

Modified Bessel Functions of Purely Imaginary Order

$K_{is}(x)$, $I_{is}(x)$ and their Related Functions

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The solutions of the Bessel equation

$$\frac{d^2y}{dx^2} + \frac{dy}{x dx} - (1 - s^2/x^2)y = 0$$

are discussed in detail. The modified Bessel Functions $K_\nu(x)$ and $I_\nu(x)$ are the two independent solutions of above equation when the order is purely imaginary, i.e., $\nu = is$. The value of the function $K_{is}(x)$ is real while the value of the function $I_{is}(x)$ is complex for real s and x . Hence a new real function $M_{is}(x)$ is introduced in place of the complex function $I_{is}(x)$.

Some series expansions for $K_{is}(x)$ and $M_{is}(x)$ are given. The possibility to compute the value of these functions and their derivatives by the use of their series expansions are discussed and a practical procedure with the accuracy of the eight decimal places is presented. Short tables for $K_{is}(x)$ and $M_{is}(x)$ and their related functions are given.

Some related formulas with $K_{is}(x)$ are also collected.

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§1 Introduction

A special form of the modified Bessel functions $I_\nu(x)$ and $K_\nu(x)$ when ν is imaginary, plays important role in the analysis of some kinds of boundary value problems in the potential theory⁽¹⁾.

J. Dougall⁽²⁾ has given three expressions of Green's function for several regions constructed in the cylindrical coordinates. The expression of the φ -form, one of them, for the region bounded by two parallel planes and a cylinder is given as follows:

$$V = \frac{8}{\pi c} \sum_{p=1}^{\infty} \sin(p\pi z/c) \sin(p\pi z'/c) \int_0^{\infty} \cosh s(\pi - |\varphi - \varphi'|) \frac{f(r)f(r')}{I_{is}(p\pi a/c)I_{-is}(p\pi a/c)} ds, \quad (1.1)$$

where

$$f(r) = I_{is}(p\pi a/c)K_{is}(p\pi r/c) - I_{is}(p\pi r/c)K_{is}(p\pi a/c). \quad (1.2)$$

The other expressions of the z - and r -forms are often used to analyze some practical problems. However the expression of the φ -form has not been used because that the method to compute the functions $I_{is}(x)$ and $K_{is}(x)$ has not been established yet.

In the previous paper⁽³⁾, the authors discussed the possibilities to compute the values of $K_{is}(x)$ starting from the integral representation for this function and proposed a procedure based on such an algorithm. However the algorithm is too time-consuming for some kinds of combination of s and x . Further, we must established the method to compute the values of $I_{is}(x)$ if we want to use the expression of the φ -form.

In this paper, the possibilities to compute the values of these functions and their derivatives by the use of the series expansions are discussed and a practical procedure with the accuracy of the eight decimal places is presented. Functions $I_{is}(x)$ and $K_{is}(x)$ are the two independent solutions of the Bessel equation

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \left(1 - \frac{s^2}{x^2}\right) y = 0. \quad (1.3)$$

It is well known that $I_{is}(x)$ and $I_{-is}(x)$ are also the two independent solutions. Hence which set should be adopted is a problem.

When s and x are both real, the value of $K_{is}(x)$ is real while the value of $I_{is}(x)$ is complex and the imaginary part of $I_{is}(x)$ can be expressed by $K_{is}(x)$. Since the function having a complex value is unwieldy, we introduce a function $M_{is}(x)$ which is a real part of $I_{is}(x)$ multiplied by $\pi/\cosh(s\pi)$, the relations between $I_{is}(x)$, $K_{is}(x)$ and $M_{is}(x)$ are

$$I_{is}(x) = \frac{\cosh(s\pi)}{\pi} M_{is}(x) - i \frac{\sinh(s\pi)}{\pi} K_{is}(x), \quad (1.4)$$

$$I_{-is}(x) = \frac{\cosh(s\pi)}{\pi} M_{is}(x)i + \frac{\sinh(s\pi)}{\pi} K_{is}(x), \tag{1.5}$$

$$K_{is}(x) = \frac{(\pi/2)}{i\sinh(s\pi)} \{I_{-is}(x) - I_{is}(x)\}, \tag{1.6}$$

and

$$M_{is}(x) = \frac{(\pi/2)}{\cosh(s\pi)} \{I_{-is}(x) + I_{is}(x)\}. \tag{1.7}$$

The reasons of introducing the function $M_{is}(x)$ are 1) by the real functions of $K_{is}(x)$ and $M_{is}(x)$, the expression of the φ -form involving the complex functions $I_{is}(x)$ and $I_{-is}(x)$ is simplified in the form taking the above expression as an example

$$\begin{aligned} & \int_0^\infty \cosh s (\pi - |\varphi - \varphi'|) \frac{f(r)f(r')}{I_{is}(p\pi a/c)I_{-is}(p\pi a/c)} ds \\ &= \int_0^\infty \cosh s (\pi - |\varphi - \varphi'|) \frac{g(r)g(r')}{M_{is}^2(p\pi a/c) + \tanh(s\pi)^2 K_{is}^2(p\pi a/c)} ds \end{aligned} \tag{1.8}$$

where

$$g(r) = M_{is}(p\pi a/c)K_{is}(p\pi r/c) - M_{is}(p\pi r/c)K_{is}(p\pi a/c), \tag{1.9}$$

and 2) in the region of oscillation, the amplitudes of envelope of $K_{is}(x)$ and $M_{is}(x)$ are comparable to each other as following form:

$$K_{is}(x) \simeq \sqrt{\frac{2\pi}{s}} e^{-s\pi/2} \sin(\pi/4 - s + s\log(2s/x)), \tag{1.10}$$

$$M_{is}(x) \simeq \sqrt{\frac{2\pi}{s}} e^{-s\pi/2} \cos(\pi/4 - s + s\log(2s/x)). \tag{1.11}$$

The procedure presented here is made to compute both values of $K_{is}(x)$ and $M_{is}(x)$, simultaneously. It is because that since the contents of computations of these two values are almost the same as each other, therefore if we make the procedure to compute these two values simultaneously, the amount of computations can be reduced to a large extent. It is also because that for the problems we aim to analyze the two values are usually necessary.

The well known recurrence technique for the calculation of Bessel functions is not applicable for this case because that the orders of Bessel functions are not real.

The derivatives of the functions $K_{is}(x)$ and $M_{is}(x)$ with respect to the variable x are also discussed. These derivatives are necessary to analyze the problem with Neuman-type boundary condition. In general if the Bessel function with real order is given, the derivative of the Bessel function can be derived by the use of recurrence formula. However it is not applicable for this case therefore the procedure to compute the two values of $K'_{is}(x)$ and $M'_{is}(x)$

must be provided separately.

At the end of this paper, some important formulas for $K_{is}(x)$ and $M_{is}(x)$ are given. In these formulas, the Fourier-type series expansions are involved. This fact shows that these functions have the large possibility to be used for the analysis of various boundary value problems.

§2 Series Expansions of the Functions $K_{is}(x)$ and $M_{is}(x)$

2.1 Series Expansion Available for Large Order

It is well known that the modified Bessel function of the second kind $K_\nu(x)$ is defined as follows:

$$K_\nu(x) = \frac{\pi \{I_{-\nu}(x) - I_\nu(x)\}}{2\sin(\nu\pi)}, \quad (2.1)$$

and the modified Bessel function of the first kind $I_\nu(x)$ is expressed⁶⁾ by

$$I_\nu(x) = \frac{(x/2)^\nu}{\Gamma(\nu+1)} \left\{ 1 + \frac{(x/2)^2}{\nu+1} + \frac{(x/2)^4}{2!(1+\nu)(2+\nu)} + \frac{(x/2)^6}{3!(1+\nu)(2+\nu)(3+\nu)} + \dots \right\}. \quad (2.2)$$

Since $\Gamma(\nu+1)\Gamma(1-\nu) = \nu\pi/\sin(\nu\pi)$, we obtain

$$I_\nu(x) = \frac{\sin(\nu\pi)}{\nu\pi} \Pi(-\nu)(x/2)^\nu \left\{ 1 + \frac{(x/2)^2}{1+\nu} + \frac{(x/2)^4}{2!(1+\nu)(2+\nu)} + \dots \right\}. \quad (2.3)$$

and for $\nu=is$, where s is real, we obtain

$$I_{is}(x) = \frac{\sinh(s\pi)}{s\pi} \Pi(-is)(x/2)^{is} \left\{ 1 + \frac{(x/2)^2}{1+is} + \frac{(x/2)^4}{2!(1+is)(2+is)} + \dots \right\}. \quad (2.4)$$

Here, we put

$$\Pi(-is) = A + iB, \quad (2.5)$$

$$F(s, x) = 1 + \frac{(x/2)^2}{1+is} + \frac{(x/2)^4}{2!(1+is)(2+is)} + \dots = C - iD. \quad (2.6)$$

Hence we obtain

$$I_{is}(x) = \frac{\sinh(s\pi)}{s\pi} (A + iB) e^{is \log(x/2)} (C - iD), \quad (2.7)$$

and

$$I_{-is}(x) = \frac{\sinh(s\pi)}{s\pi} (A - iB) e^{-is \log(x/2)} (C + iD). \quad (2.8)$$

Substituting (2. 7) and (2. 8) into (2. 1), we obtain

$$\begin{aligned}
 K_{is}(x) &= \frac{(\pi/2)}{i \sinh(s\pi)} \{I_{-is}(x) - I_{is}(x)\} \\
 &= -\frac{1}{s} \{\sin \alpha(AC + BD) + \cos \alpha(BC - AD)\}, \tag{2.9}
 \end{aligned}$$

and

$$\begin{aligned}
 M_{is}(x) &= \frac{(\pi/2)}{\cosh(s\pi)} \{I_{-is}(x) + I_{is}(x)\} \\
 &= \frac{\tanh(s\pi)}{s} \{\cos \alpha(AC + BD) - \sin \alpha(BC - AD)\}, \tag{2.10}
 \end{aligned}$$

where $\alpha = s \log(x/2)$.

When $s \gg 1 \gg x$, $F(s, x) \simeq 1$ and $\Pi(is)$ is expressed as follows

$$\Pi(is) \simeq (is/e)^{is} (2\pi is)^{1/2} = (2\pi e)^{1/2} e^{-s\pi/2} e^{i(\pi/4 - s \log s - s)} \tag{2.11}$$

from the Stirling's formula.

Hence, we obtain for $s \gg 1 \gg x$

$$K_{is}(x) = (2\pi/s)^{1/2} e^{-s\pi/2} \sin(\pi/4 - s + s \log(2s/x)), \tag{2.12}$$

$$M_{is}(x) \simeq (2\pi/s)^{1/2} e^{-s\pi/2} \cos(\pi/4 - s + s \log(2s/x)). \tag{2.13}$$

The method of computing the values of $K_{is}(x)$ and $M_{is}(x)$ given above is discussed later.

This method is available for the region of oscillation of $K_{is}(x)$ and $M_{is}(x)$, i.e., $s > x$.

2.2 The Hankel Asymptotic Series

From Erdelyi⁷⁾, we get

$$\begin{aligned}
 I_\nu(z) &= (2\pi z)^{-1/2} \left[e^z \left\{ \sum_{m=0}^{M-1} (-1)^m (v, m) (2z)^{-m} + O(|z|^{-M}) \right\} \right. \\
 &\quad \left. + i e^{-z + i\nu\pi} \left\{ \sum_{m=0}^{M-1} (v, m) (2z)^{-m} + O(|z|^{-M}) \right\} \right], \quad -\pi/2 < \arg z < 3\pi/2, \tag{2.14}
 \end{aligned}$$

where

$$(v, m) = 2^{-2m} (4v^2 - 1)(4v^2 - 3^2)(4v^2 - 5^2) \dots [4v^2 - (2m - 1)^2] / m!. \tag{2.15}$$

From the definitions of $K_{is}(x)$ and $M_{is}(x)$ and substituting $\nu = is$ into (2.14) and (2.15), we obtain

$$\begin{aligned}
 K_{is}(x) &= (\pi/2x)^{1/2} e^{-x} \left\{ 1 - \frac{4s^2 + 1}{1!8x} + \frac{(4s^2 + 1)(4s^2 + 3^2)}{(8x)^2 2!} \right. \\
 &\quad \left. - \frac{(4s^2 + 1)(4s^2 + 3^2)(4s^2 + 5^2)}{(8x)^3 3!} + \dots \right\} \tag{2.16}
 \end{aligned}$$

and

$$M_{is}(x) = \frac{1}{\cosh(s\pi)} (\pi/2x)^{1/2} e^x \left\{ 1 + \frac{4s^2 + 1}{1!8x} + \frac{(4s^2 + 1)(4s^2 + 3^2)}{(8x)^2 2!} + \dots \right\} \quad (2.17)$$

2.3 Other Asymptotic Expansions

The following asymptotic expansion are given in reference⁷⁾

$$K_{is}(x) = 2^{-1/2} (x^2 - s^2)^{-1/4} \exp[-(x^2 - s^2)^{1/2} - s \sin^{-1}(s/x)] \\ \times \left[\sum_{m=0}^{M-1} (-2)^m b_m \Gamma\left(m + \frac{1}{2}\right) (x^2 - s^2)^{-m/2} + O(x^{-M}) \right], \quad x > s > 0, \quad (2.18)$$

where

$$b_0 = 1, \quad b_1 = \frac{1}{8} - \frac{5}{24} (1 - x^2/s^2)^{-1}, \\ b_2 = \frac{3}{128} - \frac{77}{576} (1 - x^2/s^2)^{-1} + \frac{385}{3456} (1 - x^2/s^2)^2, \quad (2.19)$$

$$K_{is}(x) = 2^{+1/2} (s^2 - x^2)^{-1/4} e^{-s\pi/2} \times \left[\sum_{m=0}^{M-1} 2^m b_m \Gamma(m + 1/2) (s^2 - x^2)^{-m/2} \right. \\ \left. \times \sin\{m\pi/2 + s \cosh^{-1}(s/x) - (s^2 - x^2)^{1/2} + \pi/4\} + O(x^{-M}) \right], \quad s > x > 0, \quad (2.20)$$

$$K_{is}(x) \approx \pi/3 e^{-s\pi/2} \sum_{m=0}^{\infty} (-1)^m C_m(\varepsilon x) \sin\{(m+1)\pi/3\} \times \Gamma(m/2 + 1/3) (x/6)^{-(m+1)/3}, \\ \text{for } s > x, \quad s, x > 0, \quad \varepsilon = 1 - s/x, \quad \varepsilon = O(x^{-2/3}), \quad (2.21)$$

where

$$C_0(X) = 1, \quad C_1(X) = X, \quad C_2(X) = X^2/2 + \frac{1}{20}, \quad C_3(X) = \frac{X^3}{6} + \frac{X}{15}, \\ C_4(x) = \left(X^4 - X^2 + \frac{1}{20} \right) \frac{1}{24}, \quad C_5(X) = \frac{X^5}{120} + \frac{X^3}{60} + \frac{43X}{4800}. \quad (2.22)$$

The expression of $M_{is}(x)$ corresponding to (2. 18) is

$$M_{is}(x) = \frac{1}{\cosh(s\pi)} 2^{+1/2} (x^2 - s^2)^{-1/4} \exp\{(x^2 - s^2)^{1/2} + s \sin^{-1}(s/x)\} \\ \times \left[\sum_{m=0}^{M-1} 2^m b_m \Gamma(m + 1/2) (x^2 - s^2)^{-m/2} + O(x^{-M}) \right]. \quad (2.23)$$

§ 3 Numerical Computations Based on the Hankel Asmptotic Series

It is clear that the individual terms of (2. 16) and (2. 17) at first decrease and begin to increase after number of terms exceeds a certain number N_m . The number N_m is determined as the maximum value of N which fulfils the condition

$$\frac{4s^2 + (2N-1)^2}{8xN} < 1. \quad (3.1)$$

It is

$$N_m = [0.5 + x + \sqrt{x^2 + x - s^2}]. \quad (3.2)$$

For (2. 16) and (2.17) to be significant, they must be as follows

$$K_{is}(x) = (\pi/2x)^{1/2} e^{-x} \left[1 + \sum_{n=1}^{n \leq N_m} \frac{(4s^2 + 1)(4s^2 + 3^2) \dots \{4s^2 + (2n-1)^2\}}{n!(-8x)^n} \right], \quad (3.3)$$

$$M_{is}(x) = \frac{1}{\cosh(s\pi)} (\pi/2x)^{1/2} e^x \left[1 + \sum_{n=1}^{n \leq N_m} \frac{(4s^2 + 1)(4s^2 + 3^2) \dots \{4s^2 + (2n-1)^2\}}{n!(8x)^n} \right]. \quad (3.4)$$

There is an upper limit on the accuracy obtained by the use of (3. 3) and (3. 4). From preliminary numerical experiments, we can roughly conclude that this method based on the Hankel asymptotic series (3. 3) and (3. 4) yields the accuracy of eight decimal places for

$$\begin{aligned} x &\geq 1.5s + 8.0 && \text{for } s \leq 6, \\ x &\geq 2.0s + 5.0 && \text{for } s \geq 6. \end{aligned} \quad (3.5)$$

In Table 1, the number of terms necessary to obtain the accuracy of eight decimal places by the use of the Hankel asymptotic series is given.

Table 1. Numbers of terms necessary to obtain the accuracy of eight decimal places by the use of the Hankel asymptotic series (3.3).

| $x \backslash s$ | 0.0 | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------|-----|-----|----|-----|----|----|----|----|----|----|----|
| 9 | 13 | 16 | | | | | | | | | |
| 10 | 11 | 12 | 15 | | | | | | | | |
| 11 | 10 | 11 | 13 | 16 | | | | | | | |
| 12 | 9 | 10 | 11 | 13 | 16 | | | | | | |
| 13 | 8 | 9 | 10 | 12 | 14 | 21 | | | | | |
| 14 | 8 | 9 | 10 | 11 | 13 | 17 | | | | | |
| 15 | 8 | 8 | 9 | 10 | 12 | 15 | 21 | | | | |
| 16 | 7 | 8 | 9 | 10 | 11 | 14 | 18 | 26 | | | |
| 17 | 7 | 8 | 8 | 9 | 11 | 13 | 17 | 22 | | | |
| 18 | 7 | 7 | 8 | 9 | 10 | 12 | 15 | 19 | 26 | | |
| 19 | 7 | 7 | 8 | 9 | 10 | 12 | 15 | 18 | 23 | 33 | |
| 20 | 6 | 7 | 8 | 9 | 9 | 11 | 14 | 17 | 21 | 27 | |
| 21 | 6 | 7 | 7 | 8 | 9 | 10 | 14 | 17 | 19 | 21 | 38 |
| 22 | 6 | 6 | 7 | 8 | 9 | 10 | 13 | 16 | 17 | 19 | 30 |

§ 4 Numerical Computations Based on the Series Expansions Available for Large Order

In this section a method based on (2. 9) and (2. 10) is discussed. It consists of two parts, one is the estimation of a complex series $F(s, x)$ and other is the estimation of the Pai function of imaginary variable $\Pi(is)$.

4.1 Estimation of a Complex Series $F(s, x)$

A complex series

$$F(s, x) = 1 + \frac{(x/2)^2}{1+is} + \frac{(x/2)^4}{2!(1+is)(2+is)} + \frac{(x/2)^6}{3!(1+is)(2+is)(3+is)} + \dots \quad (4.1)$$

converges for any real s and x although it increases rapidly as x increases. As the number of terms n increases, the values of individual terms increase at first but they begin to decrease when n exceeds the value

$$n_0 = [(\sqrt{s^4 + x^4/4} - s^2)/2]^{1/2}.$$

In Fig. 1, the numbers of the terms necessary to obtain the accuracy of ten-decimal places are shown. In the figure the numbers don't depend on s strongly, hence we put

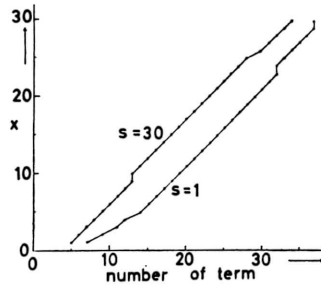


Fig. 1. Numbers of terms necessary to estimate $F(s, x)$ with the accuracy of ten decimal places.

$$n = 6 + [2x] \quad x < 5.0, \quad (4.2)$$

$$n = 10 + [x] \quad x \geq 5.0.$$

Using the value n given by (4.2), $F(s, x)$ is computed by

$$F(s, x) = 1 + \sum_{p=1}^n \frac{(x/2)^{2p}}{p!(1+is)(2+is)(3+is)\dots(p+is)} \quad (4.3)$$

4.2 Estimation of Pai function of Purely Imaginary Variable

A method to evaluate the Pai function will be to use Stirling's formula. The formula is given⁸⁾ as follows

$$\begin{aligned} \Pi(z) = \Gamma(z+1) = (z/e)^z (2\pi z)^{1/2} & \left[1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} + \frac{571}{2488320z^4} \right. \\ & \left. + \frac{1}{2090} \frac{63879}{18880z^5} + \frac{52}{7} \frac{46819}{52467} \frac{1}{96800z^6} - \frac{5347}{90} \frac{03531}{29615} \frac{1}{61600z^7} + \dots \right]. \end{aligned} \quad (4.4)$$

When $z = is$, we get

$$\Pi(is) = (2\pi s)^{1/2} e^{-s\pi/2} e^{i(\pi/4 - s \log s - s)} \left\{ \left(1 - \frac{1}{288s^2} + \frac{571}{24} \frac{1}{88320s^4} + \frac{52}{7} \frac{46819}{52467} \frac{1}{96800s^6} \right) \right\}$$

$$-i\left(\frac{1}{12s} + \frac{139}{51840s^3} - \frac{1\ 63879}{2090\ 18880s^5} + \frac{5347\ 03531}{90\ 29615\ 61600s^7}\right)\}. \tag{4.5}$$

When s is small, (4. 5) does not yield high accuracy. Then it must be reduced to the form which yields high accuracy when we use the Stirling's formula.

From the recurrence formula of Gamma function, we obtain

$$\begin{aligned} \Pi(is) &= \frac{\Gamma(7+is)}{(1+is)(2+is)(3+is)(4+is)(5+is)(6+is)} \\ &= \frac{\Gamma(7+is)}{(720 - 1624s^2 + 175s^4 - s^6) + is(1764 - 735s^2 + 21s^4)} \end{aligned} \tag{4.6}$$

where the part of Gamma function $\Pi(7+is)$ in (4. 6) is computed by the Stirling's formula (4. 4). In this case, the argument is complex $z=6+is$ hence additional complex division is necessary.

This method yeilds as high accuracy as required if we use the recurrence formula repeatedly. Another method of evaluating the Pai function is to use the Taylor series expansions. Following formula is given by Luke⁹⁾

$$[\Gamma(z+1)]^{-1} = \sum_{n=0}^{\infty} a_n z^n, \quad |z| < \infty, \tag{4.7}$$

| n | a_n | | | |
|-----|----------|-------|-------|-------|
| 0 | 1.00000 | 00000 | 00000 | 00000 |
| 1 | 0.57721 | 56649 | 01532 | 86061 |
| 2 | -0.65587 | 80715 | 20253 | 88108 |
| 3 | -0.04200 | 26350 | 34095 | 23553 |
| 4 | 0.16653 | 86113 | 82291 | 48950 |
| 5 | -0.04219 | 77345 | 55544 | 33675 |
| 6 | -0.00962 | 19715 | 27876 | 97356 |
| 7 | 0.00721 | 89432 | 46663 | 09954 |
| 8 | -0.00116 | 51675 | 91859 | 06511 |
| 9 | -0.00021 | 52416 | 74114 | 95097 |
| 10 | 0.00012 | 80502 | 82388 | 11619 |
| 11 | -0.00002 | 01348 | 54780 | 78824 |
| 12 | -0.00000 | 12504 | 93482 | 14267 |
| 13 | 0.00000 | 11330 | 27231 | 98170 |
| 14 | -0.00000 | 02056 | 33841 | 69776 |
| 15 | 0.00000 | 00061 | 16095 | 10448 |
| 16 | 0.00000 | 00050 | 02007 | 64447 |
| 17 | -0.00000 | 00011 | 81274 | 57049 |
| 18 | 0.00000 | 00001 | 04342 | 67117 |
| 19 | 0.00000 | 00000 | 07782 | 26344 |
| 20 | -0.00000 | 00000 | 03696 | 80562 |
| 21 | 0.00000 | 00000 | 00510 | 03703 |
| 22 | -0.00000 | 00000 | 00020 | 00020 |
| 23 | -0.00000 | 00000 | 00005 | 34812 |
| 24 | 0.00000 | 00000 | 00001 | 22678 |
| 25 | -0.00000 | 00000 | 00000 | 11813 |
| 26 | 0.00000 | 00000 | 00000 | 00119 |
| 27 | 0.00000 | 00000 | 00000 | 00141 |
| 28 | -0.00000 | 00000 | 00000 | 00023 |
| 29 | 0.00000 | 00000 | 00000 | 00002 |

Using (4. 7), we obtain

$$II(is) = \Gamma(is + 1) = \left\{ \sum_{n=0}^{14} (a_{2n} + is a_{2n+1}) (-s^2)^n \right\}^{-1}. \quad (4.8)$$

In Table 2, the values obtained by above three methods based on (4. 5), (4. 6) and (4. 8) are compared with the exact values. In the table the absolute values of Pai function are listed to the decimal places where the discrepancy appears.

Table 2 Comparison of three methods to compute the Pai function of imaginary vairable $II(is)$

| s | by (4.5) | by (4.6) | by (4.8) | Exact values by (4.9) |
|------|-------------------|--------------------|------------------------|-------------------------|
| 0.5 | 0.82 | 0.82617 76142 | 0.82617 76142 76045 23 | 0.82617 76142 76045 232 |
| 1.0 | 0.521 | 0.52156 40468 65 | 0.52156 40468 64939 8 | 0.52156 40468 64939 849 |
| 1.5 | 0.2909 | 0.29098 51478 2 | 0.29098 51478 15861 | 0.29098 51478 15861 831 |
| 2.0 | 0.15318 9 | 0.15318 96187 9 | 0.15318 96187 9124 | 0.15318 96187 91234 621 |
| 3.0 | 0.39001 924 | 0.39001 92404 4 | 0.39001 924 | 0.39001 92404 47059 543 |
| 4.0 | 0.93619 6951 | 0.93619 69505 1 | 0.93619 | 0.93619 69505 16372 486 |
| 5.0 | 0.21758 75548 | 0.21758 75548 18 | 0.217 | 0.21758 75548 18708 343 |
| 6.0 | 0.49549 18299 | 0.49549 18298 882 | 0.5 | 0.49549 18298 88537 677 |
| 7.0 | 0.11125 55578 66 | 0.11125 55578 647 | | 0.11125 55578 64641 210 |
| 8.0 | 0.24724 61355 48 | 0.24724 61355 477 | | 0.24724 61355 47541 997 |
| 10.0 | 0.11945 60541 104 | 0.11945 60541 1036 | | 0.11945 60541 10345 570 |

The absolute value of Pai function is computed accurately by

$$|II(is)|^2 = \Gamma(1 + is)\Gamma(1 - is) = \frac{\pi s}{\sinh(\pi s)}. \quad (4.9)$$

The computing times of the three methods based on (4. 5), (4. 6) and (4. 8) are less than 1, 4 and 2 msec, respectively, according to the results directly measured. Hence we can conclude as follows

$$II(is) \text{ must be computed } \begin{cases} \text{by (4. 8) for } & s \leq 2.0, \\ \text{by (4. 6) for } & 2.0 < s < 7.0, \\ \text{by (4. 5) for } & s \geq 7.0. \end{cases} \quad (4.10)$$

In Fig. 2, the behavior of $II(is)$ is shown.

4.3 Compensation of Cancellation

Now we can obtain the values of $K_{is}(x)$ and $M_{is}(x)$ from the values of $F(s, x)$ and $II(is)$ computed by the methods discussed heretofore. The values of $K_{is}(x)$ and $M_{is}(x)$ are accurate for $s > x$ or for a region of oscillation. However for large x , the values of $K_{is}(x)$ are not accurate because of a violent cancellation. The reason of the violent cancellation in the computation of $K_{is}(x)$ is easily understandable from Fig. 3. After the oscillations end, $M_{is}(x)$ increases rapidly while $K_{is}(x)$ tends to zero monotonously as x increases.

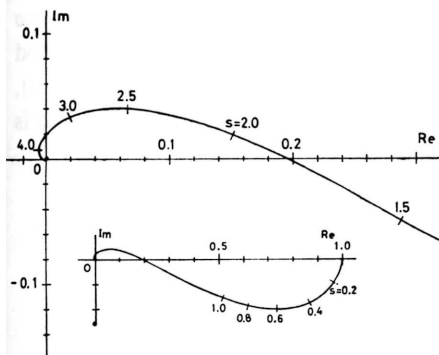


Fig. 2. Behaviors of $H(is)$.

