $x(k)$, the variance $K(k,k)=K(0)$ of the state vector, the nonlinear function on the observation mechanism and the variance of the observation noise. Since the extended recursive Wiener estimators do not use the input noise variance in the state equation, they might be superior in estimation accuracy to the extended Kalman estimators [4]. In [5], the robust extended Kalman filter is proposed for the discrete-time nonlinear systems with norm-bounded parameter uncertainties in Krein space.

In [6], [7], for input noise signals with the bounded energies, the H-infinity estimation problem is considered based on the discrete-time state-space model in Krein spaces. The H-infinity estimators are designed so as to be more robust and less sensitive for parameter variations. Also, the H-infinity recursive least-squares (RLS) Wiener fixed-point smoother and filter are presented in linear discrete-time stochastic systems [8]. The criterion is provided with an inequality that the maximum value of the ratio of the energy by the filtering error to the sum of the weighted square values of the input variables is smaller than γ^2.

The purpose of this paper, at first, is to design the H-infinity RLS Wiener fixed-point smoother and filter for the observation equation (1) with the linear modulation of the signal in discrete-time wide-sense stationary stochastic systems. Then, the extended H-infinity recursive Wiener estimators are designed for the observation equation (25) with the nonlinear modulation of the signal and with the additional white observation noise. In the estimators, the system matrix Φ for the state vector $x(k)$, the observation vector C for the state vector, the variance $K(k,k)=K(0)$ of the state vector, γ, the nonlinear observation function and the variance of the white observation noise are used. Φ, C and $K(0)$ are calculated from the auto-covariance data of the signal.

In [Theorem 1], by using the the information of Φ, C, $K(0)$ and R, the H-infinity RLS Wiener fixed-point smoother and filter are presented. The estimators are derived, based on the estimation technique and the algorithms in [4], [8], for the observation equation (1) with the linear modulation. In [Theorem 2], the extended H-infinity recursive Wiener fixed-point smoother and filter are proposed in discrete-time wide-sense stationary stochastic systems. The estimators in [Theorem 2] are obtained by extending the linear H-infinity RLS Wiener estimators in [Theorem 1] similarly as the derivation of the extended Kalman filter from the Kalman filter.

A simulation example on the estimation of a speech signal, concerning the phase demodulation problem, shows that the extended H-infinity recursive Wiener estimators are superior in estimation accuracy to the extended recursive Wiener estimators [4].

2. H-infinity smoothing problem for linear modulation

2.1 Krein-space observation equation

Let a scalar observation equation be given by

$$y_1(k) = H(k)z_1(k) + v_1(k), \quad z_1(k) = Cx(k), \tag{1}$$

in linear discrete-time stochastic systems. Here, $z_1(k)$ is a scalar signal, $H(k)$ is a linear modulation function of $z_1(k)$ and $x(k)$ is an $n \times 1$ state vector with the wide-sense stationary property. C is a $1 \times n$ observation vector that transforms $x(k)$ to $z_1(k)$. $v_1(k)$ is white observation noise. Also, let the state equation for the state vector $x(k)$ be expressed by

$$x(k+1) = \Phi x(k) + u(k+1), \tag{2}$$

where Φ is the state-transition matrix and $u(k)$ is white noise input. It is assumed that the signal and the observation noise are mutually independent and are zero mean. Let the auto-covariance function of $v_1(k)$ and $u(k)$ be expressed by

$$E[v_1(k)v_1^*(j)] = R\delta_K(k-j), \quad R > 0, \tag{3}$$

$$E[u(k)u^*(j)] = \Pi_0\delta_K(k-j), \quad \Pi_0 > 0. \tag{4}$$

Here, $\delta_K(\cdot)$ denotes the Kronecker δ function and the asterisk denotes the complex conjugation. Also, it is assumed that the mean and the variance matrix of the initial value $x(0)(=x_0)$ are given by

$$E[x_0] = 0, \quad E[x_0x_0^*] = Q_0, \quad Q_0 > 0. \tag{5}$$

In general, we consider to estimate some arbitrary linear combination of the state as

$$z_2(k) = D(k)Ux(k) \tag{6}$$

in terms of the observed value $y_1(k)$, where U is given $1 \times n$ matrix. From (1) and (6), for $D(k)=H(k)$, $U=C$, the problem to estimate $z_2(k)$ is reduced to the estimation of $z_1(k)$.

