Diameters of Planar Nets of the Regular Dodecahedron

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Abstract

Incongruent planar nets of the regular dodecahedron are classied according to their diameters, and their frequncy distribution is shown.

1 Introduction

Planar nets of a polyhedron have atracted acute interest from ancient times (for example, see Pedoe (1976)). It has been well-known that the cube (the regular hexahedron) has the number 11 kinds of incongruent planar nets. It is also known that, by the duality between the cube and the regular octahedron, the regular octahedron has the same number of incongruent planar nets.

How about the regular dodecahedron and its dual, i.e., the regular icosahedron? Hippenmeyer(1979) solved the problem: both regular solids have the number 43,380 kinds of incongruent planar net. He obtained the number by use of the Matrix-Tree theorem and the Burnside formula.

Although the number 43,380 has been determined, there is little knowledge about shapes of these nets themselves. Only one exception is a research by Horiyama and Shoji (2011). They shows that these nets never overlap.

In this paper we classify incongruent planar nets of the regular dodecahedron according to their diameters. The diameter of a finite point sets, say $V = \{P_1, P_2, \cdots, P_n\}$, is defined as the maximum of Euclidean distances of all pairs of points, i.e.,

$$\text{diamater of } V = \max_{1 \leqq i < j \leqq n} d(P_i, P_j),$$

where d denotes the Euclidean distance of two points. In other words, the diameter of a finite point sets is the minimum of diamters of circles that contain all the points.

2 Diameters

For every planar net of the regular dodecahedron, we construct a graph if we regard faces of the net as vertices of a graph and draw edges of a graph between adjacent faces of the net. Obviously the graph is a

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tree, and its number of vertices is given by 12 × 5 − (12 − 1) × 2 = 38. We compute the diameter of these 38 vertices of the net.

The following table gives the distribution of diameters for all the incongruent planar nets of the regular dodecahedron of unit edge length: the leftmost column shows exact values of diameters, where τ denotes the golden ratio $(\sqrt{5} + 1)=2$; the middle column shows their aproximate values; and the rightmost column shows frequncies of nets. Of course the sum of frequncies is equal to 43,380.