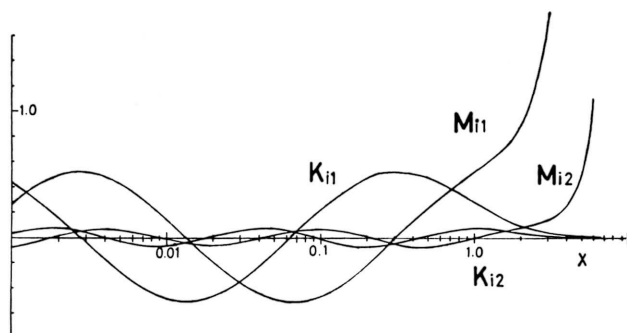


Fig. 3. Behaviors of $K_{is}(x)$ and $M_{is}(x)$.

Such a cancellation can be roughly estimated by comparing the values of $K_{is}(x)$ and $M_{is}(x)$. The estimations of the number of decimal places lost by cancellation are listed in Table 3.

Table 3 The number of decimal places lost in the computation of $K_{is}(x)$ by (2.9).

| $s \backslash x$ | 0.5 | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
|------------------|-----|---|---|---|---|---|---|----|----|----|----|----|----|----|
| 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.5 | 1 | 0 | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | |
| 2 | 1 | 1 | 0 | | | | | | | | | | | |
| 3 | 2 | 2 | 1 | 0 | | | | | | | | | | |
| 4 | 3 | 3 | 2 | 1 | 0 | | | | | | | | | |
| 5 | 4 | 3 | 2 | 1 | 1 | 0 | | | | | | | | |
| 6 | 4 | 4 | 4 | 2 | 2 | 0 | | | | | | | | |
| 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | | | | | | | |
| 8 | 6 | 6 | 4 | 4 | 3 | 1 | 0 | | | | | | | |
| 9 | 8 | 6 | 6 | 4 | 3 | 2 | 0 | | | | | | | |
| 10 | | 7 | 6 | 6 | 5 | 2 | 1 | 0 | | | | | | |
| 12 | | | 7 | 6 | 6 | 3 | 2 | 1 | 0 | | | | | |
| 14 | | | | 7 | 6 | 4 | 3 | 2 | 1 | 0 | | | | |
| 16 | | | | | 8 | 6 | 4 | 3 | 2 | 1 | 0 | | | |
| 18 | | | | | | 8 | 6 | 5 | 4 | 2 | 1 | 0 | | |
| 20 | | | | | | | 8 | 6 | 4 | 3 | 2 | 1 | 0 | |
| 22 | | | | | | | | 8 | 6 | 5 | 3 | 2 | 1 | 0 |
| 24 | | | | | | | | | 9 | 6 | 4 | 3 | 1 | 0 |
| 26 | | | | | | | | | | 9 | 7 | 4 | 3 | 2 |
| 28 | | | | | | | | | | | 9 | 7 | 5 | 3 |
| 30 | | | | | | | | | | | | 11 | 8 | 7 |

From the Table 3, the numbers of decimal places to be compensated are at most six for $s < 9$ and nine for $s > 9$. Hence if $F(s, x)$ and $H(is)$ are evaluated at the accuracy of seventeen decimal places, the cancellation will be compensated and the accuracy of eight decimal places will be obtained.

To evaluate the complex series $F(s, x)$ with high accuracy, we have only to continue the computation beyond n given by (4. 2) until the required accuracy is obtained. A method to compute the values of $\Pi(is)$ with high accuracy will be to modify the method based on (4. 6), *i.e.*, to apply the recurrence formula of Gamma function repeatedly. A following method is examined:

$$\Pi(is) = \frac{\Gamma(N+1+is)}{(1+is)(2+is)(3+is)\cdots(N+is)}, \quad (4.11)$$

where

$$N = \begin{cases} 30 & \text{for } s < 11 \\ 24 & \text{for } s < 20 \\ 18 & \text{for } s < 25 \\ 10 & \text{for } s < 30. \end{cases} \quad (4.12)$$

A procedure based on (4. 11) can compute the values of $\Pi(is)$ accurately to at least 15 decimal digits for $0 < s < 30$. By the use of (4. 11) and by evaluating $F(s, x)$ with high accuracy the revised computation were carried out. The results are given in Table 4 together with the values obtained by the Hankel asymptotic series (2. 16). For small s less than about 10, the cancellations are compensated by 5 or 6 decimal digits and the range where the accuracy of

Table 4 The values of $K_{\epsilon, \epsilon}(x)$ computed by several methods.

| $s=0.5$ x | by (2.9) | by revised (2.9) | by (2.16) | by a method using an integral representation |
|----------------|-------------|------------------|-------------|---|
| 3.0 | 0.33495 853 | | | 0.33495 853 E-1 |
| 3.5 | 0.18986 305 | | | 0.18986 305 E-1 |
| 4.0 | 0.10850 043 | | 0.1085 | 0.10850 042 E-1 |
| 4.5 | 0.62401 86 | 0.62401 847 | 0.62401 | 0.62401 847 E-2 |
| 5.0 | 0.36074 28 | 0.36074 271 | 0.36074 | 0.36074 217 E-2 |
| 6.0 | 0.12201 5 | 0.12201 479 | 0.12201 5 | 0.12201 479 E-2 |
| 7.0 | 0.41775 | 0.41774 012 | 0.41774 02 | 0.41774 012 E-3 |
| 8.0 | 0.14432 | 0.14432 424 | 0.14432 424 | 0.14432 424 E-3 |
| 9.0 | 0.502 | 0.50214 132 | 0.50214 132 | 0.50214 132 E-4 |
| 10.0 | 0.176 | 0.17569 102 | 0.17569 108 | 0.17569 108 E-4 |
| 12.0 | 0.1 | 0.21788 8 | 0.21788 865 | 0.21788 865 E-5 |
| $s=5.0$ x | by (2.9) | by revised (2.9) | by (2.16) | by a method using an integral representation |
| 4.0 | 0.48966 527 | | | 0.48966 527 E-3 |
| 5.0 | 0.31859 102 | | | 0.31859 103 E-3 |
| 6.0 | 0.16387 417 | | | 0.16387 417 E-3 |
| 7.0 | 0.75060 449 | | | 0.75060 449 E-4 |
| 8.0 | 0.32161 473 | | 0.3 | 0.32161 473 E-4 |
| 9.0 | 0.13213 430 | 0.13273 431 | 0.132 | 0.13213 431 E-4 |
| 10.0 | 0.52781 2 | 0.52781 218 | 0.528 | 0.52781 218 E-5 |
| 12.0 | 0.79817 | 0.79817 117 | 0.79817 2 | 0.79817 117 E-6 |
| 14.0 | 0.115 | 0.11554 514 | 0.11554 516 | 0.11554 514 E-6 |
| 16.0 | 0.2 | 0.16303 194 | 0.16303 194 | 0.16303 194 E-7 |
| 18.0 | | 0.22636 742 | 0.22636 742 | 0.22636 742 E-8 |
| 20.0 | | 0.31100 7 | 0.31100 591 | 0.31100 591 E-9 |

8 decimal places attained is enlarged and is connected continuously to the range computable by (2. 16).

However for s larger than about 10, the situation becomes different. In this case, the range exists where the accuracy of 8 decimal places is not obtained by revised (2. 9) and also by (2. 16). The range is expressed by following simplified form

$$\begin{aligned} x < 2.0s + 5.0 \\ x > s + 11. \end{aligned} \tag{4. 13}$$

In this range, the accuracy is reduced to 5 or 6 decimal places.

The computation of $K_{is}(x)$ in this range is computed by Debye's asymptotic expansion (2. 18) which is discussed in next sub-section.

For the values of $M_{is}(x)$, the cancellation does not occur. In Table 5, the values of real part of $I_{is}(x)$ computed by (2. 10), (2. 17) and method based on an integral representation are shown. The real part of $I_{is}(x)$ is equal to $M_{is}(x)$ if it is multiplied by $\pi/\cosh(s\pi)$ as stated in introduction.

Table 5 Real part of $I_{is}(x)$ computed by (2.10), (2.17) and method based on an integral representation.

| $s=0.05$ x | by (2.10) | by (2.17) | by a method using an integral representation |
|-----------------|--------------|-------------|--|
| 2.0 | 0.22816 1621 | 0.228 | 0.22816 1621 E 1 |
| 3.0 | 0.48834 1972 | 0.488 | 0.48834 1971 E 1 |
| 4.0 | 0.11306 1254 | 0.11306 | 0.11306 1254 E 2 |
| 5.0 | 0.27247 6070 | 0.27247 | 0.27247 6070 E 2 |
| 6.0 | 0.67249 8918 | 0.67249 9 | 0.67249 8917 E 2 |
| 7.0 | 0.16862 6619 | 0.16862 663 | 0.16862 6619 E 3 |
| 8.0 | 0.42763 5832 | 0.42763 583 | 0.42763 5831 E 3 |
| 9.0 | 0.10937 5002 | 0.10937 499 | 0.10937 5002 E 4 |
| 10.0 | 0.28160 8852 | 0.28160 885 | 0.28160 8852 E 4 |
| $s=0.5$ x | by (2.10) | by (2.17) | by a method using an integral representation |
| 3.0 | 0.51518 4791 | 0.515 | 0.51518 4790 E 1 |
| 4.0 | 0.11731 5663 | 0.1173 | 0.11731 5663 E 2 |
| 5.0 | 0.28025 8537 | 0.28026 | 0.28025 8536 E 2 |
| 6.0 | 0.68802 3969 | 0.68802 | 0.68802 3968 E 2 |
| 7.0 | 0.17189 8908 | 0.17189 89 | 0.17180 8908 E 3 |
| 8.0 | 0.43479 9165 | 0.43479 917 | 0.43479 9164 E 3 |
| 9.0 | 0.11098 7301 | 0.11098 730 | 0.11098 7301 E 4 |
| 10.0 | 0.28531 6075 | 0.28531 607 | 0.28531 6074 E 4 |
| $s=2.0$ x | by (2.10) | by (2.17) | by a method using an integral representation |
| 4.0 | 0.21109 0816 | 0.212 | 0.21109 0816 E 2 |
| 5.0 | 0.43539 4202 | 0.435 | 0.43539 4201 E 2 |
| 6.0 | 0.97868 4534 | 0.9788 | 0.97868 4532 E 2 |
| 7.0 | 0.23080 8550 | 0.23081 | 0.23080 8549 E 3 |
| 8.0 | 0.56039 5570 | 0.56039 7 | 0.56039 5568 E 3 |
| 9.0 | 0.13872 0851 | 0.13872 09 | 0.13872 0851 E 4 |
| 10.0 | 0.34816 2792 | 0.34816 28 | 0.34816 2792 E 4 |

§ 5 Possibilities of Methods Based on the Other Asymptotic Expansions

5.1 Debye's Asymptotic Expansion (2. 18)

The steepest descent method to obtain the coefficients b_m in (2. 18) is elucidated by several authors⁷⁾¹⁰⁾¹¹⁾. W. Sibagaki¹¹⁾ has given the coefficients from b_0 to b_4 as follows

$$\begin{aligned} b_0 &= 1, \quad b_1 = \frac{1}{8} \left(1 - \frac{5}{3} \lambda \right), \quad b_2 = \frac{1}{8^2} \left(\frac{3}{2} - \frac{77}{9} \lambda + \frac{385}{54} \lambda^2 \right), \\ b_3 &= \frac{1}{8^3} \left(\frac{5}{2} - \frac{1521}{50} + \frac{17017}{270} \lambda^2 - \frac{17017}{486} \lambda^3 \right), \\ b_4 &= \frac{1}{8^4} \left(\frac{35}{8} - \frac{96833}{105} \lambda + \frac{144001}{420} \lambda^2 - \frac{1062347}{2430} \lambda^3 + \frac{1062347}{5832} \lambda^4 \right), \end{aligned} \quad (5.1)$$

where $\lambda = (1 - x^2/s^2)^{-1}$

If we write the m -th coefficient in the form

$$b_m = \sum_{n=0}^m B_{m,n} (-\lambda)^n, \quad (5.2)$$

we obtain the recurrence formula

$$\begin{aligned} B_{0,0} &= 1, \quad B_{m,n} = 0 \quad (\text{for } n > m \text{ or } n < 0), \\ B_{m+1,n+1} &= \frac{(2k+5)}{(2m+1)(k+3)} \{ (2k+1)B_{m,n} + (2k+5)B_{m,n+1} \} \frac{1}{8}, \\ k &= m + 2n. \end{aligned} \quad (5.3)$$

The expressions (2. 18) and (2. 23) can be written as follows

$$K_{is}(x) = \sqrt{\frac{\pi}{2\beta}} \exp\{-\beta - s \sin^{-1}(s/x)\} \left\{ 1 - \frac{1}{\beta} b_1 + \frac{3}{\beta^2} b_2 - \frac{1 \cdot 3 \cdot 5}{\beta^3} b_3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{\beta^4} b_4 - \right\}, \quad (5.4)$$

$$M_{is}(x) = \frac{1}{\cosh(s\pi)} \sqrt{\frac{\pi}{2\beta}} \exp\{\beta + s \sin^{-1}(s/x)\} \left\{ 1 + \frac{1}{\beta} b_1 + \frac{1 \cdot 3}{\beta^2} b_2 + \frac{1 \cdot 3 \cdot 5}{\beta^3} b_3 \right\}, \quad (5.5)$$

where $\beta = (x^2 - s^2)^{1/2}$. As these series are asymptotic expansions, in practical calculation the series must be stopped when the terms no longer decrease. In Table 6, the numbers of terms necessary to obtain the accuracy of eight decimal digits by (5. 4) are shown.

This table shows that for instance when $s=1$ and $x=10$, we must estimate b_0 to b_{13} to obtain the accuracy of eight decimal digits. The applicable region of (5. 4) is slightly wider than that of the Hankel's asymptotic expansion (2. 16) previously discussed. Considering the fact that the computation of b_n becomes too time-consuming as n increases, the computing times are always larger than that of (2. 16).

The merit of the Debye asymptotic expansion is the wider applicable region and this point

Table 6 Numbers of terms necessary to obtain the accuracy of eight decimal digits by Debye's series (2.18) or (5.4)

| $x \backslash s$ | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|------------------|----|----|----|----|----|----|----|----|----|----|----|----|
| 10 | 13 | | | | | | | | | | | |
| 12 | 9 | 11 | | | | | | | | | | |
| 14 | 8 | 9 | 10 | 14 | | | | | | | | |
| 16 | 7 | 8 | 8 | 10 | | | | | | | | |
| 18 | 7 | 7 | 8 | 8 | 10 | | | | | | | |
| 20 | 7 | 7 | 7 | 7 | 9 | 11 | | | | | | |
| 22 | 6 | 6 | 6 | 7 | 8 | 9 | 12 | | | | | |
| 24 | 6 | 6 | 6 | 6 | 7 | 8 | 9 | 14 | | | | |
| 26 | 6 | 6 | 6 | 6 | 7 | 7 | 8 | 10 | | | | |
| 28 | 6 | 6 | 6 | 6 | 6 | 7 | 7 | 8 | 10 | | | |
| 30 | 5 | 6 | 6 | 6 | 6 | 6 | 7 | 8 | 9 | 11 | | |
| 35 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 8 | 9 | 10 |
| 40 | 5 | 5 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 | 7 | 7 |

is emphasized when the high accuracy is not required.

The part of the region where the accuracy of eight decimal digits is not obtained by (2. 16) or revised (2. 9) is covered by these Debye's asymptotic expansions.

5.2 Asymptotic Expansions Available for $x \sim s$.

The asymptotic expansion (2. 21), which is also derived by Debye, is available for $x \sim s$. The higher coefficients $C_n(X)$ are given by Airey²⁾ as follows:

$$\begin{aligned}
 C_6(X) &= \frac{X^6}{720} - \frac{7X^4}{1440} + \frac{X^2}{288} - \frac{1}{3600}, \\
 C_7(X) &= \frac{X^7}{5040} - \frac{X^5}{900} + \frac{19X^3}{12600} - \frac{13X}{31500}, \\
 C_9(X) &= \frac{X^9}{362880} - \frac{X^7}{30240} + \frac{71X^5}{604800} - \frac{121X^3}{907200} + \frac{7939X}{2328480000},
 \end{aligned}
 \tag{5.6}$$

for the calculation, C_2, C_5, C_8 etc. are not necessary.

This asymptotic expansion (2. 21) is not suitable for the calculation with high accuracy. Because even for the case of $s=x$, it is the most suitable case of (2. 21), the series is as follows

$$K_{is}(x) \simeq \frac{1}{3} e^{-s\pi/2} (6/x)^{1/3} \frac{\sqrt{3}}{2} \left\{ 1 - \Gamma\left(\frac{7}{3}\right) \frac{1}{280} (6/x)^{5/3} + \Gamma\left(\frac{10}{3}\right) \frac{1}{3600} (6/x)^2 \right\}, \tag{5.7}$$

the convergence of (5. 7) is very slow unless the variable x is extremely large. For example, for the case $s=x=30$, only 4 significant decimal digits are obtained by taking account of the coefficients from C_0 to C_9 .

5.3 Asymptotic Expansion Available for $s > x$.

Another asymptotic expansion involving the Debye's coefficients b_n (2. 20) is available in the region of oscillation. From the simple consideration, it is found that the convergence characteristic of this expansion is almost the same as that of (2. 18). Hence the data in Table

6 are used also for this expansion if we exchange s and x in the table.

It can be proved that the limiting form of this expansion for $s \gg x$ is coincides with (2. 12), *i.e.*, the corresponding limiting form of (2. 9). However this expansion is inferior to the series expansion (2. 9) at computing time, the applicable region and the complication of making program. For the calculation with low accuracy, this expansion will become available as used for evaluation of K_a in the previous paper³⁾.

§6 Derivatives of $K_{is}(x)$ and $M_{is}(x)$

If the Bessel functions of real order are known, the derivatives of the Bessel functions can be computed by the use of recurrence formula. For the case of pure imaginary order, however, the recurrence formula is not applicable because the Bessel functions of complex order $is \pm 1$ appear. Hence the procedure to compute the derivatives of $K_{is}(x)$ and $M_{is}(x)$ must be provided.

6.1 Series Expansions for $K'_{is}(x)$ and $M'_{is}(x)$

From (2. 4) and (2. 6), we obtain

$$I'_{is}(x) = i \frac{\sinh(s\pi)}{x} \Pi(-is)(x/2)^{is} G(s, x), \quad (6.1)$$

where

$$\begin{aligned} G(s, x) &= F(s, x) + F(s, x)' \frac{x}{is} \\ &= 1 + \left(1 + \frac{2}{is}\right) \frac{(x/2)^2}{1+is} + \left(1 + \frac{4}{is}\right) \frac{(x/2)^4}{2!(1+is)(2+is)} + \left(1 + \frac{6}{is}\right) \\ &\times \frac{(x/2)^6}{3!(1+is)(2+is)(3+is)} + \dots = E + iF. \end{aligned} \quad (6.2)$$

Hence

$$I'_{is}(x) = i \frac{\sinh(s\pi)}{x} (A + iB) e^{is \log(x/2)} (E + iF), \quad (6.3)$$

$$I'_{-is}(x) = -i \frac{\sinh(s\pi)}{x} (A - iB) e^{-is \log(x/2)} (E - iF), \quad (6.4)$$

Substituting (6. 3) and (6. 4) into the definitions of $K_{is}(x)$ and $M_{is}(x)$, we get

$$K'_{is}(x) = -x \{ \cos \alpha (AE - BF) - \sin \alpha (BE + AF) \}, \quad (6. 5)$$

$$M'_{is}(x) = -\tanh(s\pi)x \{ \cos \alpha (BE + AF) + \sin \alpha (AE - BF) \}. \quad (6. 6)$$

For the region of oscillation, *i.e.*, $s > x$, we obtain

$$K'_{is}(x) \simeq -\frac{(2\pi s)^{1/2}}{x} e^{-s\pi/2} \cos(\pi/4 - s + s \log(2s/x)), \quad (6.7)$$

$$M'_{is}(x) \simeq \frac{(2\pi s)^{1/2}}{x} e^{-s\pi/2} \sin(\pi/4 - s + s \log(2s/x)). \tag{6.8}$$

The derivatives of the asymptotic expansions (2. 16) and (2. 17) are

$$K'_{is}(x) = (\pi/2x)^{1/2} e^{-x} \left[-(1 + 1/2x) \left\{ 1 - \frac{4s^2 + 1}{1!8x} + \frac{(4s^2 + 1)(4s^2 + 3^2)}{2!(8x)^2} + \dots \right\} - 1/x \left\{ \frac{4s^2 + 1}{1!8x} - \frac{2(4s^2 + 1)(4s^2 + 3^2)}{2!(8x)^2} + \frac{3(4s^2 + 1)(4s^2 + 3^2)(4s^2 + 5^2)}{3!(8x)^3} - \dots \right\} \right], \tag{6.9}$$

$$M'_{is}(x) = \frac{1}{\cosh(s\pi)} (\pi/2x)^{1/2} e^x \left[(1 - 1/2x) \left\{ 1 + \frac{4s^2 + 1}{1!8x} + \frac{(4s^2 + 1)(4s^2 + 3^2)}{2!(8x)^2} \right\} + - 1/x \left\{ \frac{4s^2 + 1}{1!8x} + \frac{2(4s^2 + 1)(4s^2 + 3^2)}{2!(8x)^2} + \frac{3(4s^2 + 1)(4s^2 + 3^2)(4s^2 + 5^2)}{3!(8x)^3} + \dots \right\} \right]. \tag{6.10}$$

6.2 Computation of $K'_{is}(x)$ and $M'_{is}(x)$

For the region of oscillation, i.e., $s > x$, a method based on the series expansions (6. 5) and (6. 6) is available. This method is consists of the two parts, one is the estimation of the complex series $G(s, x)$ and the other is the estimation of Pai function of imaginary variable $\Pi(is)$. Although the convergence of $G(s, x)$ is slightly slower than that of $F(s, x)$ previously discussed, the estimation of $G(s, x)$ can be carried out by almost the same method used for the estimation of $F(s, x)$.

Similarly to the computation of $K_{is}(x)$, the computation of $K'_{is}(x)$ becomes difficult because of violent cancellation in the region of monotonous decay, i.e., $x > s$. For $x \gg s$, Hankel's asymptotic expansions (6. 9) and (6. 10) are available. For the remaining part of $s < x$, revised method based on (6. 9) and (6. 10), which is to estimate $\Pi(is)$ and $G(s, x)$ with high accuracy, is necessary.

Table 7 The values of $K'_{is}(x)$ computed by (6.5), (6.5) revised and the asymptotic expansion (6.9).

| $s=0.5$ x | by (6.5) | by (6.5) compensated | by (6.9) |
|----------------|--------------|----------------------|------------------|
| 4.0 | -0.12067 659 | -0.12067 660 | -0.1207 E-1 |
| 5.0 | -0.39376 38 | -0.39376 394 | -0.39378 E-2 |
| 6.0 | -0.13144 78 | -0.13144 801 | -0.13144 8 E-2 |
| 7.0 | -0.44569 3 | -0.44569 74 | -0.44569 74 E-3 |
| 8.0 | -0.15283 | -0.15283 981 | -0.15283 98 E-3 |
| 9.0 | -0.5285 | -0.52863 515 | -0.52863 516 E-4 |
| 10.0 | -0.184 | -0.18407 442 | -0.18407 442 E-4 |
| 12.0 | -0.22 | -0.22661 70 | -0.22661 695 E-5 |
| $s=5.0$ x | by (6.5) | by (6.5) compensated | by (6.9) |
| 8.0 | -0.28028 390 | -0.28028 390 | -0.25 E-4 |
| 9.0 | -0.11962 743 | -0.11962 742 | -0.118 E-4 |
| 10.0 | -0.49013 30 | -0.49013 281 | -0.489 E-5 |
| 12.0 | -0.76421 6 | -0.76421 536 | -0.76419 E-6 |
| 14.0 | -0.11253 | -0.11251 596 | -0.11251 6 E-7 |
| 16.0 | -0.160 | -0.16038 296 | -0.16038 296 E-8 |
| 18.0 | | -0.22414 345 | -0.22414 345 E-8 |

Similarly to the computation of $K_{is}(x)$, the region exists where the accuracy of eight decimal places is not obtained by (6. 9) or revised (6. 5). For the computation in this region, the Debye's asymptotic expansion must be used.

In Table 7, the values of $K'_{is}(x)$ computed by (6. 5), revised (6. 5) and asymptotic expansion (6. 9) are shown. In Table 8, the real part of $I'_{is}(x)$ computed by (6. 6) and (6. 10) are shown. The computation of the real part of $I'_{is}(x)$ is easier than that of the imaginary part of $I'_{is}(x)$, i.e., $K'_{is}(x)$ because that the cancellation does not occur.

Table 8 Real part of $I'_{is}(x)$.

| $s=0.05$ x | by (6.6) | by (6.10) | by a method using an integral representation |
|-----------------|--------------|--------------|--|
| 2.0 | 0.15909 2294 | 0.155 | 0.15909 2294 E 1 |
| 3.0 | 0.39543 5383 | 0.3945 | 0.39543 5382 E 1 |
| 4.0 | 0.97617 8470 | 0.9759 | 0.97617 8468 E 1 |
| 5.0 | 0.24340 7377 | 0.24340 | 0.24340 7377 E 2 |
| 6.0 | 0.61353 1564 | 0.61352 9 | 0.61353 1562 E 2 |
| 7.0 | 0.15606 4230 | 0.15606 41 | 0.15606 4230 E 3 |
| 8.0 | 0.39993 0508 | 0.39993 048 | 0.39993 0507 E 3 |
| 10.0 | 0.26713 0162 | 0.26713 0157 | 0.26713 0161 E 4 |
| $s=0.5$ x | by (6.6) | by (6.10) | by a method using an integral representation |
| 3.0 | 0.40520 0882 | 0.403 | 0.40520 0881 E 1 |
| 4.0 | 0.99932 7464 | 0.9987 | 0.99932 7462 E 1 |
| 5.0 | 0.24849 9254 | 0.24848 | 0.24849 9253 E 2 |
| 6.0 | 0.62473 8060 | 0.62473 | 0.42673 8056 E 2 |
| 7.0 | 0.15857 2968 | 0.15857 28 | 0.15857 2967 E 3 |
| 8.0 | 0.40565 1736 | 0.40565 168 | 0.40565 1735 E 3 |
| 10.0 | 0.27025 0555 | 0.27025 0555 | 0.27025 0554 E 4 |
| $s=2.0$ x | by (6.6) | by (6.10) | by a method using an integral representation |
| 4.0 | 0.13969 1060 | 0.135 | 0.13969 1060 E 2 |
| 5.0 | 0.33764 0222 | 0.336 | 0.33764 0222 E 2 |
| 6.0 | 0.82042 3066 | 0.8199 | 0.82042 3064 E 2 |
| 7.0 | 0.20187 8399 | 0.20186 | 0.20187 8398 E 3 |
| 8.0 | 0.50318 1128 | 0.50317 6 | 0.50318 1127 E 3 |
| 10.0 | 0.32234 1506 | 0.32234 145 | 0.32234 1505 E 4 |

§ 7 Methods based on Integral Representations

To check the accuracy of the values obtained by the methods heretofore discussed, different computing methods based on the integral representations are provided.

From Watson¹⁰⁾

$$I_\nu(z) = \frac{1}{\pi} \int_0^\pi e^{z \cos \theta} \cos(\nu \theta) d\theta - \frac{\sin(\nu \pi)}{\pi} \int_0^\infty e^{-z \cosh t - \nu t} dt, \quad (|\arg z| < \pi/2). \quad (7.1)$$

Hence

$$K_{is}(x) = \frac{\pi/2}{i\sin(s\pi)} \{I_{-is}(x) - I_{is}(x)\} = \int_0^\infty e^{-x\cosh t} \cos(st) dt, \tag{7.2}$$

and

$$\begin{aligned} M_{is}(x) &= \frac{\pi/2}{\cosh(s\pi)} \{I_{-is}(x) + I_{is}(x)\} \\ &= \frac{1}{\cosh(s\pi)} \int_0^\pi e^{x\cos\theta} \cosh(s\theta) d\theta - \tanh(s\pi) \int_0^\infty e^{-x\cosh t} \sin(st) dt. \end{aligned} \tag{7.3}$$

The derivatives of above functions are

$$K'_{is}(x) = - \int_0^\infty \cosh t e^{-x\cosh t} \cos(st) dt, \tag{7.4}$$

$$M'_{is}(x) = \frac{1}{\cosh(s\pi)} \int_0^\pi e^{x\cos\theta} \cosh(s\theta) \cos\theta d\theta + \tanh(s\pi) \int_0^\infty e^{-x\cosh t} \sin(st) \cosh t dt. \tag{7.5}$$

Although there are some other different integral representations, they seem to be unsuitable for numerical calculation.

A method to compute the values of $K_{is}(x)$ based on (7. 2) is already presented in the previous paper³⁾. As discussed in the paper, the methods based on this integral representations are too time-consuming in the region of oscillation. Hence we state the methods briefly.