Let $\breve{z}(j)$ be a filtering estimate of $z_2(j)$. Here, $\breve{z}(j)$ is also called a fictitious observed value of $z_2(j)$. In the finite-horizon H-infinity suboptimal estimation problem for $z_2(j)$, the estimators are designed so as to obtain the filtering estimate $\breve{z}(k)$ which achieves the performance criterion

$$\sup_{\{x_0,u,v_1\}} \frac{A(L)}{M(L)} < \gamma^2,\ \gamma > 0,$$

$$A(L) = \sum_{j=0}^{L} e_f^*(j) e_f(j),$$

$$M(L) = (x_0 - \breve{x}_0)^* Q_0^{-1} (x_0 - \breve{x}_0) + \sum_{j=0}^{L} u^*(j) \Pi_0^{-1} u(j) + \sum_{j=0}^{L} v_1^*(j) R^{-1} v_1(j), \tag{7}$$

for the input noise signals, $u(j)$ and $v_1(j)$, $j=0,\ 1,\ \cdots,\ L$, with the bounded energies. Here, Q_0, Π_0 and R are positive weighting matrices. Q_0 reflects a priori knowledge as to how close x_0 is to its initial guess $\breve{x}_0$. (7) means that the maximum value for the ratio of the energy of the filtering error $e_f(j) = \breve{z}(j) - Ux(j)$ to the sum of the energies by the input variables $x_0 - \breve{x}_0$, $u(j)$ and $v_1(j)$ is smaller than γ^2. The H-infinity estimation algorithms are robust and less sensitive to parameter variations. For $L=\infty$, the performance criterion (7) is reduced to that in the infinite-horizon H-infinity estimation problem.

By referring to [8], the H-infinity estimation problem described above in the linear modulation is transformed into the linear least-squares estimation of $z(j)$, which consists of $z_1(j) = Cx(j)$ and $z_2(j) = Ux(j)$,

$$z(j) = Tx(j) = \begin{bmatrix} z_1(j) \\ z_2(j) \end{bmatrix},\ T = \begin{bmatrix} C \\ U \end{bmatrix}, \tag{8}$$

for the observation equation

$$y(j) = \begin{bmatrix} y_1(j) \\ \breve{z}(j) \end{bmatrix} = \begin{bmatrix} H(j)Cx(j) + v_1(j) \\ D(j)Ux(j) + v_2(j) \end{bmatrix} = \mathrm{H}(j)x(j) + v(j),\ \mathrm{H}(j) = \begin{bmatrix} H(j)C \\ D(j)U \end{bmatrix},$$

$$v(j) = \begin{bmatrix} v_1(j) \\ v_2(j) \end{bmatrix}, \tag{9}$$

$$[Ev(j)v^*(s)] = \Xi\delta_K(j-s),\ \ \Xi = \begin{bmatrix} R & 0 \\ 0 & -\gamma^2 I \end{bmatrix}. \quad (10)$$

Here, $[Ev(j)v^*(s)]$, $0 \le j,s \le L$, represents the auto-covariance function of $v(\cdot)$ in Krein spaces [6],[7]. The variance Ξ of the observation noise $v(j)$ in the Krein spaces is indefinite.

2.2 Least-squares estimation of $x(k)$ based on Krein-space observation equation

Let a fixed-point smoothing estimate $\hat{x}(k|L)$ of $x(k)$ be expressed by

$$\hat{x}(k|L) = \sum_{i=1}^{L} h(k,i,L)y(i),\ \ 1 \le k \le L, \quad (11)$$

as a linear transformation of the observed values $y(i)$, $1 \le i \le L$. In (11), $h(k,i,L)$ is a time-varying impulse response function and k is the fixed point respectively. The fixed-point smoothing estimate $\hat{z}(k|L)$ of the signal $z(k)$ is given by $\hat{z}(k|L) = \mathrm{H}(k)\hat{x}(k|L)$.

Let us consider the estimation problem, which minimizes the mean-square value

$$J = E[\| x(k) - \hat{x}(k|L) \|^2] \quad (12)$$

of the fixed-point smoothing error. From an orthogonal projection lemma [1]

$$x(k) - \sum_{i=1}^{L} h(k,i,L)y(i) \perp y(i),\ \ 0 \le j,k \le L, \quad (13)$$

the optimal impulse response function satisfies the Wiener-Hopf equation

$$[Ex(k)y^*(s)] = \sum_{i=1}^{L} h(k,i,L)E[y(i)y^*(s)]. \quad (14)$$

Here, '$\perp$' denotes the notation of the orthogonality. Let $K(\cdot,\cdot)$ represent the auto-covariance function of $x(\cdot)$. Substituting (9) and (10) into (14), we obtain

$$h(k,s,L)\Xi = K(k,s)H^*(s) - \sum_{i=1}^{L} h(k,i,L)\mathrm{H}(i)K(i,s)\mathrm{H}^*(s). \quad (15)$$

Let $K_{z_1}(k,s)$ represent the auto-covariance function of the signal $z_1(k)$. $K_{z_1}(k,s)$ is expressed as

$$K_{z_1}(k,s) = C\Phi^{k-s}K(s,s)C^*1(k-s) + CK^*(k,k)(\Phi^*)^{s-k}C^*1(s-k), \quad (16)$$

where $1(k-s)$ represents the unit step function. In wide-sense stationary stochastic systems [1], the variance of $x(k)$ satisfies $K(s,s) = K(0)$.

3. RLS Wiener fixed-point smoothing and filtering algorithms in case of linear modulation

According to the linear H-infinity estimation problem of the signal $z(k)$ in Section 2, [Theorem 1] shows the H-infinity recursive Wiener fixed-point smoothing and filtering algorithms, which use the covariance information of the signal and observation noise.