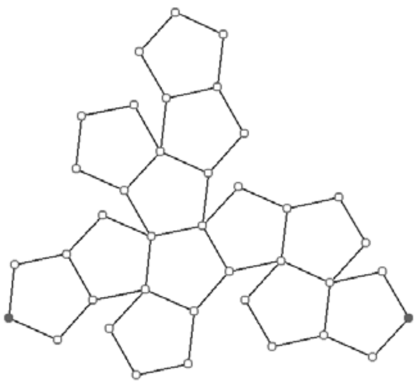
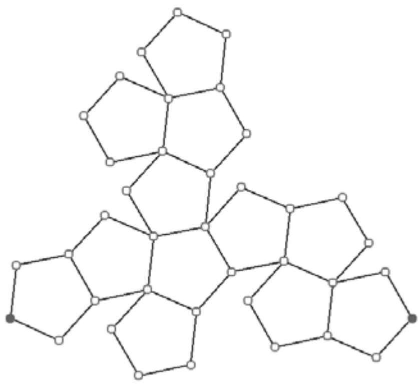
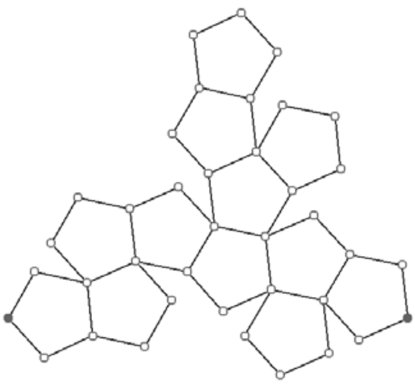
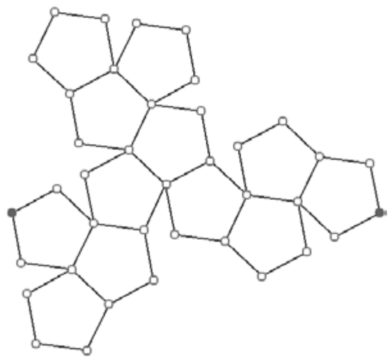
exact	appr.	freq.
$\sqrt{25\tau+16}$	7.513	20
$\sqrt{26\tau+17}$	7.686	415
$\sqrt{25\tau+19}$	7.710	176
$\sqrt{25\tau+20}$	7.775	81
$\sqrt{25\tau+21}$	7.839	67
$\sqrt{27\tau+20}$	7.980	95
$\sqrt{29\tau+18}$	8.057	1404
$\sqrt{25\tau+25}$	8.090	3
$\sqrt{29\tau+19}$	8.119	2105
$\sqrt{30\tau+19}$	8.218	4077
$\sqrt{27\tau+25}$	8.288	16
$\sqrt{29\tau+22}$	8.302	469
$\sqrt{26\tau+27}$	8.311	2
$\sqrt{18\tau+25}$	8.385	32
$\sqrt{31\tau+21}$	8.436	2
$\sqrt{32\tau+20}$	8.472	3396
$\sqrt{31\tau+23}$	8.553	39
$\sqrt{29\tau+27}$	8.598	12
$\sqrt{32\tau+23}$	8.647	837
$\sqrt{33\tau+22}$	8.683	3843
$\sqrt{31\tau+26}$	8.727	191
$\sqrt{31\tau+27}$	8.784	10
$\sqrt{34\tau+23}$	8.833	3367
$\sqrt{30\tau+31}$	8.919	10
$\sqrt{35\tau+24}$	8.979	1006
$\sqrt{36\tau+23}$	9.014	477
$\sqrt{35\tau+25}$	9.035	437
$\sqrt{35\tau+26}$	9.090	329
$\sqrt{37\tau+23}$	9.103	5418
$\sqrt{34\tau+29}$	9.166	48
$\sqrt{36\tau+27}$	9.233	246
$\sqrt{35\tau+29}$	9.254	83
$\sqrt{36\tau+28}$	9.287	294
$\sqrt{38\tau+25}$	9.300	2804
$\sqrt{37\tau+28}$	9.374	336
$\sqrt{36\tau+31}$	9.447	45
$\sqrt{40\tau+25}$	9.472	463
$\sqrt{39\tau+28}$	9.545	11
$\sqrt{39\tau+29}$	9.597	208
$\sqrt{41\tau+26}$	9.609	2003
$\sqrt{41\tau+27}$	9.661	238
$\sqrt{41\tau+28}$	9.713	1098
$\sqrt{40\tau+31}$	9.784	26
$\sqrt{38\tau+35}$	9.823	7
$\sqrt{43\tau+27}$	9.827	2192
$\sqrt{39\tau+34}$	9.854	14
$\sqrt{41\tau+32}$	9.917	72
$\sqrt{38\tau+37}$	9.924	2
$\sqrt{44\tau+28}$	9.960	15
$\sqrt{40\tau+35}$	9.986	10
$\sqrt{44\tau+29}$	10.010	470
$\sqrt{45\tau+29}$	10.090	929
$\sqrt{41\tau+36}$	10.116	2
$\sqrt{44\tau+32}$	10.158	159
$\sqrt{45\tau+31}$	10.189	115
$\sqrt{44\tau+33}$	10.208	59
$\sqrt{41\tau+38}$	10.215	1
$\sqrt{46\tau+31}$	10.268	404
$\sqrt{47\tau+30}$	10.298	459
$\sqrt{46\tau+33}$	10.365	67
$\sqrt{44\tau+37}$	10.402	5
$\sqrt{47\tau+33}$	10.443	133
$\sqrt{49\tau+32}$	10.549	40
$\sqrt{50\tau+31}$	10.578	591
$\sqrt{49\tau+33}$	10.596	127
$\sqrt{48\tau+35}$	10.614	40
$\sqrt{51\tau+32}$	10.701	699
$\sqrt{48\tau+37}$	10.708	2
$\sqrt{47\tau+40}$	10.773	1
$\sqrt{52\tau+33}$	10.823	83
$\sqrt{52\tau+35}$	10.915	99
$\sqrt{53\tau+35}$	10.989	11
$\sqrt{50\tau+41}$	11.041	1
$\sqrt{55\tau+34}$	11.090	44
$\sqrt{55\tau+35}$	11.135	50
$\sqrt{54\tau+37}$	11.152	21
$\sqrt{55\tau+36}$	11.180	36
$\sqrt{56\tau+36}$	11.252	58
$\sqrt{56\tau+37}$	11.296	22
$\sqrt{55\tau+39}$	11.313	1
$\sqrt{58\tau+37}$	11.439	52
$\sqrt{59\tau+37}$	11.509	10
$\sqrt{59\tau+38}$	11.553	4
$\sqrt{59\tau+39}$	11.596	11
$\sqrt{61\tau+38}$	11.692	5
$\sqrt{59\tau+42}$	11.725	1
$\sqrt{60\tau+41}$	11.751	3
$\sqrt{65\tau+41}$	12.090	2
$\sqrt{64\tau+43}$	12.106	1
$\sqrt{66\tau+41}$	12.157	6
$\sqrt{65\tau+44}$	12.214	1
$\sqrt{66\tau+47}$	12.401	1
$\sqrt{69\tau+43}$	12.436	1
$\sqrt{72\tau+45}$	12.708	1
$\sqrt{74\tau+47}$	12.913	1

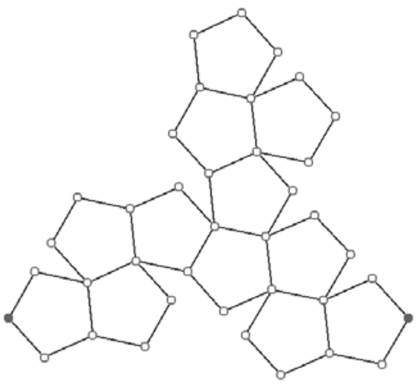
3 Incongruent planar nets of the minimal diameter

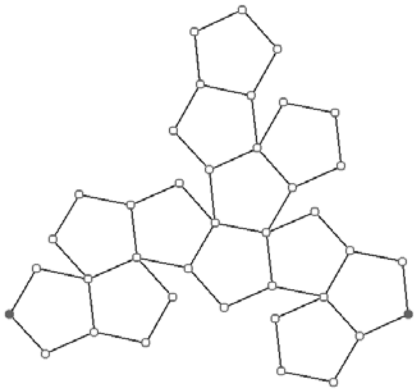
The following figures show all the incongruent planar nets of the minimal diameter $\sqrt{25\tau + 16}$.