A method to evaluate the values of $K'_{is}(x)$ based on (7. 4) is provided by modifying the procedure $K_{itr}(s, x)$ discussed in the previous paper.³⁾

The envelope of the integrand of (7. 4), putting $t = \frac{\pi}{2s} z$.

$$g(z) = \cosh\left(\frac{\pi}{2s} z\right) \exp\left\{-x\cosh\left(\frac{\pi}{2s} z\right)\right\}, \tag{7.6}$$

vanishes more slowly than that of $K_{is}(x)$ because of the factor $\cosh(t)$, however, for large t , the double exponential factor is superior to $\cosh(t)$, hence the situation of rapid decay is not changed. Therefore the main process in the procedure needs not be modified except for the cut off point b (the part larger than this can be neglected). The cut off point is determined as the root of

$$g(z)/g(0) = 10^{-N}, \tag{7.7}$$

or

$$\cosh\left(\frac{\pi}{2s} z\right) = 1 + N \log_e 10/x + \log\left\{\cosh\left(\frac{\pi}{2s} z\right)\right\} / x, \tag{7.8}$$

This is not solved explicitly, hence we use the following virtual iteration

$$\begin{aligned} c_0 &= 1 + N \log_e 10/y = 1 + 2.3N/x, \\ c_{n+1} &= c_n + \log c_n / x, \\ c &= \lim_{n \rightarrow \infty} c_n, \end{aligned} \tag{7.9}$$

$$b = \frac{2s}{\pi} \log_e \{c + \sqrt{c^2 - 1}\}.$$

It is sufficient for almost all s and x treated in this paper to apply this iteration only once or twice.

The other integrals having double exponential factor in (7.3) and (7.5) can be estimated by the use of the idea in the paper. The finite integrals in the right hand side of (7.3) and (7.5) are suitable forms for the Gauss Legendre quadrature formula. For example, the use of 12-point formula yields the accuracy of eight decimal digit for fairly large range of s and x .

§ 8 Practical Procedure and Computing Times

Based on the discussions hitherto given, as an useful procedure for the analyses of potential problems by the use of the expressions of the φ -form, a following procedure is provided (see Fig. 4)

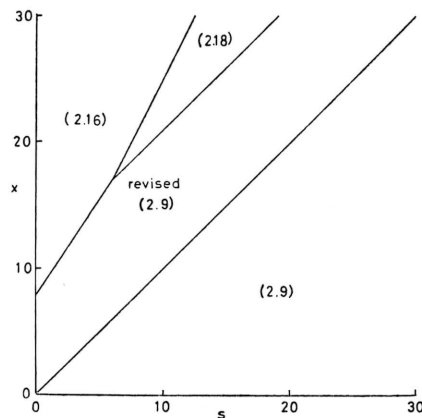


Fig. 4. Computable area of $K_{is}(x)$ by various series expansions.

- 1) for $x=0$, $K_{is}(x)=0$ and $M_{is}(x)=\text{undefined}$,
- 2) for $s=0$, $K_{is}(x)=K_0(x)$ and $M_{is}(x)=\pi I_0(x)$,
- 3) for $x < s$ $K_{is}(x)$ is computed by (2.9),
 $M_{is}(x)$ is computed by (2.10),
- 4) for $x > 1.5a + 8.0$ and $x > 2.0s + 5.0$
 $K_{is}(x)$ is computed by (2.16),
 $M_{is}(x)$ is computed by (2.17),
- 5) for $x < 2.0s + 5.0$ and $x > s + 11$
 $K_{is}(x)$ is computed by (2.18) or (5.4)
 $M_{is}(x)$ is computed by (2.23) or (5.5)
- 6) for otherwise, $K_{is}(x)$ is computed by revised (2.9)
 $M_{is}(x)$ is computed by revised (2.10)

Above procedure is made to compute the values of $K_{is}(x)$ and $M_{is}(x)$ simulataneously. It is because that the contents of computation of these two values are almost the same as each other and in the problem we aim to analyze, the two values are usually required. As stated already in the computation of $M_{is}(x)$ the cancellation does not occur. Hence the values of $M_{is}(x)$ obtained by revised calculation of(2. 10) are accurate to about 15 decimal places.

The computing times of above procedure on Facom 230-60 of Data processing center at Kyoto University are shown in Table 9. The unit for this table is ms.

Table 9 Computing times of both values of $K_{is}(x)$ and $M_{is}(x)$.

| $x \backslash s$ | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 27 | 30 |
|------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 8 | 8 | 8 | 8 | 8 | 8 | 7 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 4 | 4 |
| 2 | 12 | 12 | 12 | 12 | 12 | 11 | 9 | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 | 6 | 5 |
| 3 | 18 | 17 | 14 | 14 | 13 | 12 | 11 | 10 | 10 | 10 | 9 | 8 | 7 | 7 | 7 | 6 | 6 |
| 4 | 19 | 19 | 19 | 14 | 14 | 13 | 11 | 10 | 10 | 10 | 9 | 9 | 9 | 9 | 9 | 8 | 7 |
| 5 | 22 | 21 | 20 | 19 | 14 | 14 | 12 | 11 | 11 | 11 | 10 | 10 | 10 | 10 | 10 | 9 | 8 |
| 6 | 22 | 21 | 21 | 21 | 20 | 14 | 13 | 13 | 12 | 12 | 11 | 11 | 11 | 11 | 11 | 10 | 10 |
| 8 | 25 | 25 | 24 | 23 | 23 | 23 | 15 | 14 | 14 | 14 | 14 | 13 | 13 | 13 | 13 | 13 | 12 |
| 10 | 8 | 30 | 28 | 25 | 25 | 25 | 25 | 16 | 16 | 16 | 16 | 15 | 14 | 13 | 13 | 13 | 13 |
| 12 | 6 | 7 | 29 | 28 | 27 | 27 | 27 | 27 | 18 | 18 | 18 | 17 | 16 | 15 | 14 | 14 | 13 |
| 14 | 5 | 6 | 8 | 9 | 29 | 29 | 29 | 28 | 27 | 20 | 20 | 19 | 18 | 17 | 16 | 15 | 14 |
| 16 | 5 | 6 | 7 | 9 | 10 | 31 | 30 | 29 | 29 | 28 | 21 | 20 | 19 | 19 | 18 | 17 | 16 |
| 18 | 5 | 6 | 7 | 8 | 9 | 10 | 26 | 31 | 31 | 30 | 30 | 21 | 21 | 20 | 20 | 19 | 18 |
| 20 | 5 | 5 | 6 | 7 | 7 | 8 | 21 | 32 | 32 | 32 | 31 | 30 | 24 | 23 | 21 | 20 | 20 |
| 22 | 5 | 5 | 6 | 6 | 7 | 7 | 9 | 21 | 35 | 35 | 33 | 32 | 32 | 25 | 25 | 23 | 22 |
| 24 | 4 | 5 | 6 | 6 | 6 | 7 | 8 | 18 | 22 | 35 | 35 | 34 | 34 | 34 | 25 | 25 | 24 |
| 27 | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 10 | 18 | 21 | 24 | 38 | 37 | 36 | 35 | 25 | 25 |
| 30 | 4 | 4 | 5 | 5 | 5 | 6 | 7 | 8 | 10 | 18 | 21 | 25 | 38 | 37 | 35 | 34 | 26 |

(unit: ms)

§ 9 Behaviors of $K_{is}(x)$, $M_{is}(x)$, $I_{is}(x)$ and their Derivatives

9.1 Exact Tables

The exact tables of $K_{is}(x)$ and $M_{is}(x)$ accurate to 8 decimal digits are made for a fairly large range of s and x . For the region of oscillation, the values are computed by revised(2. 9) and (2. 10). Hence the values must be accurate to 14 or 15 decimal digits. However they are rounded to 8 decimal digits, considering the accuracy obtained in the other region.

The values in the region where the accuracy of 8 decimal places is not obtained by series expansion are computed by a elaborated method based on the integral representation.

The short tables of $K'_{is}(x)$ and $M'_{is}(x)$ are also provided by similar method to that of $K_{is}(x)$ and $M_{is}(x)$.

The short tables of $I_{is}(x)$ are provided. The real and imaginary parts of $I_{is}(x)$ are obtained by multiplying the values of $M_{is}(x)$ and $K_{is}(x)$ by $\cosh(s\pi)/\pi$ and $\sinh(s\pi)/\pi$, respectively, as stated in Introduction.

All these tables are shown in **Appendix**.

9.2 Three Dimensional Representation

A three dimensional representation of the behavior of $K_{is}(x)$ is shown in Fig. 5. A corresponding representation of $M_{is}(x)$ is shown in Fig. 6.

9.3 Zeros of Functions $K_{is}(x)$ and $M_{is}(x)$

It has been proved¹⁾ that $K_v(x)$, with real x , has zeros only for pure imaginary values of v . There are no zeros of $K_v(x)$ if v is real or complex.

We state here the solutions of the equation

$$K_{is}(x) = 0. \quad (9.1)$$

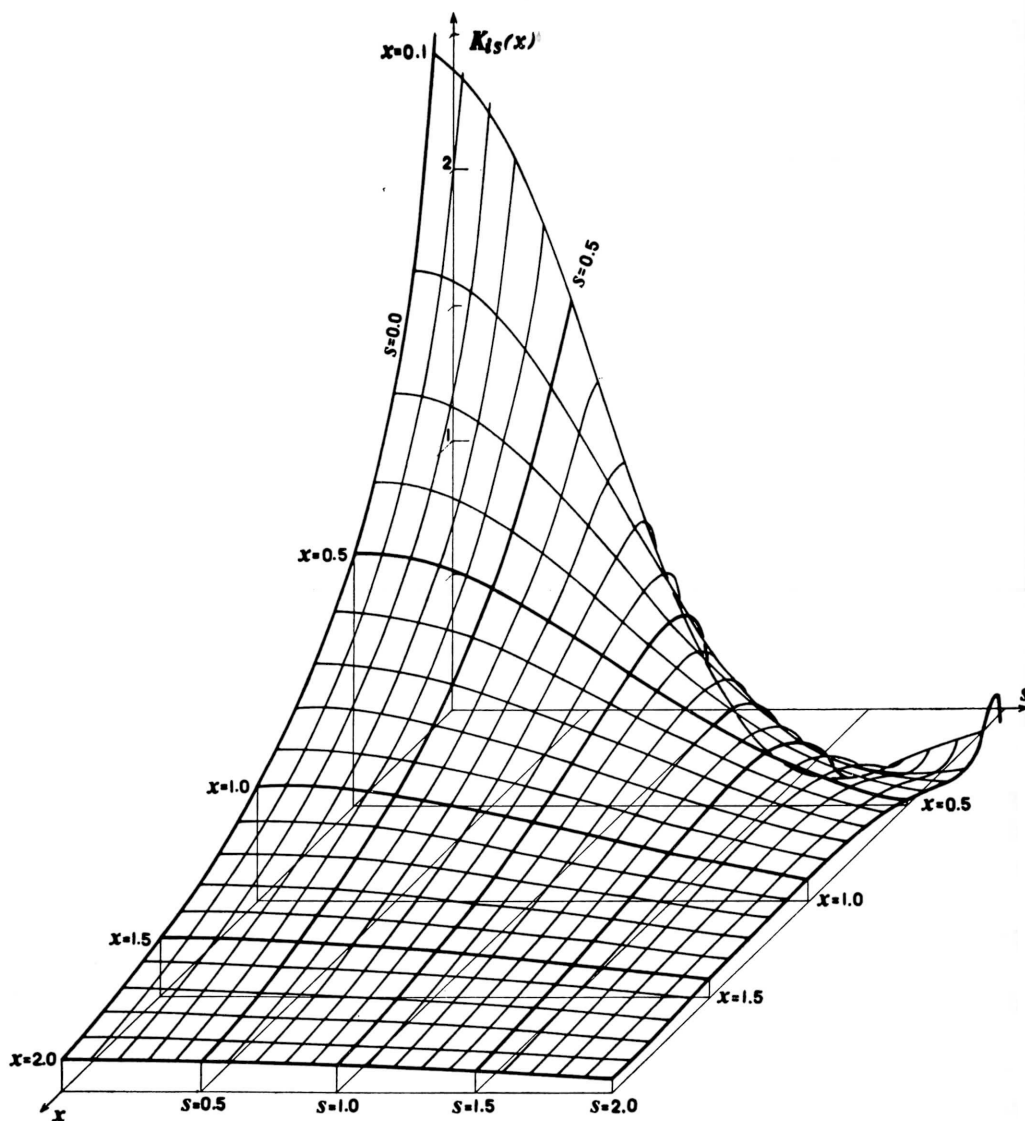


Fig. 5. $K_{is}(x)$ as a function of s and x .

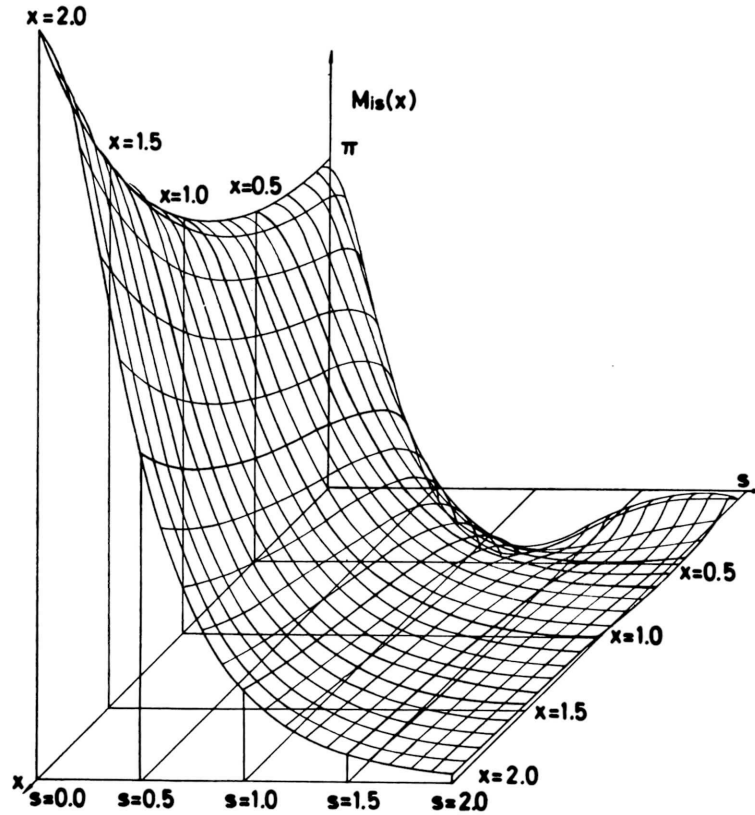


Fig. 6. $M_{is}(x)$ as a function of s and x .

As $K_{is}(x)$ is an even function of s . (9.1) defines a family of curves

$$x = x_n(s) \tag{9.2}$$

in the x - s plane, which are symmetric with respect to the x -axis. These curves cannot cross or touch each other except possibly at $x = 0$, and none of them can have points of maximum or minimum values or stop at any other value of x . It has been shown¹⁾ that for any real x , $K_{is}(x)$ regarded as a function of s has an infinite number of zeros. Thus the curves defined by (9.2) extend themselves to infinitely in the s direction of the x - s plane, so that for any value of x a line parallel to s -axis crosses an infinite number of curves.

The first fifteen zeros in the interval $x < 15$, $0 < s < 20$ are shown in Fig. 7. As shown in the figure, the s -axis is tangent to all curves at the origin, and for any given s , $x = 0$ is an accumulation point for the zeros. For all roots it is always $|s| > x$.

Hence the roots exist periodically in the graph of logarithmic scale as shown in Fig. 3 the ratio of a zero to an adjacent zero is nearly equal to constant as follows:

$$x_{n+1}(s) \simeq e^{-\pi/s} x_n(s). \tag{9.3}$$

The corresponding function $M_{is}(x)$ has similar properties to that of $K_{is}(x)$ with respect to its

zeros.

The zeros of $M_{is}(x)$ are shown in Fig. 8. As shown in the figure, they fall on the middle of the adjacent zeros of $K_{is}(x)$, in other words the oscillations of $K_{is}(x)$ and $M_{is}(x)$ differ in

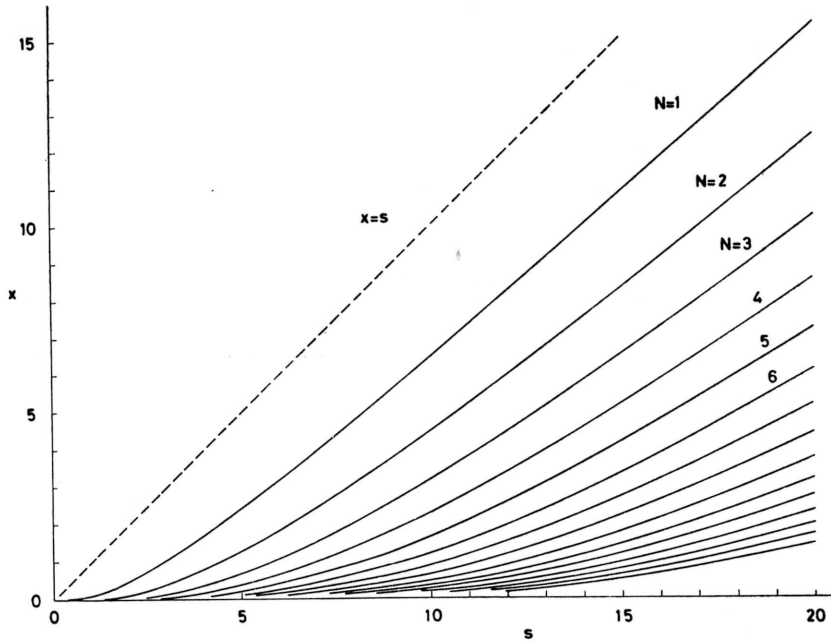


Fig. 7. Zeros of $K_{is}(x)$.

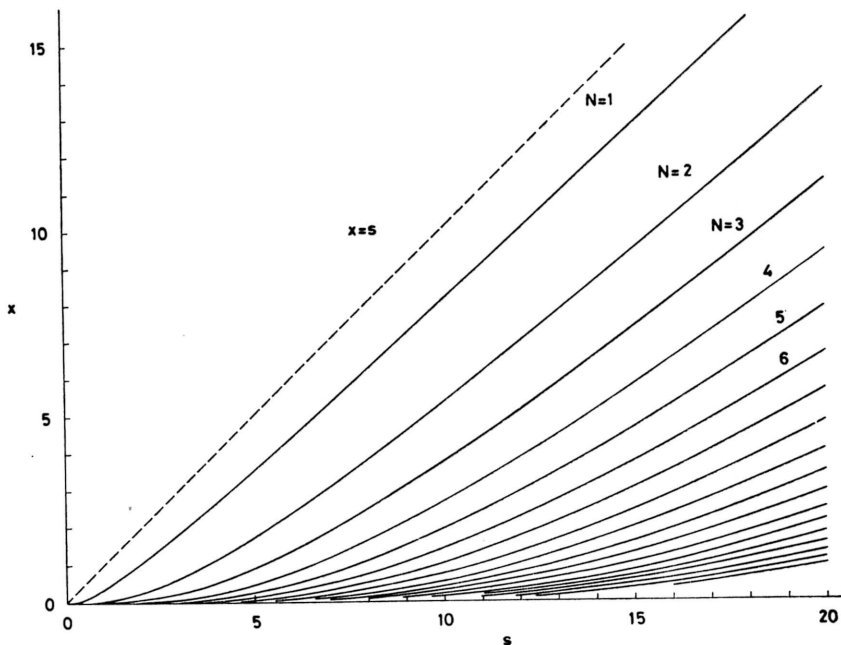


Fig. 8. Zeros of $M_{is}(x)$.

phase by one-fourth of period in logarithmic scale. The zeros of $M_{is}(x)$ are nearly equal to the solution of

$$\frac{d}{dx} K_{is}(x) = 0, \tag{9.4}$$

but not exactly. The exact values of the zeros of $K_{is}(x)$ and $M_{is}(x)$ are shown in **Appendix**.

§ 10 Some Important Formulas related with the Functions $K_{is}(x)$ and $M_{is}(x)$
Infinite integrals

$$\begin{aligned} & \Gamma(0.5 + \mu) \frac{1}{\sqrt{\pi}} (2\beta)^{-\nu} (\alpha + \beta)^{\nu + \mu} \int_0^\infty e^{-\alpha t} K_\nu(\beta t) t^{\mu-1} dt \\ &= \Gamma(\mu + \nu) \Gamma(\mu - \nu) {}_2F_1[v + \mu, \nu + 0.5; \mu + 0.5; (\alpha - \beta)/(\alpha + \beta)] \\ &= [(\alpha + \beta)/(2\alpha)]^{2\nu + 2\mu} \Gamma(\mu + \nu) \Gamma(\mu - \nu) {}_2F_1(\nu + \mu, \mu; 2\nu + 2\mu; 1 - \beta^2/\alpha^2) \end{aligned} \tag{10.1}$$

$$Re(\mu \pm \nu) > 0, Re(\alpha + \beta) > 0.$$

For $\alpha=0$, we have

$$\int_0^\infty K_\nu(\beta t) t^{\mu-1} dt = 2^{\mu-2} \beta^{-\mu} \Gamma\left(\frac{\mu + \nu}{2}\right) \Gamma\left(\frac{\mu - \nu}{2}\right) \tag{10.2}$$

$$Re(\mu \pm \nu) > 0, Re \beta > 0.$$

Hence

$$\int_0^\infty K_{is}(t) dt = \frac{\pi/2}{\cosh(\pi s/2)}, \quad \int_0^\infty t K_{is}(t) dt = \frac{\pi s/2}{\sinh(\pi s/2)}. \tag{10.3}$$

$$\begin{aligned} & 2^{e+1} \alpha^{\nu-e+1} \Gamma(\nu + 1) \int_0^\infty K_\mu(\alpha t) J_\nu(\beta t) t^{-e} dt \\ &= \beta^\nu \Gamma\left(\frac{\nu - \rho + \mu + 1}{2}\right) \Gamma\left(\frac{\nu - \rho - \mu + 1}{2}\right) {}_2F_1\left(\frac{\nu - \rho + \mu + 1}{2}, \frac{\nu - \rho - \mu + 1}{2}; \nu + 1; -\beta^2/\alpha^2\right), \end{aligned} \tag{10.4}$$

$$Re(\alpha + i\beta) > 0, Re(\nu - \rho + 1 \pm \mu) > 0.$$

$$\begin{aligned} & 2^{e+1} \Gamma(\nu + 1) \alpha^{\nu+1-e} \int_0^\infty K_\nu(\alpha t) I_\nu(\beta t) t^{-e} dt \\ &= \beta^\nu \Gamma\left(\frac{1 - \rho + \mu + \nu}{2}\right) \Gamma\left(\frac{1 - \rho - \mu + \nu}{2}\right) {}_2F_1\left(\frac{1 - \rho + \mu + \nu}{2}, \frac{1 - \rho - \mu + \nu}{2}; \nu + 1; \beta^2/\alpha^2\right), \end{aligned} \tag{10.5}$$

$$Re(\nu - \rho + 1 \pm \mu) > 0, \alpha > \beta.$$

$$2^{e+2} \Gamma(1 - \rho) \int_0^\infty K_\mu(\alpha t) K_\nu(\beta t) t^{-e} dt$$

$$= \alpha^{e-v-1} \beta^v {}_2F_1\left(\frac{1+v+\mu-\rho}{2}, \frac{1+v-\mu-\rho}{2}; 1-\rho; 1-\beta^2/\alpha^2\right) \\ \times \Gamma\left(\frac{1+v+\mu-\rho}{2}\right) \Gamma\left(\frac{1+v-\mu-\rho}{2}\right) \Gamma\left(\frac{1-v+\mu-\rho}{2}\right) \Gamma\left(\frac{1-v-\mu-\rho}{2}\right) \quad (10.6)$$

$$\operatorname{Re}(\alpha + \beta) > 0, \operatorname{Re}(\rho \pm \mu \pm v + 1) > 0.$$

$$\int_0^\infty K_{is}^2(t) dt = \frac{\pi^2}{4 \cosh(s\pi)} \quad (10.7)$$

Macdonald's and Nicholson's formulas

$$\int_0^\infty \exp[-t/2 - (z^2 + Z^2)/2t] K_\nu(Zz/t) \frac{dt}{t} = 2K_\nu(z)K_\nu(Z), \quad (10.8)$$

$$\arg z < \pi, \arg Z < \pi, \arg(Z+z) < \frac{\pi}{4},$$

$$\int_0^\infty [-t/2 - (x^2 + X^2)/2t] I_\nu(xX/t) \frac{dt}{t} = \begin{cases} 2I_\nu(x)K_\nu(X) & \text{for } X > x \\ 2K_\nu(x)I_\nu(X) & \text{for } x > X, \end{cases} \quad (10.9)$$

$$\int_0^\infty \exp[-t/2 - (x^2 + X^2)/2t] K_{is}(xX/t) \frac{dt}{t} = 2K_{is}(x)K_{is}(X), \quad (10.10)$$

$$\int_0^\infty \exp[-t/2 - (x^2 + X^2)/2t] M_{is}(xX/t) \frac{dt}{t} = \begin{cases} 2M_{is}(x)K_{is}(X) & \text{for } X > x \\ 2K_{is}(x)M_{is}(X) & \text{for } x < X, \end{cases} \quad (10.11)$$

$$K_{is}(x) = K_{-is}(x), \quad M_{is}(x) = M_{-is}(x). \quad (10.12)$$

Integral with respect to the order

$$K_0[(a^2 + b^2 - 2ab \cos \varphi)^{1/2}] = \frac{2}{\pi} \int_0^\infty K_{is}(a)K_{is}(b) \cosh[(\pi - \varphi)s] ds, \quad (10.13)$$

$$\int_0^\infty K_{is}(a) \cos(sy) ds = -\frac{\pi}{2} e^{-a \cosh(y)}, \quad (10.14)$$

$$\int_0^\infty K_{is}(a) \cosh(\pi s/2) \cos(sy) ds = -\frac{\pi}{2} \cos(asinh(y)), \quad (10.15)$$

$$\int_0^\infty K_{is}(a) \sinh(\pi s/2) \sin(sy) ds = \frac{\pi}{2} \sin(asinh(y)), \quad (10.16)$$

$$\int_{-\infty}^\infty K_{i(\xi+\eta)}(a) K_{i(\xi+\eta)}(b) e^{(\pi-C)\eta} d\eta = K_{i(\xi-\zeta)}(c) e^{-\xi B - \zeta A}, \quad (10.17)$$

where A, B, C, are the angles of the triangle whose sides are of lengths a, b, c .

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} [I_\nu(xt) + I_{-\nu}(xt)] \sqrt{xt} dt \int_0^\infty K_\nu(vt) \sqrt{vt} f(v) dv, \quad (10.18)$$

$$f(x) = \frac{\cosh(s\pi)}{2i} \int_{c-i\infty}^{c+i\infty} M_{is}(xt) \sqrt{xt} dt \int_0^\infty K_{is}(vt) \sqrt{vt} f(v) dv, \tag{1.19}$$

$$f(x) = \frac{1}{\pi^2} \int_{-\infty}^\infty e^{\pi(x+t)/2} K_{i(x+t)}(a) dt \int_{-\infty}^\infty e^{\pi(t+v)/2} K_{i(t+v)}(a) f(v) dv, \tag{10.20}$$

$$xf(x) = \frac{2}{\pi^2} \int_0^\infty K_{it}(x) t \sinh(\pi t) dt \int_0^\infty K_{it}(v) f(v) dv. \tag{10.21}$$

Series Expansions

From McLachlan, we obtain

$$\int_z^\infty z K_v^2(kz) dz = \frac{z^2}{2} \left\{ K_v'^2(zk) = \left(1 + \frac{v^2}{k^2 z^2} \right) K_v^2(kz) \right\} \tag{10.22}$$

and

$$\int_z^\infty z K_v(kz) K_v(lz) dz = \frac{z}{k^2 - l^2} \{ l K_v(kz) K_v'(lz) - k K_v(lz) K_v'(kz) \}. \tag{10.23}$$

Supposed that γ_m is the m -th root of the equation $K_{is}'(x) = 0$ and α_m is the m -th root of the equation $K_{is}(x) = 0$, we obtain the following orthogonality relations:

$$\int_1^\infty x K_{is}(\gamma_m x) K_{is}(\gamma_n x) dx = \begin{cases} 0 & m \neq n \\ \frac{s^2 - \gamma_m^2}{2\gamma_m^2} K_{is}(\gamma_m)^2 & m = n, \end{cases} \tag{10.24}$$

$$\int_1^\infty x K_{is}(\alpha_m x) K_{is}(\alpha_n x) dx = \begin{cases} 0 & m \neq n \\ -\frac{1}{2} K_{is}'(\alpha_m)^2 & m = n. \end{cases} \tag{10.25}$$

Using above relations, we can obtain the following Fourier type series expansion formula:

$$f(x) = \sum_{n=1}^\infty a_n K_{is}(\gamma_n x) \quad \text{for } 1 < x < \infty \tag{10.26}$$

where a_n is

$$a_n = \frac{2\gamma_n^2}{(s^2 - \gamma_n^2)} \frac{\int_1^\infty x f(x) K_{is}(\gamma_n x) dx}{K_{is}^2(\gamma_n)}. \tag{10.27}$$

§ 11 Conclusion

Possibilities to compute the modified Bessel functions of first and second kinds of purely imaginary order $I_{is}(x)$, $K_{is}(x)$ and their related functions are discussed based on their series expansions. The methods to compute the values of $I_{is}(x)$ and $K_{is}(x)$ accurately to eight decimal digits for fairly large range of s and x are established, although the methods presented

here will have some margins for improvement and refinement.