[Theorem 1]

Let the observation equation, concerned with the linear modulation for the signal $z_1(k)$, be given by (1). Let the auto-covariance function of the signal be given by (16) and let the variance of white observation noise $v_1(k)$ be R in wide-sense stationary stochastic systems. Then, the H-infinity recursive Wiener algorithms, using the information of the system matrix Φ, the variance $K(0)$ of the state vector, the observation vectors C and U and the linear modulation functions $H(k)$ and $D(k)$, for the fixed-point smoothing and filtering estimates of $z(k)$ consist of (17)-(24).

Fixed-point smoothing estimate of the signal $z_1(k) = Cx(k)$ at the fixed point k: $\hat{z}_1(k,L)$

$$\hat{z}_1(k,L) = \hat{z}_1(k,L-1) + Ch_1(k,L,L)(y_1(L) - H(L)\hat{z}_1(L,L-1)) + Ch_2(k,L,L)(\breve{z}(L) - D(L)\hat{z}_2(L,L-1)),$$
$$\hat{z}_1(L,L-1) = C\Phi\hat{x}(L-1,L-1),$$
$$\hat{z}_2(L,L-1) = U\Phi\hat{x}(L-1,L-1) \tag{17}$$

Fixed-point smoothing estimate of the signal $z_2(k) = Ux(k)$ at the fixed point k: $\hat{z}_2(k,L)$

$$\hat{z}_2(k,L) = \hat{z}_2(k,L-1) + Uh_1(k,L,L)(y_1(L) - H(L)\hat{z}_1(L,L-1)) + Uh_2(k,L,L)(\breve{z}(L) - D(L)\hat{z}_2(L,L-1)) \tag{18}$$

Smoother gain:

$$[h_1(k,L,L) \quad h_2(k,L,L)] = [K(k,k)(\Phi^*)^{L-k}C^*H^*(L) - q(k,L-1)\Phi^*C^*H^*(L) \quad K(k,k)(\Phi^*)^{L-k}U^*D^*(L) - q(k,L-1)\Phi^*U^*D^*(L)]R_{e,L}^{-1} \tag{19}$$

$$R_{e,L} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix},$$

$$\Omega_{11} = R + H(L)CK(L,L)C^*H^*(L) - H(L)C\Phi S(L-1)\Phi^*C^*H^*(L),$$
$$\Omega_{12} = H(L)CK(L,L)C^*H^*(L) - H(L)C\Phi S(L-1)\Phi^*U^*D^*(L),$$

$\Omega_{21} = D(L)UK(L,L)C^*H^*(L) - D(L)U\Phi S(L-1)\Phi^* C^* H^*(L)$,
$\Omega_{22} = -\gamma^2 I + D(L)UK(L,L)U^*D^*(L) - D(L)U\Phi S(L-1)\Phi^* U^* D^*(L)$

Auto-variance function of the fixed-point smoothing estimate $\hat{x}(k|L)$: $q(k,L)$

$$q(k|L) = q(k|L-1)\Phi^* + h_1(k,L,L)H(L)C(K(L,L) - \Phi S(L-1)\Phi^*) + h_2(k,L,L)D(L)U(K(L,L) - \Phi S(L-1)\Phi^*),\quad q(L|L) = S(L) \tag{20}$$

Auto-variance function of the filtering estimate $\hat{x}(k,k)$: $S(k)$

$$S(k) = \Phi S(k-1)\Phi^* + (G_1(k)H(k)C + G_2(k)D(k)U)(K(k,k) - \Phi S(L-1)\Phi^*),$$
$$[G_1(k) \quad G_2(k)] = [K(k,k)C^*H^*(k) - \Phi S(k-1)\Phi^* C^* H^*(k) \quad K(k,k)U^*D^*(k) - \Phi S(k-1)\Phi^* U^* D^*(k)]R_{e,L}^{-1},$$
$$S(0) = 0 \tag{21}$$

Filtering estimate of the signal $z_1(k)(=Cx(k))$: $\hat{z}_1(k,k)$

$$\hat{z}_1(k,k) = C\hat{x}(k,k)$$

Filtering estimate of the signal $z_2(k)(=Ux(k))$: $\hat{z}_2(k,k)$

$$\hat{z}_2(k,k) = U\hat{x}(k,k) \tag{22}$$

Fictitious observed value: $\breve{z}(k)$

$$\breve{z}(k) = D(k)\hat{z}_2(k,k) \tag{23}$$

Filtering estimate of state vector $x(k)$: $\hat{x}(k|k)$

$$\hat{x}(k|k) = \Phi\hat{x}(k-1|k-1) + (K(k,k) - \Phi S(k-1)\Phi^*)C^*H^*(k)(R + H(k)C(K(k,k) - \Phi S(k-1)\Phi^*)C^*H^*(k))^{-1}(y_1(k) - H(k)C\Phi\hat{x}(k-1|k-1)),\quad \hat{x}(0,0) = 0 \tag{24}$$