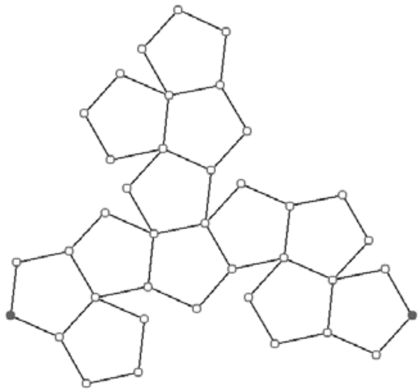
They are the most 'compact' planar nets. In each figure the distance between two filled points corresponds to the minimal diameter.

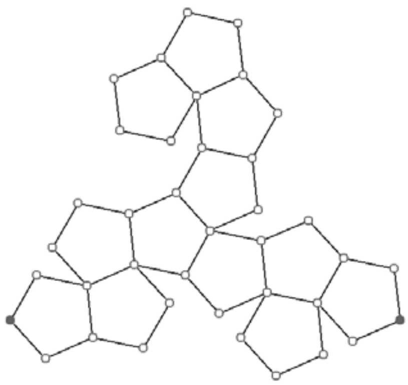


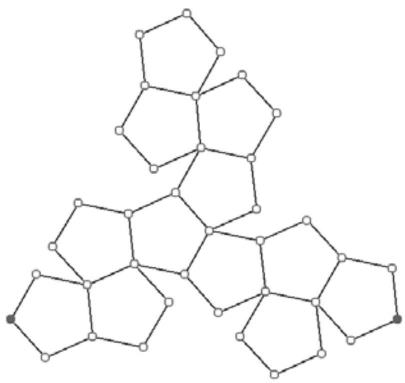





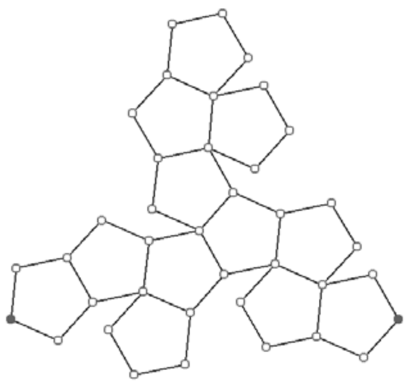


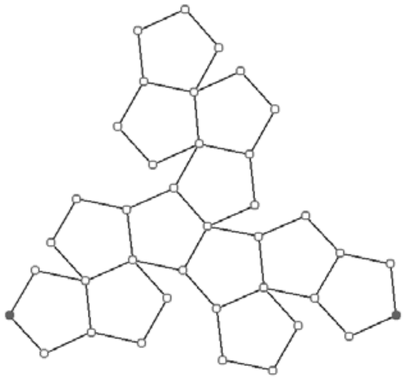


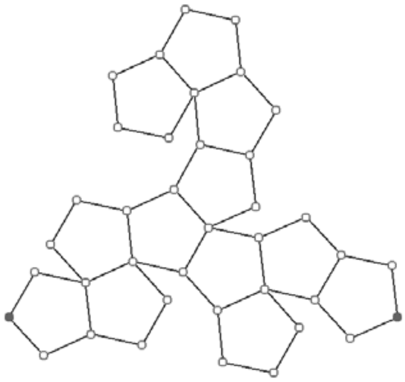






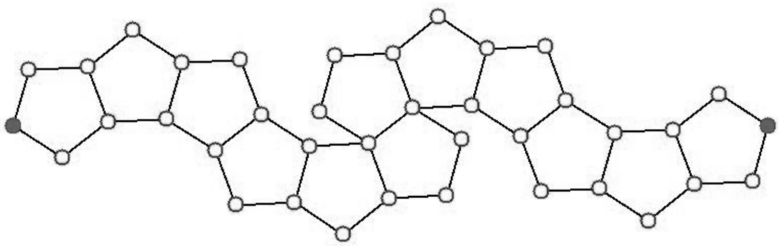






4 Unique planar net of the maximal diameter

The following figure shows the unique incongruent planar net of the maximal diameter $\sqrt{74\tau + 47}$. This is the most 'long' planar net. Again the distance between two filled points corresponds to the maximal diameter.



References

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Hippenmeyer,C. (1979) "Die Anzahl der inkongruenten ebenen Netze eines regularen Ikosaeders",*Elem. Math.*, Vol.34, 61-63.
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