For the convenience of practical use, the procedures presented here is made to compute the values of $K_{is}(x)$ and $M_{is}(x)$, which is introduced to express the real part of $I_{is}(x)$ multiplied by $\pi/\cosh(s\pi)$, simultaneously.

If a procedure to compute only the value of $K_{is}(x)$ or that of $M_{is}(x)$ is necessary the re-making of the procedure is easy.

The behaviors of $K_{is}(x)$ and $I_{is}(x)$ are clarified in detail by giving the short tables for $K_{is}(x)$ and $M_{is}(x)$ and some graphical representations.

It is hoped that the studies of applications of $K_{is}(x)$ and $I_{is}(x)$ to the analyses of boundary value problems are developed to a great extent.

The computations in this paper were carried out on the Facom 230-60 of the Data Processing Center at Kyoto University.

REFERENCES

- 1) Gray, A and T. M. MacRobert: A Treatise on Bessel Functions and Their Applications to Physics. Dover Publication Inc. New York (1966).
- 2) Ferira, E. M., and J. Sesma: Zeros of the Modified Hankel Function. Numer. Math., **16** (1970) pp. 278-284.
- 3) Kiyono, T. and S. Murashima: A Method of Evaluation of the Function $K_{is}(x)$, Memoir of the Faculty of Engineering, Kyoto University **35** (1973) pp. 102-127.
- 4) Yamauch, J., S. Moriguchi and S. Hitotsumatsu: Suuchi-keisan-ho II for computer. Baifuukan (1964) pp. 103-121.
- 5) Murashima, S.: Neumann Function for Laplace's Equation for a circular Cylinder of Finite Length. Japan J. appl. Phys. **12** (1973) pp. 1232-1243.
- 6) McLachlan, N. W.: Bessel Functions for Engineers, Oxford University Press. (1934).
- 7) Erdelyi, Magnus, Oberhettinger and Tricomi: Higher Transcendental Function. vol. 2, McGraw-Hill Book Co., Inc. (1953).
- 8) Moriguchi, S., K. Utagawa and S. Hitotsumatsu: Sugaku Koshiki II, Iwanami Shoten, (1957).
- 9) Luke, Y. L.: The Special Functions and Their Apporximations, Academic Press New York, (1969).
- 10) Watson, N. G.: Bessel Functions, pp. 181-183 (1922).
- 11) Sibagaki, W.: 0.01% Tables of Modified Bessel Functions., Baifukan (1955).
- 12) Airey: Bessel and Neumann Functions of equal order and argument., Phyllos. Mag. **31** (1916) pp. 520-530.

APPENDIX

TABLES OF MODIFIED BESSEL FUNCTIONS OF PURELY IMAGINARY ORDER $K_{is}(x)$, $I_{is}(x)$, $M_{is}(x)$ AND THEIR RELATED FUNCTIONS

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DEFINITIONS

$$I_{is}(x) = \frac{\cosh(s\pi)}{\pi} M_{is}(x) - i \frac{\sinh(s\pi)}{\pi} K_{is}(x)$$

$$I_{is}(x) = \frac{\cosh(s\pi)}{\pi} M_{is}(x) + i \frac{\sinh(s\pi)}{\pi} K_{is}(x)$$

$$K_{is}(x) = \frac{\pi/2}{i \sinh(s\pi)} \{I_{-is}(x) - I_{is}(x)\}$$

$$M_{is}(x) = \frac{\pi/2}{\cosh(s\pi)} \{I_{-is}(x) + I_{is}(x)\}$$

| s | $\frac{\cosh(s\pi)}{\pi}$ | $\frac{\sinh(s\pi)}{\pi}$ | s | $\frac{\cosh(s\pi)}{\pi}$ | $\frac{\sinh(s\pi)}{\pi}$ |
|------|---------------------------|---------------------------|-------|---------------------------|---------------------------|
| 0.0 | 0.318309886E 00 | 0.0 | 1.80 | 0.454676917E 02 | 0.454665774E 02 |
| 0.01 | 0.318466979E 00 | 0.100016450E-01 | 1.90 | 0.622498130E 02 | 0.622489991E 02 |
| 0.02 | 0.318938411E 00 | 0.200131621E-01 | 2.00 | 0.852264412E 02 | 0.852258467E 02 |
| 0.03 | 0.319724650E 00 | 0.300444327E-01 | 2.50 | 0.409978500E 03 | 0.409978376E 03 |
| 0.04 | 0.320826469E 00 | 0.401053590E-01 | 3.00 | 0.197219201E 04 | 0.197219199E 04 |
| 0.05 | 0.322244958E 00 | 0.502058704E-01 | 3.50 | 0.948718502E 04 | 0.948718501E 04 |
| 0.10 | 0.334147468E 00 | 0.101653070E 00 | 4.00 | 0.456378889E 05 | 0.456378889E 05 |
| 0.20 | 0.383236218E 00 | 0.213421684E 00 | 4.50 | 0.219540032E 06 | 0.219540032E 06 |
| 0.30 | 0.470460979E 00 | 0.346427986E 00 | 5.00 | 0.105609236E 07 | 0.105609236E 07 |
| 0.40 | 0.604501533E 00 | 0.513907502E 00 | 5.50 | 0.508030841E 07 | 0.508030841E 07 |
| 0.50 | 0.798696316E 00 | 0.732526192E 00 | 6.00 | 0.244387087E 08 | 0.244387087E 08 |
| 0.60 | 0.107236972E 01 | 0.102403888E 01 | 6.50 | 0.117561855E 09 | 0.117561855E 09 |
| 0.70 | 0.145275518E 01 | 0.141745420E 01 | 7.00 | 0.565528646E 09 | 0.565528646E 09 |
| 0.80 | 0.197770491E 01 | 0.195192098E 01 | 10.00 | 0.700783181E 13 | 0.700783181E 13 |
| 0.90 | 0.269945707E 01 | 0.268062442E 01 | 12.00 | 0.375263546E 16 | 0.375263546E 16 |
| 1.00 | 0.368983333E 01 | 0.367607791E 01 | 14.00 | 0.200950497E 19 | 0.200950497E 19 |
| 1.10 | 0.504738656E 01 | 0.503733957E 01 | 16.00 | 0.107607315E 22 | 0.107607315E 22 |
| 1.20 | 0.690720750E 01 | 0.689986915E 01 | 18.00 | 0.576228190E 24 | 0.576228190E 24 |
| 1.30 | 0.945436865E 01 | 0.944900869E 01 | 20.00 | 0.308565387E 27 | 0.308565387E 27 |
| 1.40 | 0.129423376E 02 | 0.129384227E 02 | 22.50 | 0.794855334E 30 | 0.794855334E 30 |
| 1.50 | 0.177182044E 02 | 0.177153450E 02 | 25.00 | 0.204752389E 34 | 0.204752389E 34 |
| 1.60 | 0.242572177E 02 | 0.242551292E 02 | 27.50 | 0.527436113E 37 | 0.527436113E 37 |
| 1.70 | 0.332100777E 02 | 0.332085522E 02 | 30.00 | 0.135865987E 41 | 0.135865987E 41 |

| X | TABLE OF K15(X) | | | |
|-------|---------------------|---------------------|---------------------|---------------------|
| | S ^m 0.01 | S ^m 0.02 | S ^m 0.03 | S ^m 0.04 |
| 0.01 | 0.47141429E 01 | 0.47128414E 01 | 0.47023515E 01 | 0.46687692E 01 |
| 0.02 | 0.40270768E 01 | 0.40229372E 01 | 0.40160445E 01 | 0.40064086E 01 |
| 0.03 | 0.36224790E 01 | 0.36193287E 01 | 0.36140825E 01 | 0.36067496E 01 |
| 0.04 | 0.33356883E 01 | 0.33331304E 01 | 0.33288701E 01 | 0.33229121E 01 |
| 0.05 | 0.31135150E 01 | 0.31113588E 01 | 0.31077673E 01 | 0.31027445E 01 |
| 0.06 | 0.29322584E 01 | 0.29303950E 01 | 0.29279282E 01 | 0.29229530E 01 |
| 0.07 | 0.27979171E 01 | 0.27976340E 01 | 0.27949059E 01 | 0.27910900E 01 |
| 0.08 | 0.26470050E 01 | 0.26455441E 01 | 0.26431137E 01 | 0.26397114E 01 |
| 0.09 | 0.25305793E 01 | 0.25292661E 01 | 0.25270786E 01 | 0.25240184E 01 |
| 0.10 | 0.24266716E 01 | 0.24254798E 01 | 0.24234944E 01 | 0.24207186E 01 |
| 0.20 | 0.17252095E 01 | 0.17319263E 01 | 0.17509551E 01 | 0.17495960E 01 |
| 0.30 | 0.13723417E 01 | 0.13719867E 01 | 0.13713935E 01 | 0.13705676E 01 |
| 0.40 | 0.11144497E 01 | 0.11142112E 01 | 0.11139139E 01 | 0.11132579E 01 |
| 0.50 | 0.92436254E 00 | 0.92416497E 00 | 0.92391041E 00 | 0.92351495E 00 |
| 0.60 | 0.77748035E 00 | 0.77735510E 00 | 0.77714640E 00 | 0.77685431E 00 |
| 0.70 | 0.66608817E 00 | 0.66603913E 00 | 0.66603974E 00 | 0.66601311E 00 |
| 0.80 | 0.56523228E 00 | 0.56524897E 00 | 0.56512631E 00 | 0.56495464E 00 |
| 0.90 | 0.48671100E 00 | 0.48665307E 00 | 0.48653656E 00 | 0.48621466E 00 |
| 1.00 | 0.42100905E 00 | 0.42096788E 00 | 0.42088399E 00 | 0.42077859E 00 |
| 1.10 | 0.36559000E 00 | 0.36555281E 00 | 0.36549082E 00 | 0.36540406E 00 |
| 1.20 | 0.31849813E 00 | 0.31846792E 00 | 0.31841753E 00 | 0.31834701E 00 |
| 1.30 | 0.27839835E 00 | 0.27837346E 00 | 0.27831733E 00 | 0.27824386E 00 |
| 1.40 | 0.24364823E 00 | 0.24362784E 00 | 0.24359382E 00 | 0.24354620E 00 |
| 1.50 | 0.21379992E 00 | 0.21378300E 00 | 0.21375479E 00 | 0.21371530E 00 |
| 1.60 | 0.18795005E 00 | 0.18793594E 00 | 0.18791244E 00 | 0.18787954E 00 |
| 1.70 | 0.16549239E 00 | 0.16548058E 00 | 0.16546090E 00 | 0.16543355E 00 |
| 1.80 | 0.14529809E 00 | 0.14528184E 00 | 0.14526163E 00 | 0.14523848E 00 |
| 1.90 | 0.12684319E 00 | 0.12682846E 00 | 0.12680434E 00 | 0.12677134E 00 |
| 2.00 | 0.11381451E 00 | 0.11380442E 00 | 0.11378726E 00 | 0.11376306E 00 |
| 2.50 | 0.62346487E-01 | 0.62343289E-01 | 0.62337959E-01 | 0.62330497E-01 |
| 3.00 | 0.44608991E-01 | 0.44607481E-01 | 0.44605971E-01 | 0.44604461E-01 |
| 3.50 | 0.31959648E-01 | 0.31959702E-01 | 0.31959659E-01 | 0.31959616E-01 |
| 4.00 | 0.23115751E-01 | 0.23115714E-01 | 0.23115677E-01 | 0.23115640E-01 |
| 4.50 | 0.16999796E-02 | 0.16999987E-02 | 0.16999755E-02 | 0.16998231E-02 |
| 5.00 | 0.36910645E-02 | 0.36909630E-02 | 0.36907938E-02 | 0.36905570E-02 |
| 6.00 | 0.12439847E-02 | 0.12439558E-02 | 0.12439077E-02 | 0.12438403E-02 |
| 7.00 | 0.42479400E-03 | 0.42478436E-03 | 0.42477444E-03 | 0.42476424E-03 |
| 8.00 | 0.14646948E-03 | 0.14646472E-03 | 0.14646292E-03 | 0.14646186E-03 |
| 9.00 | 0.50881044E-04 | 0.50880239E-04 | 0.50878896E-04 | 0.50877046E-04 |
| 10.00 | 0.17779977E-04 | 0.17778723E-04 | 0.17779299E-04 | 0.17778104E-04 |
| 12.00 | 0.22008166E-05 | 0.22007901E-05 | 0.22007461E-05 | 0.22006843E-05 |
| 14.00 | 0.27613613E-06 | 0.27613327E-06 | 0.27612850E-06 | 0.27612183E-06 |
| 16.00 | 0.34994010E-07 | 0.34993692E-07 | 0.34993161E-07 | 0.34992418E-07 |
| 18.00 | 0.44687433E-08 | 0.44687050E-08 | 0.44686464E-08 | 0.44685600E-08 |
| 20.00 | 0.57412237E-09 | 0.57411818E-09 | 0.57411181E-09 | 0.57410136E-09 |
| 22.50 | 0.44461124E-10 | 0.44460833E-10 | 0.44460300E-10 | 0.44459674E-10 |
| 25.00 | 0.34613071E-11 | 0.34613144E-11 | 0.34613006E-11 | 0.34612851E-11 |
| 27.50 | 0.27124085E-12 | 0.27123940E-12 | 0.27123698E-12 | 0.27123358E-12 |
| 30.00 | 0.21324740E-13 | 0.21324635E-13 | 0.21324460E-13 | 0.21324215E-13 |

| X | TABLE OF K15(X) | | | |
|-------|---------------------|---------------------|---------------------|---------------------|
| | S ^m 0.10 | S ^m 0.20 | S ^m 0.30 | S ^m 0.40 |
| 0.01 | 0.45141925E 01 | 0.39297807E 01 | 0.30698503E 01 | 0.20783687E 01 |
| 0.02 | 0.38922524E 01 | 0.35014801E 01 | 0.29117873E 01 | 0.22012567E 01 |
| 0.03 | 0.35189316E 01 | 0.32201779E 01 | 0.27610211E 01 | 0.21944895E 01 |
| 0.04 | 0.32520098E 01 | 0.30075497E 01 | 0.26292608E 01 | 0.21565009E 01 |
| 0.05 | 0.30429254E 01 | 0.28360376E 01 | 0.25113760E 01 | 0.21067995E 01 |
| 0.06 | 0.28745391E 01 | 0.26949192E 01 | 0.24272338E 01 | 0.20590885E 01 |
| 0.07 | 0.27255990E 01 | 0.25675788E 01 | 0.23191327E 01 | 0.20084868E 01 |
| 0.08 | 0.25991699E 01 | 0.24581047E 01 | 0.22355212E 01 | 0.19489176E 01 |
| 0.09 | 0.24875107E 01 | 0.23603157E 01 | 0.21590575E 01 | 0.18986757E 01 |
| 0.10 | 0.23875716E 01 | 0.22719527E 01 | 0.20885485E 01 | 0.18503386E 01 |
| 0.20 | 0.17339499E 01 | 0.16762884E 01 | 0.15848273E 01 | 0.14624096E 01 |
| 0.30 | 0.14046622E 01 | 0.13232770E 01 | 0.12520144E 01 | 0.11932948E 01 |
| 0.40 | 0.11066033E 01 | 0.10831012E 01 | 0.10448360E 01 | 0.99311789E 00 |
| 0.50 | 0.91878030E 00 | 0.90203482E 00 | 0.87468770E 00 | 0.83755618E 00 |
| 0.60 | 0.77356299E 00 | 0.76097114E 00 | 0.74069923E 00 | 0.71308026E 00 |
| 0.70 | 0.65735807E 00 | 0.64749471E 00 | 0.63252302E 00 | 0.61144933E 00 |
| 0.80 | 0.56289804E 00 | 0.55560531E 00 | 0.54363059E 00 | 0.52723742E 00 |
| 0.90 | 0.48440281E 00 | 0.47905987E 00 | 0.46961863E 00 | 0.45687136E 00 |
| 1.00 | 0.41948783E 00 | 0.41490728E 00 | 0.40736744E 00 | 0.39701711E 00 |
| 1.10 | 0.36436335E 00 | 0.36067222E 00 | 0.35491466E 00 | 0.34622919E 00 |
| 1.20 | 0.31750481E 00 | 0.31448938E 00 | 0.30959072E 00 | 0.30273816E 00 |
| 1.30 | 0.27742330E 00 | 0.27496323E 00 | 0.27096003E 00 | 0.26531498E 00 |
| 1.40 | 0.24297541E 00 | 0.24094636E 00 | 0.23798646E 00 | 0.23298106E 00 |
| 1.50 | 0.21324199E 00 | 0.21155924E 00 | 0.20878101E 00 | 0.20494625E 00 |
| 1.60 | 0.18748508E 00 | 0.18608237E 00 | 0.18376544E 00 | 0.18056522E 00 |
| 1.70 | 0.16510314E 00 | 0.16392866E 00 | 0.16198793E 00 | 0.15930561E 00 |
| 1.80 | 0.14540949E 00 | 0.14446163E 00 | 0.14278156E 00 | 0.14072481E 00 |
| 1.90 | 0.12856724E 00 | 0.12778130E 00 | 0.12633636E 00 | 0.12472413E 00 |
| 2.00 | 0.11365799E 00 | 0.11295299E 00 | 0.11178848E 00 | 0.11017262E 00 |
| 2.50 | 0.62241026E-01 | 0.61922451E-01 | 0.61394830E-01 | 0.60663112E-01 |
| 3.00 | 0.34688945E-01 | 0.34537678E-01 | 0.34286927E-01 | 0.33938719E-01 |
| 3.50 | 0.19740422E-01 | 0.19499654E-01 | 0.19376262E-01 | 0.19204742E-01 |
| 4.00 | 0.11413328E-01 | 0.11109584E-01 | 0.11047264E-01 | 0.10960366E-01 |
| 4.50 | 0.65933969E-02 | 0.63740523E-02 | 0.63419344E-02 | 0.62772211E-02 |
| 5.00 | 0.36877163E-02 | 0.36775880E-02 | 0.36607663E-02 | 0.36373388E-02 |
| 6.00 | 0.24032002E-02 | 0.24014491E-02 | 0.23939866E-02 | 0.23868377E-02 |
| 7.00 | 0.14245113E-03 | 0.14236591E-03 | 0.14228241E-03 | 0.14220670E-03 |
| 8.00 | 0.14638426E-03 | 0.14612512E-03 | 0.14569444E-03 | 0.14509344E-03 |
| 9.00 | 0.50854462E-04 | 0.50773990E-04 | 0.50640144E-04 | 0.50453335E-04 |
| 10.00 | 0.17771577E-04 | 0.17746145E-04 | 0.17703836E-04 | 0.17644770E-04 |
| 12.00 | 0.21999437E-05 | 0.21973007E-05 | 0.21929027E-05 | 0.21867598E-05 |
| 14.00 | 0.27641735E-06 | 0.27575597E-06 | 0.27528029E-06 | 0.27461571E-06 |
| 16.00 | 0.34687003E-07 | 0.34491685E-07 | 0.34388710E-07 | 0.34279727E-07 |
| 18.00 | 0.44675449E-08 | 0.44639209E-08 | 0.44578876E-08 | 0.44494545E-08 |
| 20.00 | 0.57348388E-09 | 0.57326403E-09 | 0.57286403E-09 | 0.57186612E-09 |
| 22.50 | 0.44453551E-10 | 0.44422352E-10 | 0.44373249E-10 | 0.44306756E-10 |
| 25.00 | 0.34648211E-11 | 0.34614445E-11 | 0.34580101E-11 | 0.34535057E-11 |
| 27.50 | 0.27119289E-12 | 0.27104760E-12 | 0.27080560E-12 | 0.27046718E-12 |
| 30.00 | 0.21321278E-13 | 0.21310792E-13 | 0.21293326E-13 | 0.21268897E-13 |

| x | TABLE OF $K_1^S(x)$ | | | | |
|-------|---------------------|-----------------|-----------------|-----------------|-----------------|
| | S = 0.60 | S = 0.70 | S = 0.80 | S = 0.90 | S = 1.00 |
| 0.01 | 0.29747097E 00 | -0.27204895E 00 | -0.57006827E 00 | -0.62365445E 00 | -0.30063372E 00 |
| 0.02 | 0.7733597E 00 | -0.8102190E 00 | -0.1894205E 00 | -0.4176608E 00 | -0.4760808E 00 |
| 0.03 | 0.9950500E 00 | 0.47623850E 00 | 0.68701550E-01 | -0.20936785E 00 | -0.35806366E 00 |
| 0.04 | 0.11759430E 01 | 0.64369281E 00 | 0.24875465E 00 | -0.46551235E 00 | -0.23578658E 00 |
| 0.05 | 0.1189357E 01 | 0.75580180E 00 | 0.37987163E 00 | 0.82444795E-01 | -0.12703551E 00 |
| 0.06 | 0.12334296E 01 | 0.83391896E 00 | 0.47825146E 00 | 0.18262170E 00 | -0.33255088E-01 |
| 0.07 | 0.1258270E 01 | 0.8850527E 00 | 0.55362885E 00 | 0.28707120E 00 | 0.47117079E-01 |
| 0.08 | 0.1272598E 01 | 0.9203605E 00 | 0.61221647E 00 | 0.33729648E 00 | 0.11577325E 00 |
| 0.09 | 0.12774433E 01 | 0.95794420E 00 | 0.65817846E 00 | 0.39348132E 00 | 0.17866625E 00 |
| 0.10 | 0.1278049E 01 | 0.97806226E 00 | 0.69441819E 00 | 0.44005226E 00 | 0.22538188E 00 |
| 0.20 | 0.11529420E 01 | 0.97964798E 00 | 0.80482531E 00 | 0.62455425E 00 | 0.47535334E 00 |
| 0.30 | 0.99581751E 00 | 0.8818149E 00 | 0.76311347E 00 | 0.64362298E 00 | 0.52711389E 00 |
| 0.40 | 0.82664235E 00 | 0.77631581E 00 | 0.69118930E 00 | 0.60377069E 00 | 0.51648739E 00 |
| 0.50 | 0.73854609E 00 | 0.67948979E 00 | 0.61617620E 00 | 0.55026405E 00 | 0.48539695E 00 |
| 0.60 | 0.63886319E 00 | 0.59415840E 00 | 0.54582211E 00 | 0.49499825E 00 | 0.44428318E 00 |
| 0.70 | 0.52447862E 00 | 0.5190064E 00 | 0.48226787E 00 | 0.44239425E 00 | 0.40110918E 00 |
| 0.80 | 0.48270504E 00 | 0.45221156E 00 | 0.4257570E 00 | 0.39042285E 00 | 0.36099256E 00 |
| 0.90 | 0.42135947E 00 | 0.39968662E 00 | 0.37587484E 00 | 0.35036006E 00 | 0.32260524E 00 |
| 1.00 | 0.36886651E 00 | 0.35122596E 00 | 0.33197136E 00 | 0.31125650E 00 | 0.28942804E 00 |
| 1.10 | 0.32327919E 00 | 0.30908388E 00 | 0.29338181E 00 | 0.27442828E 00 | 0.25049134E 00 |
| 1.20 | 0.28399421E 00 | 0.27264440E 00 | 0.2596592E 00 | 0.24596958E 00 | 0.23066505E 00 |
| 1.30 | 0.24987861E 00 | 0.24030737E 00 | 0.22964514E 00 | 0.21806669E 00 | 0.20573601E 00 |
| 1.40 | 0.22259E 00 | 0.21020403E 00 | 0.19020403E 00 | 0.1715994E 00 | 0.15435999E 00 |
| 1.50 | 0.19433389E 00 | 0.18770142E 00 | 0.18029973E 00 | 0.17222642E 00 | 0.16355999E 00 |
| 1.60 | 0.17169516E 00 | 0.16614098E 00 | 0.15993248E 00 | 0.15314754E 00 | 0.14586945E 00 |
| 1.70 | 0.15480714E 00 | 0.14480894E 00 | 0.13614037E 00 | 0.1264037E 00 | 0.1180380E 00 |
| 1.80 | 0.13442327E 00 | 0.13051189E 00 | 0.12609400E 00 | 0.1212519E 00 | 0.1160380E 00 |
| 1.90 | 0.1194900E 00 | 0.11581315E 00 | 0.11206928E 00 | 0.10795877E 00 | 0.10329264E 00 |
| 2.00 | 0.1056760E 00 | 0.10284427E 00 | 0.9961735E-01 | 0.9616347E-01 | 0.9239460E-01 |
| 2.50 | 0.58616501E-01 | 0.57320488E-01 | 0.55857870E-01 | 0.54241768E-01 | 0.52486461E-01 |
| 3.00 | 0.32964867E-01 | 0.32940945E-01 | 0.3163813E-01 | 0.30858676E-01 | 0.30008659E-01 |
| 3.50 | 0.1878948E-01 | 0.18412125E-01 | 0.18116035E-01 | 0.1787855E-01 | 0.17254357E-01 |
| 4.00 | 0.1071634E-01 | 0.10560474E-01 | 0.1038312E-01 | 0.1018353E-01 | 0.9984988E-02 |
| 4.50 | 0.6171130E-02 | 0.60904541E-02 | 0.5998069E-02 | 0.5890989E-02 | 0.5783494E-02 |
| 5.00 | 0.35711861E-02 | 0.35288021E-02 | 0.34804920E-02 | 0.34265007E-02 | 0.33671000E-02 |
| 6.00 | 0.12097947E-02 | 0.11976675E-02 | 0.11838492E-02 | 0.11683097E-02 | 0.11510599E-02 |
| 7.00 | 0.41667154E-03 | 0.41107291E-03 | 0.4065802E-03 | 0.40234240E-03 | 0.39724338E-03 |
| 8.00 | 0.1423353E-03 | 0.14072624E-03 | 0.13811330E-03 | 0.1362958E-03 | 0.13480630E-03 |
| 9.00 | 0.49923266E-04 | 0.49481816E-04 | 0.49109230E-04 | 0.48750274E-04 | 0.48403072E-04 |
| 10.00 | 0.17477060E-04 | 0.17368881E-04 | 0.17244868E-04 | 0.17105359E-04 | 0.16950736E-04 |
| 12.00 | 0.21695011E-05 | 0.21590780E-05 | 0.2148173E-05 | 0.21365150E-05 | 0.21243266E-05 |
| 14.00 | 0.2727252E-06 | 0.27105374E-06 | 0.27010045E-06 | 0.26891964E-06 | 0.26767214E-06 |
| 16.00 | 0.34614017E-07 | 0.34477758E-07 | 0.34321189E-07 | 0.34144588E-07 | 0.33944265E-07 |
| 18.00 | 0.44254461E-08 | 0.44099099E-08 | 0.43920483E-08 | 0.43718920E-08 | 0.43494721E-08 |
| 20.00 | 0.56910113E-09 | 0.56729804E-09 | 0.56522455E-09 | 0.56288361E-09 | 0.56027857E-09 |
| 22.50 | 0.44114420E-10 | 0.43989844E-10 | 0.4386533E-10 | 0.43684670E-10 | 0.43504463E-10 |
| 25.00 | 0.3493982E-11 | 0.34810216E-11 | 0.3467939E-11 | 0.3450545E-11 | 0.34296316E-11 |
| 27.50 | 0.2695255E-12 | 0.26881737E-12 | 0.2681371E-12 | 0.26734496E-12 | 0.26648394E-12 |
| 30.00 | 0.21199253E-13 | 0.21154105E-13 | 0.21102130E-13 | 0.21043377E-13 | 0.20977902E-13 |