From [8], it is found that the proposed filter that achieves the performance criterion (7) for $L = k$ exists if, and only, if,

$$R + H(j)CK(j,j)C^*H^*(j) - H(j)C\Phi S(j-1)\Phi^*C^*H^*(j) > 0$$
$$-\gamma^2 I + D(j)UK(j,j)U^*D^*(j) - D(j)U\Phi S(j-1)\Phi^*U^*D^*(j) < 0,$$
$$j = 0,\ 1,\ 2,\ \cdots,\ k.$$

Proof. The H-infinity recursive Wiener fixed-point smoother and filter [8], using the information of Φ, C, U, $K(0)$ and R correspond to the case of $H(k)=1$ in the observation equation (1) with the linear modulation of the signal $z_1(k)$. The H-infinity recursive Wiener fixed-point smoothing and filtering algorithms in [Theorem 1] are derived by applying the estimation technique in [8] to the case of the observation equation (1) with the linear modulation. (Q.E.D.).

4. Extended recursive Wiener estimation algorithms in case of nonlinear modulation

Let a scalar observation equation with the nonlinear modulation of the signal $z_1(k)$ be given by

$$y(k) = f(z_1(k), k) + v(k), \quad z_1(k) = Cx(k), \tag{25}$$

where the signal $z_1(k)$ and the observation noise $v(k)$ have the same stochastic properties as those in Section 2.

Similarly to the design of the extended Kalman filter, in the design of the extended recursive Wiener estimators using the covariance information, the modulation function is put as $H(k) = \left.\frac{\partial f(z_1(k), k)}{\partial z_1(k)}\right|_{z_1(k)=\hat{z}_1(k|k-1)}$, $D(k) = \left.\frac{\partial f(z_2(k), k)}{\partial z_2(k)}\right|_{z_2(k)=\hat{z}_2(k|k-1)}$ in [Theorem 1] after expanding the nonlinear observation function in a first-order Taylor series about $\hat{z}_1(k|k-1)$ and $\hat{z}_2(k|k-1)$ [1]. Here, $\hat{z}_1(k|k-1) = C\Phi\hat{x}(k-1|k-1)$ and $\hat{z}_2(k|k-1) = U\Phi\hat{x}(k-1|k-1)$ represent the one-step ahead prediction estimates for the signals $z_1(k)$ and $z_2(k)$ respectively. Also, $H(L)\hat{z}_1(L|L-1)$ and $H(k)C\hat{x}(k|k-1)$ in [Theorem 1] are replaced with $f(\hat{z}_1(L|L-1), L)$ and $f(\hat{z}_1(k|k-1), k)$ respectively. Similarly, $D(L)\hat{z}_2(L|L-1)$ and $D(k)U\hat{x}(k|k-1)$ in [Theorem 1] are replaced with $f(\hat{z}_2(L|L-1), L)$ and $f(\hat{z}_2(k|k-1), k)$ respectively.

As a consequence, the H-infinity recursive Wiener fixed-point smoothing and filtering algorithms in the case of the observation equation (25), with the nonlinear modulation of the signal $z_1(k)$, is summarized in [Theorem 2]. It is noted that the proposed extended recursive Wiener estimators are sub-optimal because of the Taylor series approximation of the modulation function.

[Theorem 2]

Let the observation equation, with the nonlinear modulation of the signal $z_1(k)$, be given by the (25). Let the auto-covariance function of the signal be expressed by (16) and let the variance of white observation noise $v_1(k)$ be R in wide-sense stationary stochastic systems. Then, the H-infinity recursive Wiener algorithms, using the information of the system matrix Φ, the variance $K(0)$ of the state vector, the observation vectors C and U and the linear modulation functions $H(k)$ and $D(k)$, for the fixed-point smoothing and filtering estimates of $z(k)$ consist of (26)-(35).

Fixed-point smoothing estimate of the signal $z_1(k) = Cx(k)$ at the fixed point k : $\hat{z}_1(k, L)$

$$\hat{z}_1(k,L) = \hat{z}_1(k,L-1) + Ch_1(k,L,L)(y_1(L) - f(\hat{z}_1(L,L-1),L))$$
$$+ Ch_2(k,L,L)(\breve{z}(L) - f(\hat{z}_2(L,L-1),L)),$$
$$\hat{z}_1(L,L-1) = C\Phi\hat{x}(L-1,L-1), \quad \hat{z}_2(L,L-1) = U\Phi\hat{x}(L-1,L-1) \tag{26}$$

Fixed-point smoothing estimate of the signal $z_2(k) = Ux(k)$ at the fixed point k :

$\hat{z}_2(k,L)$

$$\hat{z}_2(k,L)=\hat{z}_2(k,L-1)+Uh_1(k,L,L)(y_1(L)-f(\hat{z}_1(L,L-1),L)) + Uh_2(k,L,L)(\breve{z}(L)-f(\hat{z}_2(L,L-1),L)) \quad (27)$$