| x | TABLE OF $K_1^S(x)$ | | | | |
|-------|---------------------|-----------------|-----------------|-----------------|-----------------|
| | S = 1.10 | S = 1.20 | S = 1.30 | S = 1.40 | S = 1.50 |
| 0.01 | -0.28748368E 00 | -0.66495068E-01 | 0.10195905E 00 | 0.18837471E 00 | 0.19332416E 00 |
| 0.02 | -0.42393795E 00 | -0.29691558E 00 | -0.14562747E 00 | -0.9443019E-02 | 0.8641546E-01 |
| 0.03 | -0.39441680E 00 | -0.34693318E 00 | -0.2492950E 00 | -0.13418610E 00 | -0.28312976E-01 |
| 0.04 | -0.32573428E 00 | -0.35355401E 00 | -0.28279795E 00 | -0.19905718E 00 | -0.1059824E 00 |
| 0.05 | -0.24973831E 00 | -0.29584943E 00 | -0.28193332E 00 | -0.22781813E 00 | -0.1534864E 00 |
| 0.06 | -0.1763170E 00 | -0.24924473E 00 | -0.26369778E 00 | -0.23902633E 00 | -0.1798430E 00 |
| 0.07 | -0.10861496E 00 | -0.2008670E 00 | -0.23137845E 00 | -0.225110E 00 | -0.1917713E 00 |
| 0.08 | -0.4187899E-01 | -0.15261787E 00 | -0.2056294E 00 | -0.21565012E 00 | -0.1937787E 00 |
| 0.09 | 0.75043705E-02 | -0.10722446E 00 | -0.17316417E 00 | -0.19741419E 00 | -0.18931505E 00 |
| 0.10 | 0.56528822E-01 | -0.64738851E-01 | -0.14062455E 00 | -0.17656411E 00 | -0.18024888E 00 |
| 0.20 | 0.33216212E 00 | 0.20864980E 00 | 0.10692331E 00 | 0.2784917E-01 | 0.29840211E-01 |
| 0.30 | 0.4169856E 00 | 0.31592214E 00 | 0.22204554E 00 | 0.1487450E 00 | 0.8470670E-01 |
| 0.40 | 0.4315884E 00 | 0.35103988E 00 | 0.2765365E 00 | 0.2093650E 00 | 0.15028968E 00 |
| 0.50 | 0.41713595E 00 | 0.35292715E 00 | 0.2902159E 00 | 0.23547202E 00 | 0.18909341E 00 |
| 0.60 | 0.39046201E 00 | 0.33892281E 00 | 0.28917546E 00 | 0.24205108E 00 | 0.19823785E 00 |
| 0.70 | 0.35922161E 00 | 0.31794617E 00 | 0.2787808E 00 | 0.23798840E 00 | 0.20053124E 00 |
| 0.80 | 0.32715839E 00 | 0.29319305E 00 | 0.2596363E 00 | 0.22700140E 00 | 0.1974341E 00 |
| 0.90 | 0.29606994E 00 | 0.26820760E 00 | 0.24045262E 00 | 0.21202991E 00 | 0.1868430E 00 |
| 1.00 | 0.26684112E 00 | 0.24346624E 00 | 0.22078323E 00 | 0.19797325E 00 | 0.17571212E 00 |
| 1.10 | 0.23984528E 00 | 0.22076433E 00 | 0.20151651E 00 | 0.18235796E 00 | 0.16392776E 00 |
| 1.20 | 0.21518611E 00 | 0.19927591E 00 | 0.18314736E 00 | 0.16700586E 00 | 0.15104252E 00 |
| 1.30 | 0.1928274E 00 | 0.17949849E 00 | 0.1659327E 00 | 0.15229364E 00 | 0.13873624E 00 |
| 1.40 | 0.17263979E 00 | 0.16181364E 00 | 0.14938881E 00 | 0.13843040E 00 | 0.1268894E 00 |
| 1.50 | 0.15448179E 00 | 0.14302994E 00 | 0.13533916E 00 | 0.12531866E 00 | 0.11567397E 00 |
| 1.60 | 0.13818539E 00 | 0.13018476E 00 | 0.12195766E 00 | 0.11359327E 00 | 0.10517838E 00 |
| 1.70 | 0.12538212E 00 | 0.11679002E 00 | 0.10978709E 00 | 0.10264650E 00 | 0.95439861E-01 |
| 1.80 | 0.11051050E 00 | 0.10472887E 00 | 0.98753456E-01 | 0.92644520E-01 | 0.86461324E-01 |
| 1.90 | 0.9818768E-01 | 0.9385129E-01 | 0.88774994E-01 | 0.83338173E-01 | 0.7822384E-01 |
| 2.00 | 0.8836656E-01 | 0.84146964E-01 | 0.79767893E-01 | 0.75270066E-01 | 0.70695017E-01 |
| 2.50 | 0.50607191E-01 | 0.48619956E-01 | 0.46541293E-01 | 0.44388072E-01 | 0.4217720E-01 |
| 3.00 | 0.2909478E-01 | 0.28122917E-01 | 0.27101049E-01 | 0.26036154E-01 | 0.24935637E-01 |
| 3.50 | 0.14746598E-01 | 0.1430884E-01 | 0.1397878E-01 | 0.1362759E-01 | 0.13264184E-01 |
| 4.00 | 0.97346404E-02 | 0.9483288E-02 | 0.92183142E-02 | 0.8939193E-02 | 0.8645143E-02 |
| 4.50 | 0.56614049E-02 | 0.55040882E-02 | 0.53614380E-02 | 0.52449810E-02 | 0.50918699E-02 |
| 5.00 | 0.33025857E-02 | 0.32332758E-02 | 0.31595076E-02 | 0.30816350E-02 | 0.3000265E-02 |
| 6.00 | 0.11325804E-02 | 0.1125121E-02 | 0.10910849E-02 | 0.10683871E-02 | 0.10445114E-02 |
| 7.00 | 0.3916795E-03 | 0.38567270E-03 | 0.37924362E-03 | 0.37241603E-03 | 0.36521442E-03 |
| 8.00 | 0.1363671E-03 | 0.13451617E-03 | 0.13233825E-03 | 0.13036525E-03 | 0.1282127E-03 |
| 9.00 | 0.4773007E-04 | 0.47152864E-04 | 0.46533133E-04 | 0.45872691E-04 | 0.45173447E-04 |
| 10.00 | 0.16781417E-04 | 0.16597858E-04 | 0.16400551E-04 | 0.16190019E-04 | 0.15966818E-04 |
| 12.00 | 0.20905073E-05 | 0.20773944E-05 | 0.20645127E-05 | 0.2044440E-05 | 0.2028080E-05 |
| 14.00 | 0.26483336E-06 | 0.26273650E-06 | 0.26047585E-06 | 0.25805368E-06 | 0.25548076E-06 |
| 16.00 | 0.33732565E-07 | 0.33497866E-07 | 0.33244376E-07 | 0.32973134E-07 | 0.32684007E-07 |
| 18.00 | 0.43248237E-08 | 0.4279854E-08 | 0.4268990E-08 | 0.42379097E-08 | 0.42047653E-08 |
| 20.00 | 0.55741313E-09 | 0.55429135E-09 | 0.55091763E-09 | 0.54729672E-09 | 0.54343371E-09 |
| 22.50 | 0.43306139E-10 | 0.43089950E-10 | 0.42856169E-10 | 0.42605091E-10 | 0.4237031E-10 |
| 25.00 | 0.33846336E-11 | 0.33676663E-11 | 0.3351762E-11 | 0.33334691E-11 | 0.33145901E-11 |
| 27.50 | 0.26484069E-12 | 0.26435204E-12 | 0.2631737E-12 | 0.2619700E-12 | 0.26055314E-12 |
| 30.00 | 0.20905774E-13 | 0.20827056E-13 | 0.20741826E-13 | 0.20650164E-13 | 0.20552159E-13 |

| X | TABLE OF KIS(X) | | | |
|-------|---------------------|---------------------|---------------------|---------------------|
| | S ^m 1.60 | S ^m 1.70 | S ^m 1.80 | S ^m 2.00 |
| 0.01 | 0.14026356E 00 | 0.61438804E-01 | -0.12001668E-01 | -0.59422897E-01 |
| 0.05 | 0.13238656E 00 | 0.13258179E 00 | 0.14002889E 00 | 0.53000526E-01 |
| 0.10 | 0.12447040E-01 | 0.9494337E-01 | 0.10671610E 00 | 0.90366155E-01 |
| 0.04 | -0.2224231E-01 | 0.39999784E-01 | 0.75775487E-01 | 0.86052554E-01 |
| 0.05 | -0.76431757E-01 | -0.98434757E-02 | 0.38287794E-01 | 0.65103335E-01 |
| 0.06 | -0.11380355E 00 | -0.49852460E-01 | 0.28283798E-02 | 0.39194769E-01 |
| 0.07 | -0.1380415E 00 | -0.8012011E-01 | 0.27543144E-01 | 0.13517817E-01 |
| 0.08 | -0.13229754E 00 | -0.10199014E 00 | -0.52232196E-01 | -0.97364279E-02 |
| 0.09 | -0.15935764E 00 | -0.11697728E 00 | -0.71564374E-01 | -0.29771975E-01 |
| 0.10 | -0.16054570E 00 | -0.12644789E 00 | -0.86117172E-01 | -0.46481857E-01 |
| 0.20 | -0.6733226E-01 | -0.87479305E-01 | -0.93467687E-01 | -0.88744821E-01 |
| 0.30 | 0.3369357E-01 | -0.41674925E-02 | -0.30682741E-01 | -0.46037184E-01 |
| 0.40 | 0.10005447E 00 | 0.58528502E-01 | 0.25556250E-01 | 0.60991044E-03 |
| 0.50 | 0.1384564E 00 | 0.98888906E-01 | 0.65483302E-01 | 0.38121697E-01 |
| 0.60 | 0.15826890E 00 | 0.12251712E 00 | 0.91196796E-01 | 0.64371506E-01 |
| 0.70 | 0.16607234E 00 | 0.13456615E 00 | 0.10622841E 00 | 0.81287654E-01 |
| 0.80 | 0.16642059E 00 | 0.13883842E 00 | 0.11374538E 00 | 0.91134461E-01 |
| 0.90 | 0.16184835E 00 | 0.13797294E 00 | 0.112599104E 00 | 0.95841015E-01 |
| 1.00 | 0.15426473E 00 | 0.13386062E 00 | 0.11469088E 00 | 0.96490347E-01 |
| 1.10 | 0.14524360E 00 | 0.12719813E 00 | 0.11105422E 00 | 0.95453212E-01 |
| 1.20 | 0.1324443E 00 | 0.12036439E 00 | 0.10594591E 00 | 0.92307914E-01 |
| 1.30 | 0.12540075E 00 | 0.11244533E 00 | 0.99965498E-01 | 0.88072434E-01 |
| 1.40 | 0.11548808E 00 | 0.10433760E 00 | 0.93532733E-01 | 0.83141921E-01 |
| 1.50 | 0.10590534E 00 | 0.96306205E-01 | 0.86961852E-01 | 0.77948380E-01 |
| 1.60 | 0.96796019E-01 | 0.88524230E-01 | 0.80434963E-01 | 0.72593208E-01 |
| 1.70 | 0.88238108E-01 | 0.81100505E-01 | 0.74093755E-01 | 0.67271262E-01 |
| 1.80 | 0.80526542E-01 | 0.74091892E-01 | 0.68026542E-01 | 0.62088194E-01 |
| 1.90 | 0.72879899E-01 | 0.67552181E-01 | 0.62283979E-01 | 0.57115471E-01 |
| 2.00 | 0.66082571E-01 | 0.61471278E-01 | 0.56897739E-01 | 0.52396162E-01 |
| 2.50 | 0.39925775E-01 | 0.37650172E-01 | 0.35366564E-01 | 0.33090394E-01 |
| 3.00 | 0.23806950E-01 | 0.22857508E-01 | 0.21494616E-01 | 0.20325399E-01 |
| 3.50 | 0.14144259E-01 | 0.13524088E-01 | 0.12920039E-01 | 0.12305141E-01 |
| 4.00 | 0.81618345E-02 | 0.80379792E-02 | 0.77213535E-02 | 0.73946580E-02 |
| 4.50 | 0.49328769E-02 | 0.47667845E-02 | 0.46003982E-02 | 0.44283015E-02 |
| 5.00 | 0.29150615E-02 | 0.28271280E-02 | 0.27366197E-02 | 0.26439333E-02 |
| 6.00 | 0.18125139E-02 | 0.189361374E-02 | 0.18679103E-02 | 0.18391882E-02 |
| 7.00 | 0.35764431E-03 | 0.34979211E-03 | 0.34162494E-03 | 0.33314055E-03 |
| 8.00 | 0.12587643E-03 | 0.12343458E-03 | 0.12089472E-03 | 0.11826462E-03 |
| 9.00 | 0.44437402E-04 | 0.43666642E-04 | 0.42863328E-04 | 0.42029885E-04 |
| 10.00 | 0.15733513E-04 | 0.15484767E-04 | 0.15227157E-04 | 0.14959355E-04 |
| 12.00 | 0.19860119E-05 | 0.19598538E-05 | 0.19324818E-05 | 0.19039493E-05 |
| 14.00 | 0.25273670E-06 | 0.24988683E-06 | 0.24687659E-06 | 0.24373596E-06 |
| 16.00 | 0.32377671E-07 | 0.32054708E-07 | 0.31715048E-07 | 0.31360959E-07 |
| 18.00 | 0.41696170E-08 | 0.41325189E-08 | 0.40935262E-08 | 0.40526992E-08 |
| 20.00 | 0.53933398E-09 | 0.53500321E-09 | 0.53044738E-09 | 0.52567277E-09 |
| 22.50 | 0.42025232E-10 | 0.41751323E-10 | 0.41434401E-10 | 0.41101950E-10 |
| 25.00 | 0.32944436E-11 | 0.32731723E-11 | 0.32507600E-11 | 0.32272138E-11 |
| 27.50 | 0.25113575E-12 | 0.25075973E-12 | 0.250398327E-12 | 0.25005766E-12 |
| 30.00 | 0.20044790E-13 | 0.20037449E-13 | 0.20021046E-13 | 0.20008657E-13 |

| X | TABLE OF KIS(X) | | | |
|-------|---------------------|---------------------|---------------------|---------------------|
| | S ^m 2.50 | S ^m 3.00 | S ^m 3.50 | S ^m 4.00 |
| 0.01 | 0.29355202E-01 | -0.12297294E-01 | 0.53432316E-02 | -0.23364273E-02 |
| 0.05 | -0.15480673E-01 | -0.9272778E-02 | -0.48535239E-02 | 0.22249086E-02 |
| 0.10 | -0.3120297E-01 | 0.11505822E-01 | -0.32652491E-02 | 0.59947575E-03 |
| 0.04 | -0.24430559E-01 | 0.28774341E-02 | 0.19824023E-02 | -0.18212355E-02 |
| 0.05 | -0.10424645E-01 | -0.56105858E-02 | 0.50104726E-02 | -0.22874876E-02 |
| 0.06 | 0.3591609E-02 | -0.10891746E-01 | 0.53585282E-02 | -0.13755139E-02 |
| 0.07 | 0.14997759E-01 | -0.1245805E-01 | 0.39885902E-02 | -0.27272491E-04 |
| 0.08 | 0.2314607E-01 | -0.12477790E-01 | 0.11869904E-02 | -0.11878995E-02 |
| 0.09 | 0.2824955E-01 | -0.10442766E-01 | -0.3609129E-03 | 0.19613255E-02 |
| 0.10 | 0.30748132E-01 | -0.75188390E-02 | -0.23103322E-02 | 0.23123935E-02 |
| 0.20 | 0.6049534E-03 | 0.1293483E-01 | -0.13341055E-02 | -0.20243402E-02 |
| 0.30 | -0.2619556E-01 | 0.30810977E-02 | 0.49780385E-02 | -0.10795136E-02 |
| 0.40 | -0.3113896E-01 | -0.49567333E-02 | 0.46514034E-02 | 0.14530036E-02 |
| 0.50 | -0.24450952E-01 | -0.11362551E-01 | 0.12654449E-02 | 0.23468710E-02 |
| 0.60 | -0.13530202E-01 | -0.13116929E-01 | -0.21480823E-02 | 0.17887503E-02 |
| 0.70 | -0.21544053E-02 | -0.1171684E-01 | -0.44441654E-02 | 0.59397005E-03 |
| 0.80 | 0.7980207E-02 | -0.5612644E-02 | -0.54626741E-02 | -0.6315926E-03 |
| 0.90 | 0.16329680E-01 | -0.47778829E-02 | -0.54425856E-02 | -0.15827776E-02 |
| 1.00 | 0.22763552E-01 | -0.88614793E-03 | -0.46989229E-02 | -0.21607136E-02 |
| 1.10 | 0.27456163E-01 | 0.27125950E-02 | -0.35214242E-02 | -0.23793940E-02 |
| 1.20 | 0.30651578E-01 | 0.28384311E-02 | -0.2136511E-02 | -0.23029248E-02 |
| 1.30 | 0.32610407E-01 | 0.4248120E-02 | -0.70579491E-03 | -0.20096718E-02 |
| 1.40 | 0.33712727E-01 | 0.14871821E-01 | 0.6537372E-03 | -0.15744488E-02 |
| 1.50 | 0.33767414E-01 | 0.12016433E-01 | 0.19074048E-02 | -0.10593099E-02 |
| 1.60 | 0.33363611E-01 | 0.13113654E-01 | 0.29895289E-02 | -0.51443407E-03 |
| 1.70 | 0.3210376E-01 | 0.13203087E-01 | 0.38977878E-02 | 0.24210056E-04 |
| 1.80 | 0.3135663E-01 | 0.14212145E-01 | 0.46322329E-02 | 0.53154699E-03 |
| 1.90 | 0.2996084E-01 | 0.14332486E-01 | 0.52021931E-02 | 0.99454608E-03 |
| 2.00 | 0.28432376E-01 | 0.14238041E-01 | 0.56218634E-02 | 0.13551624E-02 |
| 2.50 | 0.20333298E-01 | 0.11924338E-01 | 0.60626894E-02 | 0.2539549E-02 |
| 3.00 | 0.1353739E-01 | 0.87304814E-02 | 0.50788616E-02 | 0.26036465E-02 |
| 3.50 | 0.8678534E-02 | 0.59791344E-02 | 0.37838833E-02 | 0.21822312E-02 |
| 4.00 | 0.2484649E-02 | 0.39463859E-02 | 0.26436067E-02 | 0.16201926E-02 |
| 4.50 | 0.3366746E-02 | 0.25185716E-02 | 0.1775175E-02 | 0.1173763E-02 |
| 5.00 | 0.20639617E-02 | 0.15891029E-02 | 0.11609487E-02 | 0.80234103E-03 |
| 6.00 | 0.76300096E-03 | 0.61323939E-03 | 0.47230830E-03 | 0.34800074E-03 |
| 7.00 | 0.27856073E-03 | 0.2308441E-03 | 0.18433481E-03 | 0.1421010E-03 |
| 8.00 | 0.10103794E-03 | 0.85676600E-04 | 0.70455987E-04 | 0.56054744E-04 |
| 9.00 | 0.36320977E-04 | 0.31528119E-04 | 0.26467600E-04 | 0.21617536E-04 |
| 10.00 | 0.13176998E-04 | 0.11540111E-04 | 0.98592047E-05 | 0.82140857E-05 |
| 12.00 | 0.1711768E-05 | 0.15320897E-05 | 0.13431190E-05 | 0.11523479E-05 |
| 14.00 | 0.22242924E-06 | 0.20217375E-06 | 0.18055471E-06 | 0.15481182E-06 |
| 16.00 | 0.28490979E-07 | 0.26615090E-07 | 0.24101982E-07 | 0.21490648E-07 |
| 18.00 | 0.37277867E-08 | 0.35014274E-08 | 0.3205313E-08 | 0.29492965E-08 |
| 20.00 | 0.49281199E-09 | 0.46073316E-09 | 0.42564497E-09 | 0.38806498E-09 |
| 22.50 | 0.38805022E-10 | 0.36547013E-10 | 0.34045075E-10 | 0.31367590E-10 |
| 25.00 | 0.30641664E-11 | 0.29029587E-11 | 0.27232120E-11 | 0.25294366E-11 |
| 27.50 | 0.2427059E-12 | 0.23092438E-12 | 0.21716716E-12 | 0.20371610E-12 |
| 30.00 | 0.19246411E-13 | 0.1839679E-13 | 0.17440198E-13 | 0.16397587E-13 |

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| x | TABLE OF $K_1^S(x)$ | | | | |
|-------|---------------------|-----------------|-----------------|-----------------|-----------------|
| | S= 5.00 | S= 6.00 | S= 7.00 | S= 8.00 | |
| 0.01 | -0.38948310E-03 | -0.31178954E-04 | 0.76307617E-05 | 0.30882002E-05 | 0.26572833E-06 |
| 0.02 | -0.13102575E-03 | 0.61441965E-04 | 0.14869390E-04 | -0.23662171E-05 | 0.24110111E-06 |
| 0.03 | -0.36179512E-03 | -0.35923033E-04 | -0.12118682E-04 | -0.25565985E-05 | -0.40073930E-06 |
| 0.04 | -0.42768742E-03 | -0.54449040E-04 | -0.34925948E-04 | 0.41278777E-06 | 0.21592117E-06 |
| 0.05 | -0.11577041E-03 | 0.47910055E-04 | 0.15473990E-04 | 0.29050422E-05 | 0.42116585E-06 |
| 0.06 | 0.26069135E-03 | 0.81722871E-04 | 0.79657129E-04 | -0.72227290E-06 | -0.46378621E-06 |
| 0.07 | 0.42978725E-03 | 0.39879460E-04 | -0.83623977E-05 | -0.30746966E-05 | -0.46772998E-06 |
| 0.08 | 0.37986958E-03 | -0.24182613E-04 | -0.15840648E-04 | -0.17557935E-05 | 0.19025121E-06 |
| 0.09 | -0.19759182E-03 | -0.65863529E-04 | -0.11712317E-04 | 0.10249261E-05 | 0.59468879E-06 |
| 0.10 | -0.23714179E-04 | -0.82412649E-04 | -0.14443733E-05 | 0.28586694E-05 | 0.44036121E-06 |
| 0.20 | -0.16035126E-03 | 0.38848723E-04 | -0.15678832E-04 | -0.13219312E-05 | 0.45893893E-06 |
| 0.30 | -0.29551276E-03 | -0.23030238E-04 | 0.15376377E-04 | 0.11956160E-05 | -0.20876717E-06 |
| 0.40 | -0.27930614E-03 | 0.41278063E-04 | -0.28951555E-05 | -0.91326299E-06 | 0.47761624E-06 |
| 0.50 | -0.42411715E-03 | 0.79333640E-04 | -0.15681239E-04 | 0.30815285E-05 | -0.53970759E-06 |
| 0.60 | -0.18137468E-03 | 0.15969917E-04 | -0.20216410E-05 | 0.62643362E-06 | -0.23928938E-06 |
| 0.70 | 0.14442374E-03 | -0.54976373E-04 | 0.12938555E-04 | -0.26471731E-05 | 0.50317706E-06 |
| 0.80 | 0.36766284E-03 | -0.32838071E-04 | 0.15204753E-04 | -0.26914508E-05 | 0.49918525E-06 |
| 0.90 | 0.43861397E-03 | -0.46412848E-04 | 0.46802616E-05 | -0.33788381E-06 | -0.58320111E-07 |
| 1.00 | 0.38046182E-03 | -0.24318213E-04 | -0.44821978E-05 | 0.20491847E-05 | -0.52162983E-06 |
| 1.10 | 0.24144487E-03 | 0.21929110E-04 | -0.12953294E-05 | 0.30916025E-05 | -0.57828110E-06 |
| 1.20 | 0.68734564E-04 | 0.38606950E-04 | -0.16004789E-04 | 0.25727070E-05 | -0.24016734E-06 |
| 1.30 | -0.10159957E-03 | 0.79266938E-04 | 0.10494255E-04 | 0.10494255E-05 | 0.14079669E-06 |
| 1.40 | -0.24630984E-03 | 0.83180559E-04 | -0.81939465E-05 | -0.74619444E-06 | 0.47461992E-06 |
| 1.50 | -0.39406010E-03 | 0.73040531E-04 | -0.10117679E-05 | -0.22036065E-05 | 0.60865877E-06 |
| 1.60 | -0.42043088E-03 | 0.53006565E-04 | 0.59407006E-05 | -0.30008374E-05 | 0.53361157E-06 |
| 1.70 | -0.44737526E-03 | 0.27469538E-04 | 0.11949266E-04 | -0.30682793E-05 | 0.30742368E-06 |
| 1.80 | -0.43963079E-03 | 0.28649636E-04 | 0.14931787E-04 | -0.25071306E-05 | 0.18640816E-06 |
| 1.90 | -0.40370162E-03 | -0.25489699E-04 | 0.16187158E-04 | -0.15244717E-05 | -0.26866714E-06 |
| 2.00 | -0.34633788E-03 | -0.47779430E-04 | 0.15413790E-04 | 0.34356893E-06 | -0.48228586E-06 |
| 2.50 | 0.62487561E-04 | -0.83271902E-04 | -0.37246029E-05 | 0.31675117E-05 | -0.17729867E-06 |
| 3.00 | 0.37941675E-03 | -0.27921892E-04 | -0.16463783E-04 | 0.52615130E-06 | 0.59603325E-06 |
| 3.50 | 0.50907219E-03 | 0.40748091E-04 | -0.11728931E-04 | -0.26749249E-05 | 0.34207096E-06 |
| 4.00 | 0.48965972E-03 | 0.78379188E-04 | 0.33679133E-05 | -0.33229889E-05 | -0.31839611E-07 |
| 4.50 | 0.41217779E-03 | 0.98810962E-04 | 0.11364059E-04 | -0.14022429E-05 | -0.64564325E-06 |
| 5.00 | 0.31859102E-03 | 0.93831314E-04 | 0.17676473E-04 | 0.88626253E-06 | -0.50800998E-06 |
| 6.00 | 0.16367417E-03 | 0.62340858E-04 | 0.14871621E-04 | 0.36336940E-06 | 0.29475931E-06 |
| 7.00 | 0.75060449E-04 | 0.33482869E-04 | 0.12312673E-04 | 0.35709323E-05 | 0.74050240E-06 |
| 8.00 | 0.32161473E-04 | 0.16031980E-04 | 0.68458801E-05 | 0.24486653E-05 | 0.70506962E-06 |
| 9.00 | 0.13213431E-04 | 0.71549807E-05 | 0.34014136E-05 | 0.14007757E-05 | 0.48944119E-06 |
| 10.00 | 0.52781218E-05 | 0.30482163E-05 | 0.15724229E-05 | 0.71823135E-06 | 0.28684153E-06 |
| 12.00 | 0.79871171E-06 | 0.20653674E-06 | 0.29380554E-06 | 0.15506822E-06 | 0.74034006E-07 |
| 14.00 | 0.11594914E-06 | 0.78336046E-07 | 0.49280807E-07 | -0.28660197E-07 | -0.15371668E-07 |
| 16.00 | 0.16303194E-07 | 0.11607864E-07 | 0.77466791E-08 | 0.48371116E-08 | 0.28211569E-08 |
| 18.00 | 0.22636742E-08 | 0.16739368E-08 | 0.11693438E-08 | 0.77074161E-09 | 0.47876411E-09 |
| 20.00 | 0.31100591E-09 | 0.23703628E-09 | 0.17169428E-09 | 0.11809677E-09 | 0.77060634E-10 |
| 22.50 | 0.25759675E-10 | 0.20233444E-10 | 0.15193887E-10 | 0.10901665E-10 | 0.74687485E-11 |
| 25.00 | 0.21819631E-11 | 0.17062967E-11 | 0.13117074E-11 | 0.97727154E-12 | 0.69581697E-12 |
| 27.50 | 0.17338440E-12 | 0.14245326E-12 | 0.11823798E-12 | 0.85813381E-13 | 0.63046007E-13 |
| 30.00 | 0.14140262E-13 | 0.11179516E-13 | 0.95155774E-14 | 0.79224315E-14 | 0.55766847E-14 |

| x | TABLE OF $K_1^S(x)$ | | | | |
|-------|---------------------|-----------------|-----------------|-----------------|-----------------|
| | S= 10.00 | S= 12.00 | S= 14.00 | S= 16.00 | S= 18.00 |
| 0.01 | -0.86737929E-07 | 0.22413751E-08 | -0.92584169E-10 | 0.55583705E-11 | -0.30438048E-12 |
| 0.02 | -0.17866405E-07 | -0.47098712E-08 | 0.43723490E-10 | 0.57166407E-11 | -0.30869425E-12 |
| 0.03 | -0.81310759E-07 | -0.56987078E-08 | 0.14048254E-09 | 0.66200864E-11 | -0.13405684E-12 |
| 0.04 | 0.55574556E-07 | 0.19721900E-08 | 0.82236535E-11 | -0.44764853E-11 | -0.31044510E-12 |
| 0.05 | 0.49394539E-07 | 0.151116727E-09 | -0.32065860E-11 | 0.15075488E-11 | 0.19436259E-12 |
| 0.06 | -0.11765550E-06 | -0.39289163E-08 | -0.10038957E-09 | -0.31325634E-11 | -0.19662738E-12 |
| 0.07 | -0.24190056E-07 | 0.35635674E-08 | 0.18847320E-09 | 0.67878525E-11 | 0.24381956E-12 |
| 0.08 | -0.14818400E-06 | 0.29459494E-08 | -0.60096900E-10 | -0.62635924E-11 | -0.30973196E-12 |
| 0.09 | 0.88292623E-07 | -0.31077648E-08 | -0.17346859E-09 | -0.16587287E-11 | 0.14309426E-12 |
| 0.10 | -0.26280913E-07 | -0.42804338E-08 | 0.56930424E-10 | 0.75796359E-11 | 0.21530206E-12 |
| 0.20 | -0.91263587E-07 | 0.36704531E-08 | -0.10419553E-09 | -0.77334073E-11 | -0.23457321E-12 |
| 0.30 | -0.11684823E-06 | -0.23579105E-08 | 0.41561844E-11 | 0.14643663E-11 | 0.48911991E-13 |
| 0.40 | -0.11931325E-06 | 0.10046674E-08 | -0.14326529E-09 | -0.75962401E-11 | 0.25218816E-12 |
| 0.50 | 0.67717247E-07 | -0.29662968E-08 | -0.14087371E-09 | 0.66526979E-11 | -0.22948121E-13 |
| 0.60 | 0.78731205E-07 | 0.47051355E-08 | 0.47111438E-10 | -0.73186010E-11 | -0.20088883E-13 |
| 0.70 | -0.87402913E-07 | -0.98288449E-09 | 0.12627712E-09 | 0.70426130E-11 | -0.92971625E-13 |
| 0.80 | -0.10081632E-06 | 0.49581392E-08 | -0.17092617E-10 | -0.12907094E-11 | 0.26895000E-12 |
| 0.90 | 0.21793132E-07 | 0.36836734E-08 | -0.66878772E-10 | 0.67611559E-11 | -0.27308877E-12 |
| 1.00 | 0.11249551E-06 | 0.45937882E-08 | 0.16903777E-09 | 0.42643007E-11 | -0.53818398E-13 |
| 1.10 | 0.98270576E-07 | 0.24909641E-08 | 0.12186876E-09 | -0.65284253E-11 | 0.31063213E-12 |
| 1.20 | -0.11648277E-07 | -0.17332918E-08 | -0.42621485E-10 | -0.27018488E-11 | 0.12422287E-13 |
| 1.30 | -0.76996288E-07 | -0.45908578E-08 | -0.18862501E-09 | -0.76161378E-11 | -0.30824249E-12 |
| 1.40 | -0.11875944E-06 | 0.29459494E-08 | 0.87317273E-10 | 0.248122510E-11 | 0.15711849E-12 |
| 1.50 | -0.10283933E-06 | -0.49585043E-09 | 0.87346989E-10 | 0.53605827E-11 | 0.22822000E-12 |
| 1.60 | -0.45700867E-07 | 0.28134098E-08 | 0.18566659E-09 | 0.74305384E-11 | 0.28528307E-12 |
| 1.70 | 0.24898396E-07 | 0.46334675E-08 | 0.15057138E-09 | 0.77733446E-11 | 0.25658239E-13 |
| 1.80 | 0.83919867E-07 | 0.41781327E-08 | 0.23834235E-10 | -0.39179117E-11 | -0.25324645E-12 |
| 1.90 | 0.11623095E-06 | 0.20121738E-08 | -0.11068549E-09 | -0.75333994E-11 | -0.28275752E-12 |
| 2.00 | 0.11735704E-06 | -0.79908213E-09 | -0.18427132E-09 | 0.61316428E-11 | -0.93652479E-13 |
| 2.50 | -0.90307340E-07 | -0.15961823E-08 | 0.18194064E-09 | 0.39170275E-11 | -0.15996495E-12 |
| 3.00 | -0.63759940E-07 | -0.46752512E-08 | -0.11376807E-09 | -0.55357382E-11 | 0.18678953E-12 |
| 3.50 | -0.17434495E-06 | 0.28012434E-08 | -0.75123565E-10 | 0.17082776E-11 | -0.70492766E-12 |
| 4.00 | 0.10436663E-06 | -0.45047648E-08 | 0.18972057E-09 | -0.40305675E-11 | -0.16770712E-12 |
| 4.50 | -0.96231354E-07 | 0.64637215E-09 | 0.32996539E-10 | -0.54542170E-11 | 0.31446997E-12 |
| 5.00 | -0.10825398E-06 | 0.47527343E-08 | -0.18295254E-09 | 0.57862933E-11 | -0.58537576E-13 |
| 6.00 | -0.75842093E-07 | -0.31054184E-09 | -0.81706438E-10 | -0.33159401E-11 | 0.54130699E-13 |
| 7.00 | 0.76597612E-07 | -0.51969246E-08 | 0.14561329E-09 | 0.35225191E-11 | 0.12026555E-12 |
| 8.00 | 0.15033345E-06 | -0.11972567E-08 | -0.14606575E-09 | -0.81096399E-11 | -0.31492788E-12 |
| 9.00 | 0.14081114E-06 | 0.40700294E-08 | 0.18658349E-09 | 0.83941457E-12 | 0.16926617E-12 |
| 10.00 | 0.98241375E-08 | 0.61805886E-08 | 0.10004479E-10 | -0.81804962E-11 | 0.28564212E-12 |
| 12.00 | 0.31746666E-07 | 0.39053494E-08 | 0.25436746E-09 | 0.22510227E-11 | -0.39686310E-12 |
| 14.00 | 0.75748816E-08 | 0.13889221E-08 | 0.16402400E-09 | 0.10501090E-10 | 0.15162777E-12 |
| 16.00 | 0.15323055E-08 | 0.36023308E-09 | 0.60508053E-10 | 0.67801273E-11 | 0.43489591E-12 |
| 18.00 | 0.27963863E-09 | 0.78739199E-10 | 0.16816119E-10 | 0.26269997E-11 | 0.28177762E-12 |
| 20.00 | 0.47649831E-10 | 0.15403376E-10 | 0.39306088E-11 | 0.77433741E-12 | 0.11187564E-12 |
| 22.50 | 0.48888626E-11 | 0.18042443E-11 | 0.59500524E-12 | 0.13275054E-12 | 0.25576514E-13 |
| 25.00 | 0.47520158E-12 | 0.19687476E-12 | 0.67052114E-13 | 0.19355373E-13 | 0.44980760E-14 |
| 27.50 | 0.44488486E-13 | 0.19707125E-13 | 0.75398126E-14 | 0.24950294E-14 | 0.67013816E-15 |
| 30.00 | 0.40778471E-14 | 0.19505104E-14 | 0.80934833E-15 | 0.28998844E-15 | 0.89188854E-16 |

| x | TABLE OF KIS(X) | | | |
|-------|-----------------|-----------------|-----------------|-----------------|
| | S = 20.00 | S = 22.50 | S = 25.00 | S = 30.00 |
| 0.01 | 0.10660524E-13 | -0.20434748E-15 | 0.94875008E-18 | 0.29288830E-19 |
| 0.02 | 0.94835114E-14 | 0.21348036E-15 | 0.43588672E-17 | 0.44980663E-19 |
| 0.03 | -0.10570672E-13 | -0.23947794E-15 | -0.37123032E-17 | -0.62087178E-19 |
| 0.04 | -0.3469143E-14 | -0.22569511E-15 | -0.5180808E-18 | 0.59681988E-19 |
| 0.05 | 0.12489167E-13 | -0.13571197E-15 | -0.32349855E-17 | 0.49434763E-19 |
| 0.06 | -0.9740117E-14 | 0.23646409E-15 | -0.24781789E-17 | 0.13362933E-20 |
| 0.07 | 0.10283734E-13 | -0.22346537E-15 | 0.42675936E-17 | -0.37301076E-19 |
| 0.08 | -0.12546309E-13 | 0.23217558E-15 | -0.44101320E-17 | 0.69708354E-19 |
| 0.09 | 0.10382913E-13 | -0.22585152E-15 | 0.42671885E-17 | -0.73766038E-19 |
| 0.10 | -0.11312447E-14 | 0.11317065E-15 | -0.31692123E-17 | 0.62361440E-19 |
| 0.20 | 0.10139256E-13 | -0.89151427E-16 | 0.29199655E-17 | 0.7490160E-19 |
| 0.30 | -0.12636113E-14 | 0.19687700E-16 | -0.45531224E-19 | -0.28875520E-19 |
| 0.40 | -0.75002774E-14 | 0.63780243E-16 | 0.34566029E-17 | 0.79364225E-19 |
| 0.50 | -0.8105608E-14 | -0.19776199E-15 | 0.85111868E-18 | 0.74913436E-19 |
| 0.60 | 0.11837397E-13 | 0.73176858E-17 | -0.44175359E-17 | -0.12059066E-19 |
| 0.70 | -0.11334767E-13 | 0.68632603E-16 | 0.32412710E-17 | -0.67586648E-19 |
| 0.80 | 0.78136979E-14 | -0.36659374E-16 | -0.25967902E-17 | 0.82707303E-19 |
| 0.90 | 0.14889076E-14 | -0.7836132E-16 | -0.32522449E-17 | -0.82053798E-19 |
| 1.00 | -0.1189908E-13 | 0.21199308E-15 | -0.43003690E-17 | 0.48661474E-19 |
| 1.10 | 0.85838656E-14 | -0.20300084E-15 | 0.38240062E-17 | -0.60015589E-19 |
| 1.20 | 0.7840370E-14 | 0.3467947E-16 | -0.33610555E-17 | 0.79701305E-19 |
| 1.30 | -0.10236782E-13 | 0.23604270E-15 | -0.3873326E-17 | 0.6435852E-19 |
| 1.40 | -0.8498745E-14 | -0.39098801E-16 | 0.3120464E-17 | -0.73817229E-19 |
| 1.50 | 0.76767406E-14 | -0.23426737E-15 | 0.26239975E-17 | -0.80925014E-19 |
| 1.60 | 0.1192173E-13 | 0.52439828E-17 | -0.36589837E-17 | 0.82656918E-19 |
| 1.70 | -0.31783651E-13 | 0.23261802E-15 | -0.26974654E-17 | -0.16270171E-19 |
| 1.80 | 0.10171209E-13 | -0.10472209E-16 | 0.4080496E-17 | 0.81547776E-19 |
| 1.90 | -0.12017443E-13 | -0.15494568E-15 | -0.37802370E-17 | 0.82840075E-19 |
| 2.00 | 0.12091367E-13 | -0.22531121E-15 | -0.11155811E-17 | 0.83011489E-19 |
| 2.50 | -0.12566731E-13 | -0.13350597E-15 | 0.20160960E-17 | -0.8113768E-19 |
| 3.00 | 0.12276058E-13 | 0.23730548E-15 | -0.35199412E-17 | 0.44161752E-19 |
| 3.50 | -0.12601939E-13 | -0.23065572E-15 | -0.4440357E-17 | -0.7509092E-19 |
| 4.00 | 0.12618242E-13 | 0.2378084E-15 | 0.4373220E-17 | 0.83386787E-19 |
| 4.50 | -0.50382437E-14 | -0.21295814E-15 | -0.46659579E-17 | -0.8315531E-19 |
| 5.00 | -0.82646567E-14 | 0.65292789E-16 | 0.36577998E-17 | 0.82342004E-19 |
| 6.00 | 0.44963705E-14 | -0.21607800E-15 | -0.34946902E-17 | -0.13149177E-19 |
| 7.00 | -0.67085971E-14 | 0.23321184E-15 | 0.1316682E-17 | -0.70611225E-19 |
| 8.00 | -0.12475588E-13 | -0.20368562E-15 | -0.11354886E-17 | 0.83463926E-19 |
| 9.00 | -0.10676880E-13 | 0.72305138E-16 | 0.26758184E-17 | -0.82886299E-19 |
| 10.00 | -0.4950845E-14 | 0.15989344E-15 | -0.44822371E-17 | 0.67642782E-19 |
| 12.00 | 0.63390772E-14 | -0.6472394E-15 | 0.17296727E-17 | -0.61631867E-19 |
| 14.00 | -0.1427767E-13 | 0.14211522E-15 | -0.10710743E-17 | 0.1683937E-19 |
| 16.00 | -0.1107395E-13 | -0.20279482E-15 | 0.32674624E-17 | -0.14064597E-19 |
| 18.00 | 0.1807610E-13 | 0.87152436E-16 | -0.52711125E-17 | 0.70988807E-19 |
| 20.00 | -0.11754807E-13 | 0.34050392E-15 | 0.31638750E-18 | -0.95278450E-19 |
| 22.00 | 0.37868519E-14 | 0.2260497E-15 | -0.4497712E-17 | 0.70590704E-19 |
| 24.00 | 0.84959103E-15 | 0.75422552E-16 | 0.4236392E-17 | -0.12413664E-19 |
| 26.00 | -0.15246832E-15 | -0.48018660E-16 | 0.14979337E-17 | 0.80860669E-19 |
| 30.00 | -0.23367689E-16 | 0.34422523E-17 | 0.37812732E-18 | 0.29676904E-19 |

| x | TABLE OF HIS(X) | | | |
|-------|-----------------|----------------|-----------------|-----------------|
| | S = 0.01 | S = 0.02 | S = 0.03 | S = 0.04 |
| 0.01 | 0.31368805E 01 | 0.31225390E 01 | 0.30987375E 01 | 0.30656266E 01 |
| 0.02 | 0.31180677E 01 | 0.31265725E 01 | 0.31071875E 01 | 0.3080922E 01 |
| 0.03 | 0.31189695E 01 | 0.31328970E 01 | 0.31223775E 01 | 0.30980229E 01 |
| 0.04 | 0.31338102E 01 | 0.31307088E 01 | 0.31155924E 01 | 0.30945442E 01 |
| 0.05 | 0.31407422E 01 | 0.3123143E 01 | 0.31181346E 01 | 0.30988209E 01 |
| 0.06 | 0.31381778E 01 | 0.31336654E 01 | 0.31207218E 01 | 0.31024144E 01 |
| 0.07 | 0.3142971E 01 | 0.31354342E 01 | 0.31229714E 01 | 0.31056110E 01 |
| 0.08 | 0.3144228E 01 | 0.31370621E 01 | 0.31251580E 01 | 0.31085778E 01 |
| 0.09 | 0.31456591E 01 | 0.31387731E 01 | 0.31273359E 01 | 0.31114084E 01 |
| 0.10 | 0.31472335E 01 | 0.31405903E 01 | 0.31295944E 01 | 0.31141796E 01 |
| 0.20 | 0.31713027E 01 | 0.31689958E 01 | 0.31571072E 01 | 0.3144694E 01 |
| 0.30 | 0.3185090E 01 | 0.3201701E 01 | 0.3187104E 01 | 0.31869777E 01 |
| 0.40 | 0.32689750E 01 | 0.32623515E 01 | 0.32546697E 01 | 0.32439655E 01 |
| 0.50 | 0.33395124E 01 | 0.33349621E 01 | 0.33274022E 01 | 0.33168879E 01 |
| 0.60 | 0.34282364E 01 | 0.34246672E 01 | 0.34170767E 01 | 0.34064996E 01 |
| 0.70 | 0.35268137E 01 | 0.3521782E 01 | 0.35244468E 01 | 0.35136741E 01 |
| 0.80 | 0.36631153E 01 | 0.36583245E 01 | 0.36503654E 01 | 0.3639753E 01 |
| 0.90 | 0.38090490E 01 | 0.3804074E 01 | 0.37988086E 01 | 0.37848974E 01 |
| 1.00 | 0.39575729E 01 | 0.39705312E 01 | 0.39618972E 01 | 0.39498763E 01 |
| 1.10 | 0.41044320E 01 | 0.41569717E 01 | 0.41499001E 01 | 0.41372607E 01 |
| 1.20 | 0.42578597E 01 | 0.43108339E 01 | 0.43125505E 01 | 0.43479166E 01 |
| 1.30 | 0.44138359E 01 | 0.44672782E 01 | 0.445975847E 01 | 0.445834520E 01 |
| 1.40 | 0.48779680E 01 | 0.48714782E 01 | 0.48606970E 01 | 0.48456757E 01 |
| 1.50 | 0.5171241E 01 | 0.51641970E 01 | 0.51526157E 01 | 0.51366933E 01 |
| 1.60 | 0.54925288E 01 | 0.54878682E 01 | 0.54759902E 01 | 0.54584841E 01 |
| 1.70 | 0.5853765E 01 | 0.58452634E 01 | 0.5832117E 01 | 0.58138006E 01 |
| 1.80 | 0.62479508E 01 | 0.6242330E 01 | 0.6229498E 01 | 0.6205312E 01 |
| 1.90 | 0.66614474E 01 | 0.66723253E 01 | 0.6657170E 01 | 0.66360375E 01 |
| 2.00 | 0.7115250E 01 | 0.71484339E 01 | 0.71321253E 01 | 0.71094037E 01 |
| 2.50 | 0.10330518E 02 | 0.10316091E 02 | 0.10292125E 02 | 0.10258735E 02 |
| 3.00 | 0.15326228E 02 | 0.15304562E 02 | 0.15268570E 02 | 0.15218426E 02 |
| 3.50 | 0.2168284E 02 | 0.2135261E 02 | 0.21040402E 02 | 0.2059793E 02 |
| 4.00 | 0.3589909E 02 | 0.35438174E 02 | 0.3515348E 02 | 0.35239605E 02 |
| 4.50 | 0.54882358E 02 | 0.54813316E 02 | 0.54662044E 02 | 0.54449159E 02 |
| 5.00 | 0.8353539E 02 | 0.85411814E 02 | 0.87206619E 02 | 0.88920742E 02 |
| 6.00 | 0.21115067E 03 | 0.21841463E 03 | 0.22105090E 03 | 0.20999716E 03 |
| 7.00 | 0.5293629E 03 | 0.5282620E 03 | 0.52734653E 03 | 0.52556400E 03 |
| 8.00 | 0.13425278E 04 | 0.13406212E 04 | 0.13373693E 04 | 0.13328390E 04 |
| 9.00 | 0.3433934E 04 | 0.34289197E 04 | 0.34205987E 04 | 0.34089824E 04 |
| 10.00 | 0.88415178E 04 | 0.88285889E 04 | 0.88071111E 04 | 0.87771892E 04 |
| 12.00 | 0.5900699E 05 | 0.59413526E 05 | 0.59268714E 05 | 0.59066698E 05 |
| 14.00 | 0.4963130E 06 | 0.49785179E 06 | 0.49799485E 06 | 0.49431511E 06 |
| 16.00 | 0.42054068E 07 | 0.42013309E 07 | 0.41945052E 07 | 0.41849709E 07 |
| 18.00 | 0.19526139E 08 | 0.19497443E 08 | 0.19449775E 08 | 0.19383366E 08 |
| 20.00 | 0.13677525E 09 | 0.13652401E 09 | 0.13624001E 09 | 0.13577456E 09 |
| 22.00 | 0.1569825E 10 | 0.15675118E 10 | 0.15636744E 10 | 0.15583295E 10 |
| 24.00 | 0.18132405E 11 | 0.18105714E 11 | 0.18061375E 11 | 0.17999604E 11 |
| 26.00 | 0.21051826E 12 | 0.21026829E 12 | 0.20969327E 12 | 0.2089783E 12 |
| 30.00 | 0.24544887E 13 | 0.24508732E 13 | 0.24444066E 13 | 0.24364944E 13 |

| X | TABLE OF H ⁵ (x) | | | | |
|-------|-----------------------------|---------------------|---------------------|---------------------|---------------------|
| | S ^{= 1.10} | S ^{= 1.20} | S ^{= 1.30} | S ^{= 1.40} | S ^{= 1.50} |
| 0.01 | 0.31215135E 00 | 0.34074773E 00 | 0.26633435E 00 | 0.14033941E 00 | 0.13456384E 00 |
| 0.02 | 0.2797497E-01 | 0.1408335E 00 | 0.23470783E 00 | 0.13760911E 00 | 0.17365901E 00 |
| 0.03 | -0.15761784E 00 | 0.20831782E-01 | 0.13875211E 00 | 0.1928307E 00 | 0.19189943E 00 |
| 0.04 | -0.27228099E 00 | -0.97653842E-01 | 0.38164354E-01 | 0.12485036E 00 | 0.16248151E 00 |
| 0.05 | -0.34312277E 00 | -0.18231705E 00 | -0.44247564E-01 | 0.57677982E-01 | 0.11867833E 00 |
| 0.06 | -0.38592114E 00 | -0.24210885E 00 | -0.10914850E 00 | -0.16642093E-02 | 0.72849412E-01 |
| 0.07 | -0.41012415E 00 | -0.28377853E 00 | -0.13940741E 00 | -0.31909317E-01 | 0.29733690E-01 |
| 0.08 | -0.42163533E 00 | -0.31211704E 00 | -0.19786771E 00 | -0.67573553E-01 | 0.89763759E-02 |
| 0.09 | -0.42429318E 00 | -0.33052435E 00 | -0.22690760E 00 | -0.12767121E 00 | -0.42675600E-01 |
| 0.10 | -0.42068654E 00 | -0.34144375E 00 | -0.24844272E 00 | -0.15527539E 00 | -0.72102414E-01 |
| 0.20 | -0.26738725E 00 | -0.27940971E 00 | -0.26552481E 00 | -0.23405943E 00 | -0.19226300E 00 |
| 0.30 | -0.10136866E 00 | -0.15223131E 00 | -0.17791637E 00 | -0.18414612E 00 | -0.17599623E 00 |
| 0.40 | 0.29725674E-01 | -0.39713545E-01 | -0.85956348E-01 | -0.11352048E 00 | -0.12633415E 00 |
| 0.50 | 0.3030979E 00 | 0.51155822E-01 | -0.64707472E-02 | -0.46521856E-01 | -0.7230575E-01 |
| 0.60 | 0.20813949E 00 | 0.12344606E 00 | 0.59122743E-01 | 0.11455425E-01 | -0.22593640E-01 |
| 0.70 | 0.26946082E 00 | 0.18124255E 00 | 0.11268909E 00 | 0.60151706E-01 | 0.20713093E-01 |
| 0.80 | 0.31890344E 00 | 0.22806377E 00 | 0.13639466E 00 | 0.10074167E 00 | 0.27642406E-01 |
| 0.90 | 0.35984233E 00 | 0.26669219E 00 | 0.19287369E 00 | 0.13457340E 00 | 0.88836260E-01 |
| 1.00 | 0.39478732E 00 | 0.29928453E 00 | 0.22337509E 00 | 0.16306305E 00 | 0.11528851E 00 |
| 1.10 | 0.42560094E 00 | 0.32751345E 00 | 0.24950198E 00 | 0.18735142E 00 | 0.13785471E 00 |
| 1.20 | 0.45372885E 00 | 0.35268757E 00 | 0.27240767E 00 | 0.20841629E 00 | 0.15732735E 00 |
| 1.30 | 0.48029784E 00 | 0.37584440E 00 | 0.29302437E 00 | 0.22707231E 00 | 0.17439980E 00 |
| 1.40 | 0.50491814E 00 | 0.39718144E 00 | 0.31111717E 00 | 0.24379307E 00 | 0.18973510E 00 |
| 1.50 | 0.5282240E 00 | 0.41969362E 00 | 0.33029631E 00 | 0.25975075E 00 | 0.2035592E 00 |
| 1.60 | 0.55889915E 00 | 0.44084388E 00 | 0.34610214E 00 | 0.27481237E 00 | 0.21658212E 00 |
| 1.70 | 0.58637959E 00 | 0.46057959E 00 | 0.3609734E 00 | 0.28919313E 00 | 0.22906877E 00 |
| 1.80 | 0.61464942E 00 | 0.48666509E 00 | 0.38430358E 00 | 0.3044171E 00 | 0.24133371E 00 |
| 1.90 | 0.64830390E 00 | 0.51052159E 00 | 0.40343021E 00 | 0.3196267E 00 | 0.25365627E 00 |
| 2.00 | 0.68268695E 00 | 0.53677413E 00 | 0.42836745E 00 | 0.33546839E 00 | 0.26627622E 00 |
| 2.50 | 0.90726154E 00 | 0.70559363E 00 | 0.55133650E 00 | 0.43276140E 00 | 0.34113944E 00 |
| 3.00 | 0.12605987E 01 | 0.96952127E 00 | 0.74891858E 00 | 0.58107987E 00 | 0.45284866E 00 |
| 3.50 | 0.318314E 00 | 0.14546459E 00 | 0.10393462E 00 | 0.13359590E 00 | 0.271764E 00 |
| 4.00 | 0.26856858E 01 | 0.20354834E 01 | 0.15458109E 01 | 0.11777505E 01 | 0.90033327E 00 |
| 4.50 | 0.40520458E 01 | 0.30547329E 01 | 0.23071533E 01 | 0.17473032E 01 | 0.13270582E 01 |
| 5.00 | 0.61997471E 01 | 0.46528536E 01 | 0.34996520E 01 | 0.26384479E 01 | 0.19940119E 01 |
| 6.00 | 0.14900182E 02 | 0.11124167E 02 | 0.83192059E 01 | 0.62328235E 01 | 0.4678497E 01 |
| 7.00 | 0.36702395E 02 | 0.2736685E 02 | 0.20344205E 02 | 0.15179475E 02 | 0.1343441E 02 |
| 8.00 | 0.91870452E 02 | 0.6738955E 02 | 0.50672631E 02 | 0.3769794E 02 | 0.28085230E 02 |
| 9.00 | 0.23275808E 03 | 0.17424218E 03 | 0.12785235E 03 | 0.9406909E 02 | 0.70520960E 02 |
| 10.00 | 0.59473180E 03 | 0.43992432E 03 | 0.32568977E 03 | 0.24134834E 03 | 0.17902876E 03 |
| 12.00 | 0.39576968E 04 | 0.29212504E 04 | 0.21570750E 04 | 0.15948744E 04 | 0.11788481E 04 |
| 14.00 | 0.26818797E 05 | 0.19765066E 05 | 0.14575348E 05 | 0.10759699E 05 | 0.7949824E 04 |
| 16.00 | 0.18407024E 06 | 0.13551188E 06 | 0.99806445E 05 | 0.73548074E 05 | 0.54229934E 05 |
| 18.00 | 0.12754044E 07 | 0.93844741E 06 | 0.69031808E 06 | 0.50819035E 06 | 0.37430541E 06 |
| 20.00 | 0.89021175E 07 | 0.65436841E 07 | 0.48115011E 07 | 0.35392572E 07 | 0.26045962E 07 |
| 22.50 | 0.10181236E 09 | 0.74788926E 08 | 0.54951285E 08 | 0.40389283E 08 | 0.2967892E 08 |
| 25.00 | 0.1172626E 10 | 0.86097148E 09 | 0.63223208E 09 | 0.46839967E 09 | 0.34123660E 09 |
| 27.50 | 0.13958776E 11 | 0.99647089E 10 | 0.73167981E 10 | 0.53748915E 10 | 0.39496724E 10 |
| 30.00 | 0.15807714E 12 | 0.1136507E 12 | 0.85082180E 11 | 0.62437757E 11 | 0.45832626E 11 |
| X | S ^{= 1.60} | S ^{= 1.70} | S ^{= 1.80} | S ^{= 1.90} | S ^{= 2.00} |
| 0.01 | -0.78058899E-01 | -0.11805756E 00 | -0.10988403E 00 | -0.70170638E-01 | -0.20377287E-01 |
| 0.02 | 0.70783112E-01 | 0.11640189E-01 | -0.46276260E-01 | -0.7540972E-01 | -0.76320887E-01 |
| 0.03 | 0.15232163E 00 | 0.93277866E-01 | 0.3243731E-01 | -0.17019806E-01 | -0.4787556E-01 |
| 0.04 | -0.5897400E 00 | 0.12694703E 00 | 0.80492556E-01 | 0.32425391E-01 | -0.76374243E-02 |
| 0.05 | 0.14117914E 00 | 0.13274241E 00 | 0.10371166E 00 | 0.64951882E-01 | 0.26002766E-01 |
| 0.06 | 0.11325797E 00 | 0.12342974E 00 | 0.11052411E 00 | 0.83197955E-01 | 0.4998641E-01 |
| 0.07 | 0.82075726E-01 | 0.10632148E 00 | 0.10708349E 00 | 0.80975799E-01 | 0.65243706E-01 |
| 0.08 | 0.69002227E-01 | 0.85457839E-01 | 0.97466333E-01 | 0.4146497E-01 | 0.73531294E-01 |
| 0.09 | -0.2363609E-01 | 0.63622089E-01 | 0.84314061E-01 | 0.8703631E-01 | 0.76531621E-01 |
| 0.10 | -0.53899074E-02 | 0.41796290E-01 | 0.69336932E-01 | 0.79394329E-01 | 0.75640630E-01 |
| 0.20 | -0.14620348E 00 | -0.10075353E 00 | -0.29501632E-01 | -0.24822871E-01 | 0.20279821E-02 |
| 0.30 | -0.15797955E 00 | -0.13379569E 00 | -0.10681938E 00 | -0.7929726E-01 | -0.54084017E-01 |
| 0.40 | -0.12784287E 00 | -0.1216666E 00 | -0.10862641E 00 | -0.275413E-01 | -0.75303217E-01 |
| 0.50 | -0.86797726E-01 | -0.9295879E-01 | -0.91105640E-01 | -0.85067949E-01 | -0.73792681E-01 |
| 0.60 | -0.45583849E-01 | -0.59625352E-01 | -0.66036866E-01 | -0.68215264E-01 | -0.65756788E-01 |
| 0.70 | -0.19995876E-02 | -0.27972616E-01 | -0.40879913E-01 | -0.48156276E-01 | -0.31037385E-01 |
| 0.80 | 0.48426399E-01 | 0.81773694E-03 | -0.16341461E-01 | -0.7839966E-01 | -0.38605835E-01 |
| 0.90 | 0.53339537E-01 | 0.26224769E-01 | 0.9781551E-02 | -0.86537751E-02 | -0.18723362E-01 |
| 1.00 | 0.7767375E-01 | 0.48345703E-01 | 0.25802726E-01 | 0.88215605E-02 | -0.36092367E-02 |
| 1.10 | 0.9854466E-01 | 0.67501601E-01 | 0.43202988E-01 | 0.24429035E-01 | 0.10185056E-01 |
| 1.20 | 0.11635494E 00 | 0.84101737E-01 | 0.58405069E-01 | 0.3822596E-01 | 0.2256538E-01 |
| 1.30 | 0.1325526E 00 | 0.98364231E-01 | 0.71694773E-01 | 0.50370794E-01 | 0.3357721E-01 |
| 1.40 | 0.14613757E 00 | 0.11128970E 00 | 0.8367412E-01 | 0.61968970E-01 | 0.43359273E-01 |
| 1.50 | 0.15863052E 00 | 0.12461560E 00 | 0.93714599E-01 | 0.70534745E-01 | 0.5148739E-01 |
| 1.60 | 0.17010567E 00 | 0.13287963E 00 | 0.10299387E 00 | 0.78978421E-01 | 0.59691529E-01 |
| 1.70 | 0.18088801E 00 | 0.14235543E 00 | 0.11144572E 00 | 0.86986809E-01 | 0.6660404E-01 |
| 1.80 | 0.19124872E 00 | 0.15128961E 00 | 0.11928359E 00 | 0.9370547E-01 | 0.7287350E-01 |
| 1.90 | 0.20144266E 00 | 0.15989974E 00 | 0.12667773E 00 | 0.10006366E 00 | 0.78638101E-01 |
| 2.00 | 0.21168466E 00 | 0.16837899E 00 | 0.13363815E 00 | 0.10622343E 00 | 0.8402438E-01 |
| 2.50 | 0.26996621E 00 | 0.21438183E 00 | 0.17074244E 00 | 0.13630500E 00 | 0.10899555E 00 |
| 3.00 | 0.3545937E 00 | 0.27860076E 00 | 0.21986387E 00 | 0.17416722E 00 | 0.13844990E 00 |
| 3.50 | 0.473482E 00 | 0.37378784E 00 | 0.2794040E 00 | 0.23066640E 00 | 0.1814133E 00 |
| 4.00 | 0.6062160E 00 | 0.53160612E 00 | 0.41065131E 00 | 0.3193501E 00 | 0.24768231E 00 |
| 4.50 | 0.10108212E 01 | 0.77223372E 00 | 0.59174919E 00 | 0.45848540E 00 | 0.33071004E 00 |
| 5.00 | 0.15107417E 01 | 0.11475164E 01 | 0.87388580E 00 | 0.6672641E 00 | 0.51086752E 00 |
| 6.00 | 0.35185779E 01 | 0.26514378E 01 | 0.2001990E 01 | 0.15146782E 00 | 0.11483344E 00 |
| 7.00 | 0.84902136E 01 | 0.6364902E 01 | 0.4779777E 01 | 0.35947152E 01 | 0.27081801E 00 |
| 8.00 | -0.2192437E 02 | 0.1564671E 02 | 0.3299229E 02 | 0.11339229E 02 | -0.6753722E 00 |
| 9.00 | 0.5247440E 02 | 0.39691612E 02 | 0.2915242E 02 | 0.2170959E 02 | 0.16276139E 02 |
| 10.00 | 0.13293920E 03 | 0.98819529E 02 | 0.75535416E 02 | 0.54779435E 02 | 0.40851500E 02 |
| 12.00 | 0.87395216E 03 | 0.44734860E 03 | 0.4813057E 03 | 0.35642147E 03 | 0.24842074E 03 |
| 14.00 | 0.58676021E 04 | 0.43387468E 04 | 0.32106213E 04 | 0.23775919E 04 | 0.17670141E 04 |
| 16.00 | 0.40010557E 05 | 0.29538248E 05 | 0.21820928E 05 | 0.16130293E 05 | 0.11931447E 05 |
| 18.00 | 0.2754234E 06 | 0.20339379E 06 | 0.15008432E 06 | 0.1107726E 06 | 0.81818074E 05 |
| 20.00 | 0.19176882E 07 | 0.14126406E 07 | 0.10411303E 07 | 0.76771505E 06 | 0.56639372E 06 |
| 22.50 | 0.2189844E 08 | 0.46076907E 08 | 0.1183689E 08 | 0.87187475E 07 | 0.64250296E 07 |
| 25.00 | 0.270482E 09 | 0.1844530E 09 | 0.13567415E 09 | 0.9864320E 08 | 0.73525224E 08 |
| 27.50 | 0.28981187E 10 | 0.2129864E 10 | 0.15137174E 10 | 0.11519259E 10 | 0.85148641E 08 |
| 30.00 | 0.33654190E 11 | 0.24719605E 11 | 0.18162990E 11 | 0.13349917E 11 | 0.98155863E 10 |