Smoother gain:

$$[h_1(k,L,L) \quad h_2(k,L,L)] = [K(k,k)(\Phi^*)^{L-k}C^*H^*(L)-q(k,L-1)\Phi^*C^*H^*(L) \quad K(k,k)(\Phi^*)^{L-k}U^*D^*(L)-q(k,L-1)\Phi^*U^*D^*(L)]R_{e,L}^{-1} \quad (28)$$

$$R_{e,L}=\begin{bmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{bmatrix},$$

$$\Omega_{11}=R+H(L)CK(L,L)C^*H^*(L)-H(L)C\Phi S(L-1)\Phi^*C^*H^*(L),$$
$$\Omega_{12}=H(L)CK(L,L)C^*H^*(L)-H(L)C\Phi S(L-1)\Phi^*U^*D^*(L),$$
$$\Omega_{21}=D(L)UK(L,L)C^*H^*(L)-D(L)U\Phi S(L-1)\Phi^*C^*H^*(L),$$
$$\Omega_{22}=-\gamma^2I+D(L)UK(L,L)U^*D^*(L)-D(L)U\Phi S(L-1)\Phi^*U^*D^*(L)$$

Auto-variance function of the fixed-point smoothing estimate $\hat{x}(k\mid L)$: $q(k,L)$

$$q(k\mid L)=q(k\mid L-1)\Phi^*+h_1(k,L,L)H(L)C(K(L,L)-\Phi S(L-1)\Phi^*) + h_2(k,L,L)D(L)U(K(L,L)-\Phi S(L-1)\Phi^*),\ q(L\mid L)=S(L) \quad (29)$$

Auto-variance function of the filtering estimate $\hat{x}(k,k)$: $S(k)$

$$S(k)=\Phi S(k-1)\Phi^*+(G_1(k)H(k)C+G_2(k)D(k)U)(K(k,k)-\Phi S(L-1)\Phi^*),$$
$$[G_1(k) \quad G_2(k)]=[K(k,k)C^*H^*(k)-\Phi S(k-1)\Phi^*C^*H^*(k) \quad K(k,k)U^*D^*(k)-\Phi S(k-1)\Phi^*U^*D^*(k)]R_{e,L}^{-1},$$
$$S(0)=0 \quad (30)$$

Filtering estimate of the signal $z_1(k)(=Cx(k))$: $\hat{z}_1(k,k)$

$$\hat{z}_1(k,k)=C\hat{x}(k,k) \quad (31)$$

Filtering estimate of the signal $z_2(k)(=Ux(k))$: $\hat{z}_2(k,k)$

$$\hat{z}_2(k,k)=U\hat{x}(k,k) \quad (32)$$

Fictitious observed value: $\breve{z}(k)$

$$\breve{z}(k)=D(k)\hat{z}_2(k,k) \quad (33)$$

Filtering estimate of state vector $x(k)$: $\hat{x}(k\mid k)$

$$\hat{x}(k\mid k)=\Phi\hat{x}(k-1\mid k-1)+(K(k,k)-\Phi S(k-1)\Phi^*)C^*H^*(k)(R+H(k)C(K(k,k) - \Phi S(k-1)\Phi^*)C^*H^*(k))^{-1}(y_1(k)-f(\hat{z}(k\mid k-1),k)),\quad \hat{x}(0,0)=0 \quad (34)$$

Here, the functions $H(k)$ and $D(k)$ are given by

$$H(k)=\left.\frac{\partial f(z_1(k),k)}{\partial z_1(k)}\right|_{z_1(k)=\hat{z}_1(k|k-1)},\quad D(k)=\left.\frac{\partial f(z_2(k),k)}{\partial z_2(k)}\right|_{z_2(k)=\hat{z}_2(k|k-1)}. \quad (35)$$

The difference of the H-infinity recursive Wiener estimators from the extended Kalman estimators is based on the information used. The H-infinity recursive Wiener estimators use the information of Φ, $K(0)$, C, U, $H(k)$ $D(k)$ and R. The extended Kalman estimators use the information of Φ, C and the variance Π_0 of the white noise input $u(k)$ in (2). Both estimators use the information of nonlinear modulation function. Since $S(k|k)$ is the auto-variance function of the filtering estimate $\hat{x}(k|k)$, the Kalman filtering algorithm for the filtering error variance function $P(k|k)$ is obtained by substituting $S(k|k)=K(0)-P(k|k)$ into (30) in the H-infinity extended recursive Wiener estimation algorithms of [Theorem 2]. For the quantities $S(k|k-1)=\Phi S(k-1|k-1)\Phi^T$ and $P(k|k-1)=\Phi P(k-1|k-1)\Phi^T$, there is a relationship $S(k|k-1)=K(0)-\Pi_0-P(k|k-1)$.

5. A numerical simulation example

Let a scalar observation equation with the nonlinear modulation of the signal $z_1(k)$ be given by

$$y(k)=f(z_1(k),k)+v_1(k),\ z_1(k)=Cx(k),$$
$$f(z_1(k),k)=\cos(2\pi f_c k\Delta+m_A z_1(k)),\ f_c=1{,}000(Hz),\ \Delta=0.0001,\ m_A=1.2. \tag{36}$$

The nonlinear function in (36) expresses the phase modulation in analogue communication systems [9]. Here, f_c, Δ and m_A represent the carrier frequency, the sampling period of the signal $z_1(k)$ and the phase sensitivity respectively. The observation function is given by

$$H(k)=\left.\frac{\partial f(z_1(k),k)}{\partial z_1(k)}\right|_{z_1(k)=\hat{z}_1(k|k-1)}=-m_A\sin(2\pi f_c k\Delta+m_A\hat{z}_1(k|k-1)). \tag{37}$$

Let $v_1(k)$ be white Gaussian observation noise having the mean zero and the variance R, which is expressed by $N(0,R)$.