| K | TABLE OF $H_1^{(x)}$ | | | |
|-------|----------------------|-----------------|-----------------|-----------------|
| | S= 2.50 | S= 3.00 | S= 3.50 | S= 4.00 |
| 0.01 | 0.10674267E-01 | -0.42183023E-02 | 0.12317203E-02 | -0.13794257E-03 |
| 0.02 | 0.27249742E-01 | -0.96451222E-02 | 0.25446328E-02 | -0.11410323E-03 |
| 0.03 | 0.47949322E-01 | -0.60268732E-02 | -0.44410780E-02 | 0.22624496E-02 |
| 0.04 | -0.19463775E-01 | 0.12678745E-01 | -0.51173931E-02 | 0.14701262E-02 |
| 0.05 | -0.29447308E-01 | 0.11728560E-01 | -0.22391356E-02 | -0.49568938E-03 |
| 0.06 | -0.31031419E-01 | 0.74004913E-02 | 0.11858071E-02 | -0.18937915E-02 |
| 0.07 | -0.27409434E-01 | 0.14968859E-02 | 0.37699222E-02 | -0.23405022E-02 |
| 0.08 | -0.20823398E-01 | -0.36572400E-02 | 0.51424009E-02 | -0.30283952E-02 |
| 0.09 | -0.13386185E-01 | -0.77481474E-02 | 0.594767423E-02 | -0.42650075E-03 |
| 0.10 | -0.55576039E-02 | -0.10609820E-01 | 0.49789103E-02 | -0.36380330E-03 |
| 0.20 | -0.31272903E-01 | -0.14302335E-02 | -0.5733172E-02 | 0.11774572E-02 |
| 0.30 | 0.17191550E-01 | 0.11627291E-01 | -0.23318774E-02 | -0.20801693E-02 |
| 0.40 | -0.41462294E-02 | 0.12075271E-01 | 0.2943374E-02 | -0.18419063E-02 |
| 0.50 | -0.19873264E-01 | 0.64845666E-02 | 0.53667387E-02 | -0.36679716E-04 |
| 0.60 | -0.28591406E-01 | -0.26346388E-03 | 0.50909301E-02 | 0.15287482E-02 |
| 0.70 | -0.31704955E-01 | -0.59936651E-02 | 0.33108248E-02 | 0.22815620E-02 |
| 0.80 | -0.40952632E-01 | -0.10052401E-01 | 0.26142000E-02 | 0.23749792E-02 |
| 0.90 | -0.74924466E-01 | -0.12380499E-01 | -0.12042136E-02 | 0.17627524E-02 |
| 1.00 | -0.2302114E-01 | -0.13311393E-01 | -0.30375904E-02 | 0.98822315E-03 |
| 1.10 | -0.17633659E-01 | -0.13139247E-01 | -0.43784771E-02 | 0.14336128E-03 |
| 1.20 | -0.12003968E-01 | -0.12172941E-01 | -0.52252081E-02 | 0.69809848E-03 |
| 1.30 | -0.6433400E-02 | -0.1069082E-01 | -0.56301457E-02 | -0.13131741E-02 |
| 1.40 | -0.1033300E-01 | -0.8304291E-02 | -0.6177400E-02 | -0.33139748E-02 |
| 1.50 | -0.38685731E-02 | -0.68105228E-02 | -0.54154105E-02 | -0.21797487E-02 |
| 1.60 | 0.8888308E-02 | -0.47215595E-02 | -0.49465968E-02 | -0.23809538E-02 |
| 1.70 | 0.1071603E-01 | -0.14261592E-02 | -0.34237764E-02 | 0.44117772E-02 |
| 1.80 | 0.16520429E-01 | -0.62702094E-03 | -0.36011795E-02 | -0.24058889E-02 |
| 1.90 | 0.1998992E-01 | 0.12913966E-02 | -0.28192438E-02 | -0.22727938E-02 |
| 2.00 | 0.25137834E-01 | 0.30933913E-02 | -0.20112495E-02 | -0.10524355E-02 |
| 2.50 | 0.35498938E-01 | 0.10277796E-01 | 0.17684085E-02 | -0.57486586E-03 |
| 3.00 | 0.8565622E-01 | 0.15244475E-01 | 0.45677640E-02 | 0.94089444E-03 |
| 3.50 | 0.57316362E-01 | -0.19411117E-01 | 0.65192791E-02 | 0.44182784E-02 |
| 4.00 | 0.10030228E 00 | 0.24273608E-01 | 0.83420463E-02 | 0.28701479E-02 |
| 5.00 | 0.14014091E 00 | 0.41369117E-01 | 0.13151475E-01 | 0.444602168E-02 |
| 6.00 | 0.2964913E 00 | 0.81182174E-01 | 0.23589938E-01 | 0.73117915E-02 |
| 7.00 | 0.67418307E 00 | 0.17539483E 00 | 0.47832160E-01 | 0.13727479E-01 |
| 8.00 | 0.15948676E 01 | 0.40121064E 00 | 0.10483943E 00 | 0.28532630E-01 |
| 9.00 | 0.38721899E 01 | 0.9094966E 00 | 0.24116633E 00 | 0.63255481E-01 |
| 10.00 | 0.9577645E 01 | 0.23084666E 01 | 0.57233973E 00 | 0.14608407E 00 |
| 12.00 | 0.60749437E 02 | 0.14261592E-02 | 0.34237764E-02 | 0.44117772E-02 |
| 14.00 | 0.39359394E 03 | 0.91782226E 02 | 0.21554519E 02 | 0.51606474E 01 |
| 16.00 | 0.26680624E 04 | 0.60648012E 03 | 0.14015347E 03 | 0.32931811E 02 |
| 18.00 | 0.18142475E 05 | 0.40815445E 04 | 0.93167995E 03 | 0.21580521E 03 |
| 20.00 | 0.12475827E 06 | 0.27838271E 05 | 0.62930507E 04 | 0.14412974E 04 |
| 22.50 | 0.8298962E 07 | 0.31117397E 06 | 0.69660393E 05 | 0.15779317E 05 |
| 25.00 | 0.16004140E 08 | 0.35175396E 07 | 0.78200435E 06 | 0.17555699E 06 |
| 27.50 | 0.18231878E 09 | 0.40149808E 08 | 0.88702725E 07 | 0.19170569E 07 |
| 30.00 | 0.21198962E 10 | 0.46175055E 09 | 0.10148389E 09 | 0.22475363E 08 |

| K | TABLE OF $H_1^{(x)}$ | | | |
|-------|----------------------|-----------------|-----------------|-----------------|
| | S= 5.00 | S= 6.00 | S= 7.00 | S= 8.00 |
| 0.01 | 0.19411507E-03 | 0.76470022E-04 | 0.13942018E-04 | 0.12120734E-04 |
| 0.02 | -0.59463699E-04 | -0.13673710E-04 | -0.26468838E-05 | -0.4984425E-05 |
| 0.03 | 0.413207E-03 | 0.6339693E-04 | 0.98012190E-05 | 0.17863883E-05 |
| 0.04 | -0.40415623E-04 | -0.62090510E-04 | -0.15905289E-04 | -0.30629052E-05 |
| 0.05 | -0.41950410E-03 | -0.67265437E-04 | -0.36292766E-05 | -0.10947880E-05 |
| 0.06 | -0.3484680E-03 | 0.11686824E-04 | 0.13753714E-04 | 0.30649293E-05 |
| 0.07 | -0.6839694E-04 | 0.72317787E-04 | 0.13516317E-04 | 0.31347268E-06 |
| 0.08 | 0.21236719E-05 | 0.74963646E-04 | 0.13031419E-05 | 0.25434866E-05 |
| 0.09 | 0.3873947E-03 | 0.44357065E-04 | -0.10744777E-04 | -0.59357822E-05 |
| 0.10 | 0.43457041E-03 | 0.55720510E-05 | -0.18628683E-04 | -0.11749078E-05 |
| 0.20 | -0.624731E-03 | -0.27898870E-05 | -0.7526100E-05 | -0.2784109E-05 |
| 0.30 | 0.32180144E-03 | -0.29907061E-04 | -0.40492775E-05 | 0.76477455E-05 |
| 0.40 | 0.33459017E-03 | -0.71628840E-04 | 0.15640694E-04 | -0.29453544E-05 |
| 0.50 | -0.10207067E-03 | 0.23432965E-04 | -0.27093625E-05 | -0.27251809E-06 |
| 0.60 | -0.39724551E-03 | 0.81228976E-04 | -0.15793516E-04 | 0.30308087E-05 |
| 0.70 | -0.41277153E-03 | 0.61991590E-04 | -0.93042986E-05 | 0.16063427E-05 |
| 0.80 | -0.23784749E-03 | 0.41630282E-05 | 0.48030082E-05 | 0.15364384E-05 |
| 0.90 | 0.28500007E-03 | 0.95228489E-04 | 0.14394843E-04 | -0.10959779E-05 |
| 1.00 | 0.21991951E-03 | -0.79513058E-04 | 0.15332332E-04 | -0.73295494E-05 |
| 1.10 | 0.36827983E-03 | -0.80330547E-04 | 0.93771073E-05 | -0.28942270E-06 |
| 1.20 | 0.43600120E-03 | -0.59341367E-04 | 0.41001679E-06 | 0.17436612E-05 |
| 1.30 | 0.43068577E-03 | 0.26407915E-04 | -0.80849626E-05 | 0.29302374E-05 |
| 1.40 | 0.3694234E-03 | 0.9526689E-05 | -0.13804340E-04 | 0.3023527E-05 |
| 1.50 | 0.26966175E-03 | 0.41247788E-04 | -0.16045409E-04 | 0.20574449E-05 |
| 1.60 | 0.15031008E-03 | 0.65225280E-04 | -0.14467704E-04 | 0.86028130E-06 |
| 1.70 | 0.232444E-03 | 0.7466816E-04 | -0.11347927E-04 | 0.60740408E-06 |
| 1.80 | 0.48448340E-04 | 0.84848035E-04 | -0.6459838E-05 | 0.18762400E-06 |
| 1.90 | -0.20222396E-03 | 0.80789066E-04 | -0.44097224E-06 | -0.2739326E-05 |
| 2.00 | -0.29268283E-03 | 0.70252310E-04 | 0.50728904E-05 | -0.31210474E-05 |
| 2.50 | -0.46104407E-03 | -0.23190154E-04 | 0.16001770E-04 | 0.10859317E-06 |
| 3.00 | -0.29646705E-03 | -0.83886465E-04 | 0.27461995E-05 | 0.31637339E-05 |
| 3.50 | -0.24990601E-04 | -0.81370549E-04 | -0.12454986E-04 | 0.8581066E-05 |
| 4.00 | 0.21612446E-03 | -0.42071159E-04 | 0.14394843E-04 | -0.10807977E-05 |
| 4.50 | 0.40440405E-03 | 0.48123973E-05 | -0.13966868E-04 | -0.30827956E-05 |
| 5.00 | 0.55351759E-03 | 0.47121236E-04 | -0.61219203E-05 | -0.33625099E-05 |
| 6.00 | 0.89202742E-03 | 0.10282832E-03 | 0.10078104E-04 | -0.4574334E-06 |
| 7.00 | 0.13379354E-02 | 0.16177532E-03 | 0.21368079E-04 | -0.21250495E-05 |
| 8.00 | 0.24178398E-02 | 0.25056931E-03 | 0.31464587E-04 | 0.42478364E-05 |
| 9.00 | 0.48512432E-02 | 0.43817363E-03 | 0.47707130E-04 | 0.61752640E-05 |
| 10.00 | 0.10430865E-01 | 0.85136740E-03 | 0.81087799E-04 | 0.91953523E-05 |
| 12.00 | 0.54527283E-01 | 0.39047473E-02 | 0.31144259E-03 | 0.28034976E-04 |
| 14.00 | 0.3101724E 00 | 0.20497942E-01 | 0.14893127E-03 | 0.11667739E-04 |
| 16.00 | 0.9124880E 01 | 0.11897748E 00 | 0.79460714E-02 | 0.47137341E-03 |
| 18.00 | 0.12103227E 02 | 0.72077546E 00 | 0.45641187E-01 | 0.30785290E-02 |
| 20.00 | 0.78648703E 02 | 0.45266196E 01 | 0.27508881E 00 | 0.12009697E-02 |
| 22.50 | 0.83809407E 03 | 0.46647751E 02 | 0.27225348E 01 | 0.16674677E 00 |
| 25.00 | 0.91273086E 04 | 0.49478585E 03 | 0.27978837E 02 | 0.16512507E 01 |
| 27.50 | 0.1001777E 06 | 0.5565667E 04 | 0.29543904E 03 | 0.16726493E 02 |
| 30.00 | 0.11320869E 07 | 0.59061306E 05 | 0.31865689E 04 | 0.17606493E 03 |

| X | TABLE OF M ⁵ (X) | | | |
|-------|-----------------------------|----------------------|----------------------|----------------------|
| | S ^m 10.00 | S ^m 12.00 | S ^m 14.00 | S ^m 16.00 |
| 0.01 | -0.8213788E-07 | 0.41451134E-08 | -0.16423623E-09 | 0.5210275E-11 |
| 0.02 | -0.1178621E-06 | 0.13055909E-08 | 0.18339479E-09 | -0.50400018E-11 |
| 0.03 | 0.8751213E-06 | 0.46778127E-08 | 0.12573819E-09 | -0.37757266E-11 |
| 0.04 | -0.1037414E-06 | -0.42798604E-08 | -0.18834243E-09 | -0.64678856E-11 |
| 0.05 | 0.1078629E-06 | 0.47099846E-08 | 0.18846346E-09 | 0.74705511E-11 |
| 0.06 | 0.20688459E-07 | -0.26020200E-08 | -0.15958828E-09 | -0.69475901E-11 |
| 0.07 | -0.11499336E-06 | -0.30609688E-08 | -0.4885521E-11 | 0.34651328E-11 |
| 0.08 | -0.3074321E-07 | 0.36781333E-08 | 0.17070170E-09 | -0.37181307E-11 |
| 0.09 | 0.8046373E-07 | 0.34861746E-08 | -0.73862520E-10 | -0.74384869E-11 |
| 0.10 | 0.11653227E-06 | -0.19711084E-08 | -0.17973517E-09 | -0.79466932E-12 |
| 0.20 | 0.7709462E-07 | -0.29489611E-08 | 0.15713770E-09 | -0.76210297E-11 |
| 0.30 | 0.2495198E-07 | -0.40809028E-08 | 0.18851049E-09 | -0.74797965E-11 |
| 0.40 | -0.6703374E-08 | 0.4653775E-08 | -0.12265032E-09 | 0.63114332E-11 |
| 0.50 | -0.9849779E-07 | -0.3664649E-08 | 0.1538955E-09 | 0.3712851E-11 |
| 0.60 | 0.8998132E-07 | -0.30973014E-09 | -0.18264780E-09 | -0.21344991E-11 |
| 0.70 | 0.8164078E-07 | 0.46121863E-08 | 0.14015611E-09 | -0.29220843E-11 |
| 0.80 | -0.6559601E-07 | -0.11170894E-08 | 0.79921708E-10 | 0.7518549E-11 |
| 0.90 | -0.1176707E-06 | -0.47045937E-08 | -0.17648197E-09 | -0.35298823E-11 |
| 1.00 | -0.39802220E-07 | -0.10665957E-08 | -0.84036564E-10 | -0.63253949E-11 |
| 1.10 | 0.68548326E-07 | 0.37198372E-08 | 0.14423261E-09 | 0.39494842E-11 |
| 1.20 | 0.11931813E-06 | 0.43947006E-08 | 0.18461238E-09 | 0.71378394E-11 |
| 1.30 | 0.9490108E-07 | -0.12242953E-08 | -0.10973750E-09 | -0.57357400E-12 |
| 1.40 | -0.1469171E-07 | -0.26385649E-08 | -0.16762812E-09 | -0.72446774E-11 |
| 1.50 | -0.6209178E-07 | -0.4704787E-08 | -0.16766586E-09 | -0.54409338E-11 |
| 1.60 | -0.11119991E-06 | -0.37305853E-08 | -0.36145122E-10 | 0.17784138E-11 |
| 1.70 | -0.11772795E-06 | -0.88116370E-09 | 0.11464074E-09 | 0.71226566E-11 |
| 1.80 | -0.86380377E-07 | 0.22363309E-08 | 0.18781157E-09 | 0.63621425E-11 |
| 1.90 | -0.3176143E-07 | 0.42741082E-08 | 0.13370177E-09 | -0.13179121E-11 |
| 2.00 | -0.8072243E-07 | 0.4677684E-08 | 0.44224485E-10 | -0.45762199E-11 |
| 2.50 | 0.81097134E-07 | -0.44892206E-08 | -0.54957944E-10 | 0.6592240E-11 |
| 3.00 | -0.10632883E-06 | 0.10348263E-08 | 0.15310969E-09 | -0.53367894E-11 |
| 3.50 | -0.7564940E-07 | 0.43420171E-08 | -0.17623170E-09 | 0.34843492E-12 |
| 4.00 | 0.6236183E-07 | -0.18021506E-08 | -0.32978302E-10 | 0.66130127E-11 |
| 4.50 | 0.12591532E-06 | -0.48495674E-08 | 0.19087866E-09 | -0.55465368E-11 |
| 5.00 | 0.68630108E-07 | -0.1346529E-08 | 0.67561413E-10 | -0.52579923E-11 |
| 6.00 | -0.1094231E-06 | 0.50327054E-08 | -0.18064051E-09 | 0.71854428E-11 |
| 7.00 | 0.1174419E-06 | -0.46985309E-09 | 0.14063492E-09 | -0.74202099E-11 |
| 8.00 | -0.1305721E-06 | 0.45316520E-08 | -0.1478492E-09 | -0.11047878E-11 |
| 9.00 | 0.92555831E-07 | -0.4054362E-08 | -0.10749408E-09 | 0.8323107E-11 |
| 10.00 | 0.17036695E-06 | 0.87107243E-10 | -0.22398369E-09 | 0.26885401E-11 |
| 12.00 | 0.35073694E-06 | 0.69271119E-09 | 0.2279939E-10 | -0.90384176E-11 |
| 14.00 | 0.98099244E-06 | 0.13710172E-07 | 0.28432701E-09 | 0.13712484E-11 |
| 16.00 | 0.37524640E-05 | 0.35748532E-07 | 0.54502036E-09 | 0.11751714E-10 |
| 18.00 | 0.17095683E-04 | 0.12707727E-06 | 0.13407078E-08 | 0.214932275E-10 |
| 20.00 | 0.86590209E-04 | 0.54234252E-06 | 0.44698728E-08 | 0.51362057E-10 |
| 22.50 | 0.72574335E-03 | 0.38858237E-05 | 0.25996204E-07 | 0.22297031E-09 |
| 25.00 | 0.65363008E-02 | 0.3145759E-04 | 0.17941846E-06 | 0.17467494E-08 |
| 27.50 | 0.62457973E-01 | 0.27061904E-03 | 0.13858071E-05 | 0.8463517E-08 |
| 30.00 | 0.6187787E 00 | 0.24853798E-02 | 0.11593055E-04 | 0.63192195E-07 |

| X | TABLE OF M ⁵ (X) | | | |
|-------|-----------------------------|----------------------|----------------------|----------------------|
| | S ^m 20.00 | S ^m 22.50 | S ^m 25.00 | S ^m 27.50 |
| 0.01 | -0.6958407E-14 | -0.11898424E-13 | 0.43169048E-17 | -0.77695434E-19 |
| 0.02 | 0.8783411E-14 | 0.95386124E-14 | -0.73217127E-18 | -0.4879536E-19 |
| 0.03 | 0.71569098E-14 | -0.26734709E-16 | -0.22979803E-17 | -0.5514967E-19 |
| 0.04 | -0.11494371E-13 | -0.70547447E-16 | -0.43699679E-17 | -0.58767665E-19 |
| 0.05 | 0.24619957E-14 | 0.19364280E-15 | -0.30117623E-17 | -0.66713049E-19 |
| 0.06 | -0.8195326E-14 | 0.26508413E-16 | 0.36595761E-17 | -0.67012766E-19 |
| 0.07 | 0.7610847E-14 | -0.76135693E-16 | -0.11594401E-17 | 0.74182662E-19 |
| 0.08 | -0.25122123E-14 | 0.44843292E-16 | 0.29437409E-18 | -0.4511307E-19 |
| 0.09 | -0.73555759E-14 | 0.70047668E-16 | -0.11517837E-17 | 0.38118505E-19 |
| 0.10 | 0.12689191E-13 | -0.20762488E-15 | 0.30809156E-17 | -0.34822347E-19 |
| 0.20 | 0.44416597E-14 | 0.21601910E-15 | 0.33181648E-17 | -0.40494124E-19 |
| 0.30 | -0.12667148E-13 | -0.23563633E-15 | -0.44198256E-17 | 0.77852599E-19 |
| 0.40 | -0.30286847E-13 | 0.44843292E-16 | 0.27550295E-17 | -0.24422605E-19 |
| 0.50 | 0.6178348E-14 | -0.12786854E-15 | -0.43588326E-17 | -0.35826399E-19 |
| 0.60 | -0.46892448E-14 | 0.23639256E-15 | -0.16318280E-18 | 0.82163437E-19 |
| 0.70 | 0.5353706E-14 | -0.22634427E-15 | 0.30062608E-17 | -0.48256523E-19 |
| 0.80 | -0.10053631E-13 | 0.23386041E-15 | -0.35780234E-17 | 0.7537494E-19 |
| 0.90 | 0.12632615E-14 | -0.22319339E-15 | 0.29952153E-17 | -0.41217032E-19 |
| 1.00 | -0.5037284E-14 | 0.11502414E-15 | -0.10287081E-17 | 0.19816603E-19 |
| 1.10 | -0.94128895E-14 | 0.12154293E-15 | -0.22207377E-17 | 0.57428733E-19 |
| 1.20 | 0.10927095E-13 | -0.23379210E-15 | 0.44096918E-17 | -0.82933359E-19 |
| 1.30 | 0.75804756E-14 | 0.17098433E-16 | -0.21353128E-17 | 0.52539945E-19 |
| 1.40 | -0.94978482E-14 | 0.23844104E-15 | -0.35251950E-17 | -0.5584808E-19 |
| 1.50 | -0.10176507E-13 | -0.34034642E-16 | 0.3552760E-17 | -0.8289409E-19 |
| 1.60 | 0.45200932E-14 | -0.23670502E-15 | 0.24875083E-17 | 0.85984013E-20 |
| 1.70 | 0.12752540E-13 | -0.44316364E-16 | -0.5070018E-17 | 0.81303167E-20 |
| 1.80 | -0.35407973E-14 | 0.21041922E-15 | -0.31774859E-17 | -0.16099184E-19 |
| 1.90 | -0.76674799E-14 | 0.17564578E-15 | 0.23026446E-17 | -0.82651064E-19 |
| 2.00 | -0.12704032E-13 | -0.73292330E-16 | 0.42841550E-17 | -0.46678991E-19 |
| 2.50 | 0.23227353E-14 | 0.19605857E-15 | -0.3948085E-17 | -0.18431480E-19 |
| 3.00 | -0.36315360E-14 | -0.10175672E-16 | 0.26995858E-17 | 0.83163997E-19 |
| 3.50 | -0.42007514E-14 | 0.15337000E-16 | -0.11292137E-17 | -0.3240051E-19 |
| 4.00 | -0.4166620E-14 | 0.1634454E-16 | 0.8155805E-18 | 0.38814453E-20 |
| 4.50 | 0.11870479E-13 | 0.10827247E-15 | 0.74667038E-18 | -0.87730992E-20 |
| 5.00 | -0.99515737E-14 | -0.23039674E-15 | -0.25609388E-17 | -0.15109456E-19 |
| 6.00 | 0.12337360E-13 | 0.10645997E-15 | 0.28708030E-17 | -0.84035975E-19 |
| 7.00 | -0.189844E-13 | -0.66649592E-16 | 0.43144495E-17 | 0.45908118E-19 |
| 8.00 | 0.4533798E-14 | 0.11537000E-16 | -0.34894909E-17 | -0.1543911E-19 |
| 9.00 | 0.8073537E-14 | -0.23414314E-15 | 0.37116531E-17 | 0.20623713E-19 |
| 10.00 | -0.12745694E-13 | 0.19190728E-15 | -0.11045779E-17 | -0.53146234E-19 |
| 12.00 | 0.22720038E-13 | -0.21819191E-15 | 0.43899901E-17 | -0.62145299E-19 |
| 14.00 | -0.4688943E-14 | 0.22606227E-15 | -0.47342761E-17 | 0.8794081E-19 |
| 16.00 | -0.1409094E-13 | -0.31749987E-16 | 0.38861598E-17 | -0.8945804E-19 |
| 18.00 | 0.43610984E-14 | -0.24905725E-15 | 0.50718507E-18 | 0.63758573E-19 |
| 20.00 | 0.20369769E-13 | 0.7097410E-17 | -0.56673813E-17 | -0.30525236E-19 |
| 22.50 | 0.42965116E-13 | 0.38587669E-15 | 0.43672763E-18 | -0.10696259E-19 |
| 25.00 | -0.1598001E-12 | 0.7908377E-15 | -0.73402883E-17 | 0.3398178E-19 |
| 27.50 | 0.56742612E-12 | 0.22538496E-14 | 0.14662472E-16 | -0.14609008E-19 |
| 30.00 | 0.31085836E-11 | 0.92672530E-14 | 0.39815547E-16 | 0.7737648E-18 |
| | | | | 0.28813946E-20 |