Let the signal $z_1(k)$ be expressed by the state vector $x(k)$, which consists of the state variables $x_1(k)=z_1(k),\ \ x_2(k)=z_1(k+1),\ \ \cdots,\ \ x_n(k)=z(k+n-1)$, as

$$z_1(k)=Cx(k),\ x(k)=[x_1(k)\ \ x_2(k)\ \ \cdots\ \ x_n(k)]^T,\ z_1(k)=x_1(k),\ C=[1\ \ 0\ \ \cdots\ \ 0]. \tag{38}$$

Let us consider to estimate a vowel signal spoken by the author. Its phonetic symbol is written as "/i:/". The sampling frequency of the voice signal is $10.025[kHz]$. The auto-covariance function of the signal is calculated in terms of the $N=350$ sampled signal data. Let the stochastic process of the vowel signal be modeled in terms of the AR process of order n.

$$z_1(k) = -a_1 z_1(k-1) - a_2 z_1(k-2) - \cdots - a_n z_1(k-n) + e(k), \quad E[e(k)e(s)] = \sigma^2 \delta_K (k-s) \tag{39}$$

Let $K_z(i)$, $i=1, \cdots, n$, represent the auto-covariance function of the signal $z_1(k)$ in wide-sense stationary stochastic systems. The AR parameters a_i, $i=1, \cdots, n$, are calculated by the Yule-Walker equations

$$\begin{bmatrix} K_z(0) & K_z(1) & \cdots & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & \cdots & K_z(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} -K_z(1) \\ -K_z(2) \\ \vdots \\ -K_z(n-1) \\ -K_z(n) \end{bmatrix}. \tag{40}$$

By referring to [4], the $1 \times n$ observation vector C, the auto-variance function $K(0)$ of the state vector $x(k)$ and the system matrix Φ are obtained in terms of the auto-covariance function of the signal as follows:

$$C = [1 \quad 0 \quad \cdots \quad 0], \tag{41}$$

$$K(0) = \begin{bmatrix} K_z(0) & K_z(1) & \cdots & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & \cdots & K_z(0) \end{bmatrix}, \tag{42}$$

$$\Phi = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & -a_2 & -a_1 \end{bmatrix}. \tag{43}$$

$K(0)$ is also called the Hankel matrix. As indicated in [10], a finite dimensional realization for $z_1(k)$ exists if and only if the rank of the Hankel matrix is n.

By substituting Φ, C and $K(0)$ into the H-infinity extended recursive Wiener estimation algorithms of [Theorem 2], the fixed-point smoothing estimate $\hat{z}_1(k \mid L)$ at the fixed point k and the filtering estimate $\hat{z}_1(k \mid k)$ of the signal are calculated recursively.

Fig.1 illustrates the signal $z_1(k)$, the filtering estimate $\hat{z}_1(k \mid k)$ and the fixed-point smoothing estimate $\hat{z}_1(k \mid k+5)$ by the extended H-infinity recursive Wiener fixed-point smoother and filter in [Theorem 2] vs. k for $SNR = 5$ [dB] and $\gamma = 2$. **Fig.2** illustrates the mean-square values (MSVs) in [dB] of the fixed-point smoothing error $z(k) - \hat{z}(k \mid k+5)$ and the filtering error $z(k) - \hat{z}(k \mid k)$ by the extended H-infinity recursive Wiener estimators vs. γ, $1.5 \le \gamma \le 1000$, for $SNR = 5$ [dB]. **Fig.2** indicates that the smoother is superior in estimation accuracy to the filter. For the large value of γ such as $\gamma = 1000$, the MSVs by the extended H-infinity recursive Wiener estimators are same as those by the extended recursive estimators [4]. The MSV of the filtering errors decreases gradually as the value of γ increases. In the fixed-point smoother, for $1.5 \le \gamma \le 2$, the MSV decreases gradually as γ increases. It might be found that the minimum value of the MSV exists around $\gamma = 2.0$. Here, the MSVs, by the dB expression, of the fixed-point smoothing errors and the filtering errors are calculated respectively by $10\log_{10} \dfrac{\sum_{k=1}^{600} (z(k) - \hat{z}(k \mid k+5))^2 / 600}{\sum_{k=1}^{600} z^2(k) / 600}$ [dB] and $10\log_{10} \dfrac{\sum_{k=1}^{600} (z(k) - \hat{z}(k \mid k))^2 / 600}{\sum_{k=1}^{600} z^2(k) / 600}$. **Fig.3** illustrates the MSVs of the filtering error $z(k) - \hat{z}(k \mid k)$ and the fixed-point smoothing error $z(k) - \hat{z}(k \mid k+5)$ by the extended H-infinity recursive Wiener filter and smoother vs. SNR [dB], $1 \le SNR \le 10$, for $\gamma = 2$. The MSVs of the fixed-point smoothing errors and filtering errors decrease, as the value of SNR increases. Also, from **Fig.3**, it is shown that the estimation accuracy of the extended H-infinity recursive Wiener fixed-point smoother is superior to that of the extended H-infinity recursive Wiener filter. **Fig.4** illustrates the MSVs of the filtering error $z(k) - \hat{z}(k \mid k)$ and the fixed-point smoothing error $z(k) - \hat{z}(k \mid k + Lag)$ by the extended H-infinity recursive Wiener fixed-point smoother and the extended recursive Wiener fixed-point smoother vs. Lag, $1 \le Lag \le 10$, for $\gamma = 2$ and $SNR = 5$ [dB]. As Lag increases, the estimation accuracies by the extended H-infinity recursive Wiener fixed-point smoother and the extended recursive Wiener fixed-smoother are improved. It is seen that the estimation accuracy of the extended H-infinity fixed-point smoother is superior to that of the extended recursive Wiener fixed-point smoother.

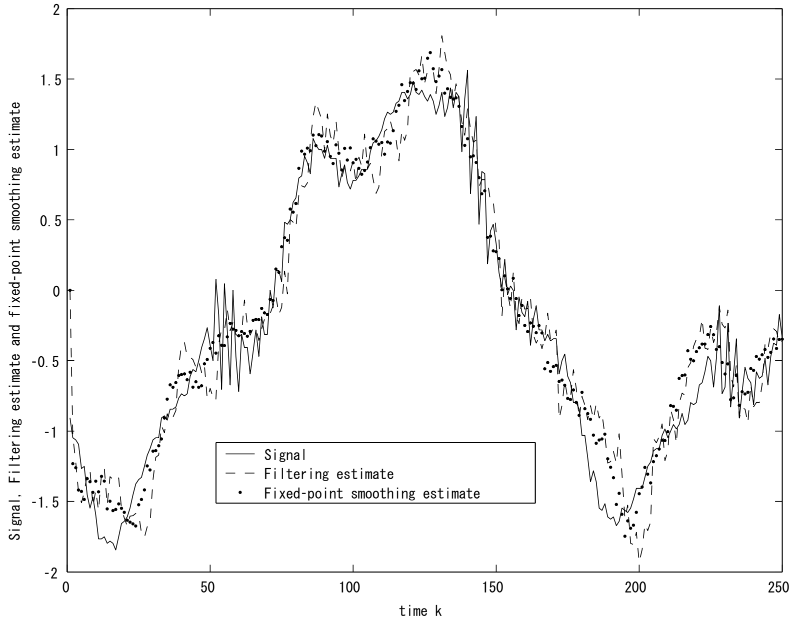

Fig.1 Signal $z_1(k)$, the filtering estimate $\hat{z}_1(k \mid k)$ and the fixed-point smoothing estimate $\hat{z}_1(k \mid k+5)$ by the extended H-infinity recursive Wiener fixed-point smoother and filter in [Theorem 2] vs. k for $SNR = 5$ [dB] and $\gamma = 2$.

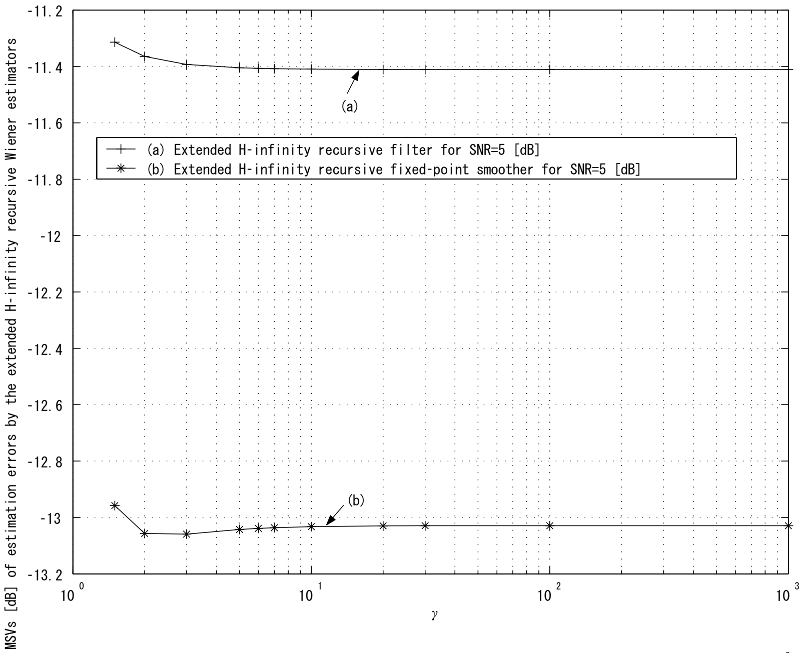

Fig.2 Mean-square values in [dB] of the fixed-point smoothing error $z(k) - \hat{z}(k \mid k+5)$ and the filtering error $z(k) - \hat{z}(k \mid k)$ by the extended H-infinity recursive Wiener estimators in [Theorem 2] vs. γ for $SNR = 5$ [dB].

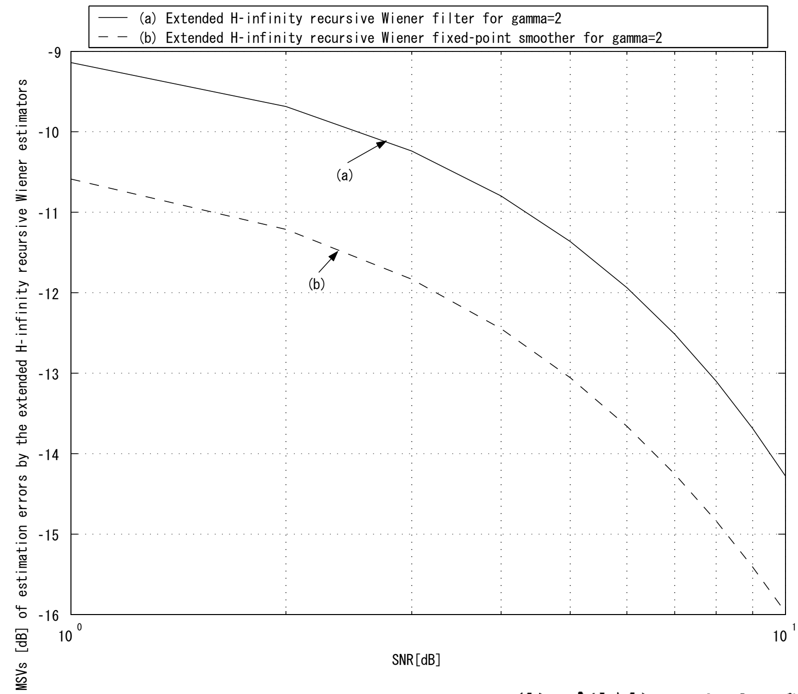

Fig.3 Mean-square values of the filtering error $z(k)-\hat{z}(k|k)$ and the fixed-point smoothing error $z(k)-\hat{z}(k|k+5)$ by the extended H-infinity recursive Wiener filter and smoother vs. SNR [dB], $1 \le SNR \le 10$, for $\gamma = 2$.

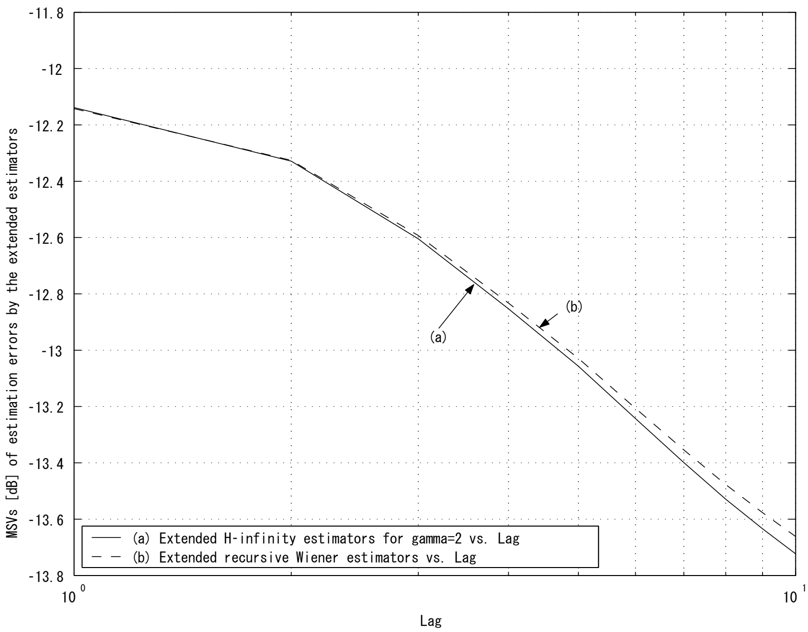

Fig.4 Mean-square values in [dB] of the filtering error $z(k)-\hat{z}(k|k)$ and the fixed-point smoothing error $z(k)-\hat{z}(k|k+Lag)$ by the extended H-infinity recursive Wiener estimators in [Theorem 2] and the extended recursive Wiener estimators vs. Lag, $1 \le Lag \le 10$, for $\gamma = 2$ and $SNR = 5$ [dB].

6. Conclusions

In this paper, the H-infinity RLS Wiener fixed-point smoother and filter for the observation equation with the linear modulation of the signal are proposed, in [Theorem 1], in discrete-time wide-sense stationary stochastic systems. Then, in [Theorem 2], the extended H-infinity recursive Wiener fixed-point smoother and filter for the observation equation with the nonlinear modulation of the signal are presented in [Theorem 2].

From the simulation example, it has been shown that the the extended H-infinity recursive Wiener fixed-point smoothing and filtering algorithms proposed in [Theorem 2] are feasible.

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