| X | TABLE OF DMIS(X) | | S = 0.08 | | S = 0.10 | |
|-------|------------------|----------------|----------------|----------------|----------------|----------------|
| | S = 0.02 | S = 0.04 | S = 0.06 | S = 0.08 | S = 0.10 | S = 0.10 |
| 0.02 | 0.28363770E 00 | 0.10322118E 01 | 0.22532845E 01 | 0.39084083E 01 | 0.59463937E 01 | 0.59463937E 01 |
| 0.04 | 0.16706885E 00 | 0.47683337E 00 | 0.98355128E 00 | 0.16733652E 01 | 0.25277870E 01 | 0.25277870E 01 |
| 0.06 | 0.15511935E 00 | 0.33603677E 00 | 0.63249537E 00 | 0.10367379E 01 | 0.15390361E 01 | 0.15390361E 01 |
| 0.08 | 0.14680123E 00 | 0.28851015E 00 | 0.48823201E 00 | 0.76108724E 00 | 0.11006903E 01 | 0.11006903E 01 |
| 0.10 | 0.18705887E 00 | 0.27570691E 00 | 0.42115161E 00 | 0.62005103E 00 | 0.86793889E 00 | 0.86793889E 00 |
| 0.20 | 0.32567526E 00 | 0.35529515E 00 | 0.40397827E 00 | 0.47073209E 00 | 0.55422914E 00 | 0.55422914E 00 |
| 0.40 | 0.64274931E 00 | 0.64305897E 00 | 0.65680517E 00 | 0.66883776E 00 | 0.68395248E 00 | 0.68395248E 00 |
| 0.60 | 0.98480191E 00 | 0.98263425E 00 | 0.97907996E 00 | 0.97422338E 00 | 0.96817687E 00 | 0.96817687E 00 |
| 0.80 | 0.13577413E 01 | 0.13513532E 01 | 0.13408580E 01 | 0.13264622E 01 | 0.13084511E 01 | 0.13084511E 01 |
| 1.00 | 0.17122480E 01 | 0.17225585E 01 | 0.17466244E 01 | 0.17247500E 01 | 0.16973759E 01 | 0.16973759E 01 |
| 1.20 | 0.22409256E 01 | 0.22281070E 01 | 0.22070235E 01 | 0.21780824E 01 | 0.21418276E 01 | 0.21418276E 01 |
| 1.40 | 0.27783338E 01 | 0.27622218E 01 | 0.27357188E 01 | 0.26993333E 01 | 0.26537439E 01 | 0.26537439E 01 |
| 1.60 | 0.34013959E 01 | 0.33816148E 01 | 0.33490747E 01 | 0.33043973E 01 | 0.32484124E 01 | 0.32484124E 01 |
| 1.80 | 0.41291515E 01 | 0.41059531E 01 | 0.40687961E 01 | 0.40122771E 01 | 0.39443521E 01 | 0.39443521E 01 |
| 2.00 | 0.4974288E 01 | 0.49585059E 01 | 0.49109250E 01 | 0.48455917E 01 | 0.47637144E 01 | 0.47637144E 01 |
| 3.00 | 0.12359597E 02 | 0.12324414E 02 | 0.12206828E 02 | 0.12045358E 02 | 0.11842990E 02 | 0.11842990E 02 |
| 4.00 | 0.30601006E 02 | 0.30424390E 02 | 0.30133834E 02 | 0.29734859E 02 | 0.29234834E 02 | 0.29234834E 02 |
| 6.00 | 0.19233723E 03 | 0.19122212E 03 | 0.18937644E 03 | 0.1868687E 03 | 0.18371178E 03 | 0.18371178E 03 |
| 8.00 | 0.12539176E 04 | 0.12464989E 04 | 0.12345018E 04 | 0.12188024E 04 | 0.11973835E 04 | 0.11973835E 04 |
| 10.00 | 0.83747781E 04 | 0.83259614E 04 | 0.82456541E 04 | 0.81355843E 04 | 0.79971930E 04 | 0.79971930E 04 |

| X | TABLE OF DKIS(X) | | S = 0.08 | | S = 0.10 | |
|-------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | S = 0.02 | S = 0.04 | S = 0.06 | S = 0.08 | S = 0.10 | S = 0.10 |
| 0.02 | -0.49776240E 02 | -0.49242402E 02 | -0.48357966E 02 | -0.47130822E 02 | -0.45571901E 02 | -0.45571901E 02 |
| 0.04 | -0.24859603E 02 | -0.24668947E 02 | -0.24352724E 02 | -0.23912952E 02 | -0.23352639E 02 | -0.23352639E 02 |
| 0.06 | -0.16529788E 02 | -0.16428124E 02 | -0.16259404E 02 | -0.16024488E 02 | -0.15724735E 02 | -0.15724735E 02 |
| 0.08 | -0.12352801E 02 | -0.12268682E 02 | -0.12182170E 02 | -0.12033790E 02 | -0.1184273E 02 | -0.1184273E 02 |
| 0.10 | -0.98390129E 01 | -0.97945828E 01 | -0.97207510E 01 | -0.96178441E 01 | -0.94863169E 01 | -0.94863169E 01 |
| 0.20 | -0.47715293E 01 | -0.47582149E 01 | -0.47360642E 01 | -0.47051502E 01 | -0.46655653E 01 | -0.46655653E 01 |
| 0.40 | -0.21819381E 01 | -0.21797143E 01 | -0.21739226E 01 | -0.21658304E 01 | -0.21554355E 01 | -0.21554355E 01 |
| 0.60 | -0.13023538E 01 | -0.13009110E 01 | -0.12985087E 01 | -0.12951505E 01 | -0.12908414E 01 | -0.12908414E 01 |
| 0.80 | -0.86153923E 00 | -0.86081249E 00 | -0.85960171E 00 | -0.85790892E 00 | -0.85573597E 00 | -0.85573597E 00 |
| 1.00 | -0.60477110E 00 | -0.60436287E 00 | -0.60068294E 00 | -0.59973200E 00 | -0.59851101E 00 | -0.59851101E 00 |
| 1.20 | -0.43451047E 00 | -0.43426476E 00 | -0.43385549E 00 | -0.43328300E 00 | -0.43254780E 00 | -0.43254780E 00 |
| 1.40 | -0.32078416E 00 | -0.32026289E 00 | -0.32037049E 00 | -0.32000887E 00 | -0.31954440E 00 | -0.31954440E 00 |
| 1.60 | -0.24000060E 00 | -0.24049854E 00 | -0.24032940E 00 | -0.24009277E 00 | -0.23978882E 00 | -0.23978882E 00 |
| 1.80 | -0.18260034E 00 | -0.18253210E 00 | -0.18241840E 00 | -0.18225932E 00 | -0.18205496E 00 | -0.18205496E 00 |
| 2.00 | -0.13985026E 00 | -0.13980339E 00 | -0.13972531E 00 | -0.13961606E 00 | -0.13947507E 00 | -0.13947507E 00 |
| 3.00 | -0.40153493E-01 | -0.40144680E-01 | -0.40129995E-01 | -0.40109444E-01 | -0.40083036E-01 | -0.40083036E-01 |
| 4.00 | -0.12482823E-01 | -0.12480797E-01 | -0.12477420E-01 | -0.12472693E-01 | -0.12466618E-01 | -0.12466618E-01 |
| 6.00 | -0.13438721E-02 | -0.13437294E-02 | -0.13434914E-02 | -0.13431584E-02 | -0.13427304E-02 | -0.13427304E-02 |
| 8.00 | -0.15536513E-03 | -0.15535270E-03 | -0.15533251E-03 | -0.15530397E-03 | -0.15526727E-03 | -0.15526727E-03 |
| 10.00 | -0.18648385E-04 | -0.18647219E-04 | -0.18645277E-04 | -0.18642558E-04 | -0.18639062E-04 | -0.18639062E-04 |

| X | TABLE OF DMIS(X) | | S = 0.80 | | S = 1.00 | |
|-------|------------------|----------------|----------------|-----------------|-----------------|-----------------|
| | S = 0.20 | S = 0.40 | S = 0.60 | S = 0.80 | S = 1.00 | S = 1.00 |
| 0.02 | 0.19515881E 02 | 0.37419792E 02 | 0.22143394E 02 | -0.74637239E 01 | -0.23839859E 02 | -0.23839859E 02 |
| 0.04 | 0.84086158E 01 | 0.18327352E 02 | 0.15989578E 02 | 0.48986790E 02 | -0.58749775E 01 | -0.58749775E 01 |
| 0.06 | 0.50531362E 01 | 0.11640381E 02 | 0.11755888E 02 | 0.62750529E 01 | -0.55947183E 01 | -0.55947183E 01 |
| 0.08 | 0.35011760E 01 | 0.82918415E 01 | 0.90872889E 01 | 0.60181328E 01 | 0.14286986E 00 | 0.14286986E 00 |
| 0.10 | 0.26329957E 01 | 0.63093672E 01 | 0.72907372E 01 | 0.54541832E 01 | 0.22280700E 01 | 0.22280700E 01 |
| 0.20 | 0.11605030E 01 | 0.25686533E 01 | 0.32945735E 01 | 0.31401657E 01 | 0.23331953E 01 | 0.23331953E 01 |
| 0.40 | 0.79629412E 00 | 0.25061810E 01 | 0.12964055E 01 | 0.13525628E 01 | 0.12486344E 01 | 0.12486344E 01 |
| 0.60 | 0.92527880E 00 | 0.83371913E 00 | 0.78588183E 00 | 0.76058486E 00 | 0.71996865E 00 | 0.71996865E 00 |
| 0.80 | 0.11773547E 01 | 0.86283422E 00 | 0.64605108E 00 | 0.53283176E 00 | 0.47131431E 00 | 0.47131431E 00 |
| 1.00 | 0.14971557E 01 | 0.10968503E 01 | 0.65230939E 00 | 0.45707789E 00 | 0.35710409E 00 | 0.35710409E 00 |
| 1.20 | 0.18762148E 01 | 0.12200179E 01 | 0.73507561E 00 | 0.45925319E 00 | 0.35161106E 00 | 0.35161106E 00 |
| 1.40 | 0.23194068E 01 | 0.16391476E 02 | 0.94139083E 01 | 0.52411610E 01 | 0.29044859E 01 | 0.29044859E 01 |
| 1.60 | 0.28376030E 01 | 0.18152606E 01 | 0.10453710E 01 | 0.59325364E 00 | 0.34770584E 00 | 0.34770584E 00 |
| 1.80 | 0.34457592E 01 | 0.22029884E 01 | 0.12631443E 01 | 0.70694770E 00 | 0.40141434E 00 | 0.40141434E 00 |
| 2.00 | 0.41625844E 01 | 0.26627906E 01 | 0.15256228E 01 | 0.84935920E 00 | 0.4703580E 00 | 0.4703580E 00 |
| 3.00 | 0.10356835E 02 | 0.66442020E 01 | 0.38192193E 01 | 0.21273898E 01 | 0.11797595E 01 | 0.11797595E 01 |
| 4.00 | 0.25529884E 02 | 0.16391476E 02 | 0.84900333E 00 | 0.50900568E 00 | 0.31670277E 00 | 0.31670277E 00 |
| 6.00 | 0.28376030E 01 | 0.18152606E 01 | 0.10453710E 01 | 0.59325364E 00 | 0.34770584E 00 | 0.34770584E 00 |
| 8.00 | 0.34457592E 01 | 0.22029884E 01 | 0.12631443E 01 | 0.70694770E 00 | 0.40141434E 00 | 0.40141434E 00 |
| 10.00 | 0.41625844E 01 | 0.26627906E 01 | 0.15256228E 01 | 0.84935920E 00 | 0.4703580E 00 | 0.4703580E 00 |

| X | TABLE OF DKIS(X) | | S = 0.80 | | S = 1.00 | |
|-------|------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| | S = 0.20 | S = 0.40 | S = 0.60 | S = 0.80 | S = 1.00 | S = 1.00 |
| 0.02 | -0.33360165E 02 | -0.28465988E 01 | 0.30444015E 02 | 0.31101737E 02 | 0.10363614E 02 | 0.10363614E 02 |
| 0.04 | -0.18901559E 02 | -0.46238180E 01 | 0.92527249E 01 | 0.15211767E 02 | 0.11626523E 02 | 0.11626523E 02 |
| 0.06 | -0.13324980E 02 | -0.53286766E 01 | 0.33144783E 01 | 0.83615327E 01 | 0.86706603E 01 | 0.86706603E 01 |
| 0.08 | -0.10339048E 02 | -0.51142933E 01 | 0.88200648E 00 | 0.51708488E 01 | 0.43541818E 01 | 0.43541818E 01 |
| 0.10 | -0.84239078E 01 | -0.47359372E 01 | -0.29676595E 00 | 0.32127660E 01 | 0.47046391E 01 | 0.47046391E 01 |
| 0.20 | -0.43426749E 01 | -0.31699773E 01 | -0.15937732E 01 | -0.40246603E-01 | 0.11176156E 01 | 0.11176156E 01 |
| 0.40 | -0.20701718E 01 | -0.17496493E 01 | -0.12834122E 01 | -0.75593440E 00 | -0.25565594E 00 | -0.25565594E 00 |
| 0.60 | -0.12553059E 01 | -0.11196674E 01 | -0.91542257E 00 | -0.67090769E 00 | -0.41803835E 00 | -0.41803835E 00 |
| 0.80 | -0.83778042E 00 | -0.78613959E 00 | -0.62352446E 00 | -0.53100368E 00 | -0.38687448E 00 | -0.38687448E 00 |
| 1.00 | -0.58840783E 00 | -0.54925357E 00 | -0.48829158E 00 | -0.41134926E 00 | -0.32945977E 00 | -0.32945977E 00 |
| 1.20 | -0.42645839E 00 | -0.40275571E 00 | -0.36549815E 00 | -0.31777355E 00 | -0.26338275E 00 | -0.26338275E 00 |
| 1.40 | -0.31569458E 00 | -0.30065969E 00 | -0.27685658E 00 | -0.24602707E 00 | -0.21034729E 00 | -0.21034729E 00 |
| 1.60 | -0.23726792E 00 | -0.22739736E 00 | -0.21168251E 00 | -0.19115288E 00 | -0.16710983E 00 | -0.16710983E 00 |
| 1.80 | -0.18039288E 00 | -0.17370598E 00 | -0.16306344E 00 | -0.14906883E 00 | -0.13282079E 00 | -0.13282079E 00 |
| 2.00 | -0.13831064E 00 | -0.133373143E 00 | -0.12638078E 00 | -0.11665678E 00 | -0.10507144E 00 | -0.10507144E 00 |
| 3.00 | -0.39863561E-01 | -0.38996227E-01 | -0.37587604E-01 | -0.35690847E-01 | -0.33376056E-01 | -0.33376056E-01 |
| 4.00 | -0.12416101E-01 | -0.12215889E-01 | -0.11888711E-01 | -0.11444026E-01 | -0.10894476E-01 | -0.10894476E-01 |
| 6.00 | -0.13391682E-02 | -0.13250080E-02 | -0.13017194E-02 | -0.12679598E-02 | -0.12297476E-02 | -0.12297476E-02 |
| 8.00 | -0.1481848E-03 | -0.14737858E-03 | -0.14573966E-03 | -0.1442822E-03 | -0.14258511E-03 | -0.14258511E-03 |
| 10.00 | -0.18609956E-04 | -0.18493972E-04 | -0.18302211E-04 | -0.18036959E-04 | -0.17701358E-04 | -0.17701358E-04 |

| X | TABLE | | OF | | DMIS(X) | |
|-------|-----------------|-----------------|-----------------|-----------------|------------------|--|
| | S= 1.20 | S= 1.40 | S= 1.60 | S= 1.80 | S= 2.00 | |
| 0.02 | -0.17793809E 02 | -0.65999357E 00 | 0.10589673E 02 | 0.90362479E 01 | 0.64820379E 00 | |
| 0.04 | -0.99935124E 01 | -0.89621680E 01 | -0.89007902E 01 | 0.34095497E 01 | 0.38102951E 01 | |
| 0.06 | -0.49789034E 01 | -0.54789721E 01 | -0.30320038E 01 | 0.82593595E-01 | 0.19347269E 01 | |
| 0.08 | -0.22897191E 01 | -0.37697922E 01 | -0.30421783E 01 | -0.11733887E 01 | 0.33852186E 00 | |
| 0.10 | -0.78445324E 00 | -0.24696025E 01 | -0.25649793E 01 | -0.15484039E 01 | -0.244480212E 00 | |
| 0.20 | 0.12288623E 01 | 0.18424690E 00 | -0.53973387E 00 | -0.83863315E 00 | -0.76410212E 00 | |
| 0.40 | 0.10146486E 01 | 0.70513023E 00 | 0.38394620E 00 | 0.10763881E 00 | -0.87042032E-01 | |
| 0.60 | 0.64394550E 00 | 0.53202622E 00 | 0.39692833E 00 | 0.25716910E 00 | 0.13079730E 00 | |
| 0.80 | 0.42318096E 00 | 0.36974809E 00 | 0.30609190E 00 | 0.23518178E 00 | 0.16348734E 00 | |
| 1.00 | 0.30165717E 00 | 0.26210824E 00 | 0.22300539E 00 | 0.18593364E 00 | 0.14486209E 00 | |
| 1.20 | 0.24011286E 00 | 0.19737149E 00 | 0.16767706E 00 | 0.14196220E 00 | 0.11681073E 00 | |
| 1.40 | 0.21621230E 00 | 0.16253310E 00 | 0.13121174E 00 | 0.10963932E 00 | 0.91790989E-01 | |
| 1.60 | 0.21756113E 00 | 0.14854807E 00 | 0.11077322E 00 | 0.88279882E-01 | 0.72827755E-01 | |
| 1.80 | 0.23735350E 00 | 0.14967467E 00 | 0.10240386E 00 | 0.75960649E-01 | 0.59949609E-01 | |
| 2.00 | 0.27204392E 00 | 0.16254035E 00 | 0.10332935E 00 | 0.70857124E-01 | 0.52365656E-01 | |
| 3.00 | 0.65672165E 00 | 0.36802299E 00 | 0.20809212E 00 | 0.11913872E 00 | 0.69977486E-01 | |
| 4.00 | 0.16153131E 01 | 0.90344480E 00 | 0.50836563E 00 | 0.28778635E 00 | 0.16390578E 00 | |
| 6.00 | 0.98655820E 01 | 0.54668361E 01 | 0.30476156E 01 | 0.17080421E 01 | 0.96263912E 00 | |
| 8.00 | 0.62881761E 02 | 0.34577027E 02 | 0.19093572E 02 | 0.10593639E 02 | 0.59040495E 01 | |
| 10.00 | 0.41374194E 03 | 0.22627183E 03 | 0.12418000E 03 | 0.68403742E 02 | 0.37821779E 02 | |

| X | TABLE | | OF | | DKIS(X) | |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|--|
| | S= 1.20 | S= 1.40 | S= 1.60 | S= 1.80 | S= 2.00 | |
| 0.02 | -0.10837976E 02 | -0.16433441E 02 | -0.72624943E 01 | 0.41650329E 01 | 0.76318489E 01 | |
| 0.04 | -0.2925032E 01 | -0.43713422E 01 | -0.63582035E 01 | -0.36212129E 01 | 0.38211770E 00 | |
| 0.06 | 0.48406866E 01 | 0.36436950E-01 | -0.30199049E 01 | -0.33143770E 01 | -0.16652514E 01 | |
| 0.08 | 0.46780906E 01 | 0.16333466E 01 | -0.10204879E 01 | -0.21918684E 01 | -0.18369464E 01 | |
| 0.10 | 0.40919557E 01 | 0.21678567E 01 | 0.83868004E-01 | -0.12476408E 01 | -0.15114335E 01 | |
| 0.20 | 0.16730974E 01 | 0.16287944E 01 | 0.53079091E 01 | 0.53079091E 00 | -0.21738666E-01 | |
| 0.40 | 0.14449349E 00 | 0.40110680E 00 | 0.50562711E 00 | 0.48085224E 00 | 0.36980848E 00 | |
| 0.60 | -0.18668907E 00 | 0.10429768E-03 | 0.12919023E 00 | 0.19813459E 00 | 0.21391779E 00 | |
| 0.80 | -0.24947701E 00 | -0.12558899E 00 | -0.25783244E-01 | 0.45353835E-01 | 0.87479940E-01 | |
| 1.00 | -0.23796198E 00 | -0.15561652E 00 | -0.83869362E-01 | -0.26361212E-01 | 0.15266351E-01 | |
| 1.20 | -0.20640993E 00 | -0.15078440E 00 | -0.99893771E-01 | -0.56291645E-01 | -0.21537003E-01 | |
| 1.40 | -0.17220715E 00 | -0.13397912E 00 | -0.97804582E-01 | -0.65421596E-01 | -0.30051419E-01 | |
| 1.60 | -0.14100695E 00 | -0.11432186E 00 | -0.88433979E-01 | -0.64520689E-01 | -0.43481939E-01 | |
| 1.80 | -0.11433349E 00 | -0.95450936E-01 | -0.76777648E-01 | -0.59114712E-01 | -0.43110312E-01 | |
| 2.00 | -0.92211010E-01 | -0.78691758E-01 | -0.65116443E-01 | -0.52032426E-01 | -0.39908307E-01 | |
| 3.00 | -0.30726341E-01 | -0.27833285E-01 | -0.24792119E-01 | -0.21696934E-01 | -0.18636257E-01 | |
| 4.00 | -0.10255334E-01 | -0.95438457E-02 | -0.87785085E-02 | -0.79783248E-02 | -0.71622021E-02 | |
| 6.00 | -0.1182450E-02 | -0.11287533E-02 | -0.10696364E-02 | -0.10061625E-02 | -0.93935695E-03 | |
| 8.00 | -0.14132872E-03 | -0.13656253E-03 | -0.13125342E-03 | -0.12547420E-03 | -0.11930203E-03 | |
| 10.00 | -0.17299341E-04 | -0.16833559E-04 | -0.16315299E-04 | -0.15744378E-04 | -0.15129048E-04 | |

| X | TABLE | | OF | | DMIS(X) | |
|-------|-----------------|-----------------|-----------------|------------------|-----------------|--|
| | S= 3.00 | S= 4.00 | S= 6.00 | S= 8.00 | S= 10.00 | |
| 0.02 | 0.14508939E 01 | 0.44577570E 00 | 0.24432454E-01 | 0.94648360E-03 | -0.97730147E-05 | |
| 0.04 | 0.21581364E 00 | -0.18211305E 00 | -0.81672131E-02 | 0.82555945E-04 | 0.13893508E-04 | |
| 0.06 | -0.54446796E 00 | -0.91694504E-01 | 0.81748990E-02 | -0.96358999E-04 | -0.19608895E-04 | |
| 0.08 | -0.46778199E 00 | 0.58379028E-01 | -0.18134537E-02 | -0.17557226E-03 | 0.13517551E-04 | |
| 0.10 | -0.22550546E 00 | 0.92467461E-01 | -0.49440981E-02 | 0.22867506E-03 | -0.26279035E-05 | |
| 0.20 | 0.19362000E 00 | -0.40432218E-01 | 0.11650288E-02 | 0.52857474E-04 | -0.45621833E-05 | |
| 0.40 | -0.36633691E-01 | 0.14439743E-01 | 0.61744241E-03 | -0.18251879E-04 | -0.29805045E-05 | |
| 0.60 | 0.64409034E-01 | 0.11825796E-01 | 0.15958513E-03 | 0.83433515E-05 | 0.13101145E-05 | |
| 0.80 | -0.31636159E-01 | -0.30430587E-02 | -0.61584865E-03 | -0.26791253E-04 | -0.12470630E-05 | |
| 1.00 | -0.32348775E-02 | -0.83561935E-02 | -0.14502794E-03 | 0.16248770E-04 | 0.11236521E-05 | |
| 1.20 | 0.12733067E-01 | -0.73720065E-02 | 0.28629271E-03 | 0.16976911E-04 | 0.97093350E-07 | |
| 1.40 | 0.19511432E-01 | 0.43166641E-02 | 0.34715272E-03 | -0.44657546E-05 | -0.84072002E-06 | |
| 1.60 | 0.20947639E-01 | -0.13134160E-02 | 0.19330688E-03 | -0.146952574E-04 | -0.28282666E-06 | |
| 1.80 | 0.19712798E-01 | 0.91787815E-03 | 0.31686989E-05 | -0.10889234E-04 | 0.45789400E-06 | |
| 2.00 | 0.17401835E-01 | 0.23054971E-02 | -0.13332963E-03 | -0.13823375E-05 | 0.57534793E-06 | |
| 3.00 | 0.86346967E-02 | 0.26213441E-02 | -0.53001564E-04 | 0.13865336E-05 | -0.20455737E-06 | |
| 4.00 | 0.11244319E-01 | 0.15127371E-02 | 0.93125529E-04 | -0.54769205E-05 | 0.24045606E-06 | |
| 6.00 | 0.59301436E-01 | 0.41260959E-02 | 0.51022684E-04 | 0.32460565E-05 | -0.10657771E-06 | |
| 8.00 | 0.34012273E 00 | 0.21942904E-01 | 0.12440508E-03 | 0.18427282E-05 | 0.11660735E-06 | |
| 10.00 | 0.20691447E 01 | 0.12456044E 00 | 0.60061886E-03 | 0.41205861E-05 | 0.69173895E-07 | |

| X | TABLE | | OF | | DKIS(X) | |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|--|
| | S= 3.00 | S= 4.00 | S= 6.00 | S= 8.00 | S= 10.00 | |
| 0.02 | 0.13029605E 01 | 0.14282028E 00 | 0.41027126E-02 | 0.79525494E-03 | 0.58922987E-04 | |
| 0.04 | -0.90587949E 00 | -0.14760078E 00 | -0.7133457E-02 | -0.61287363E-03 | 0.26332346E-04 | |
| 0.06 | -0.35499335E 00 | 0.12623697E 00 | -0.1168592E-02 | -0.40064649E-03 | -0.34447166E-05 | |
| 0.08 | 0.13705343E 00 | 0.10141042E 00 | -0.59219373E-02 | 0.42543350E-03 | 0.63429940E-05 | |
| 0.10 | 0.31809784E 00 | 0.14534565E-01 | 0.32216810E-03 | 0.93987959E-04 | -0.11652669E-05 | |
| 0.20 | 0.21540269E-01 | -0.23533361E-01 | -0.21856792E-02 | 0.11173203E-03 | -0.38540617E-05 | |
| 0.40 | -0.89939315E-01 | 0.1834942E-01 | 0.10723318E-02 | 0.59015505E-04 | -0.16768819E-06 | |
| 0.60 | 0.89087949E-03 | -0.10051340E-01 | -0.8082001E-03 | -0.40293071E-05 | 0.14967796E-06 | |
| 0.80 | 0.36053315E-01 | -0.11184793E-01 | -0.31861236E-04 | 0.15253955E-04 | 0.81695032E-06 | |
| 1.00 | 0.37876084E-01 | -0.39022335E-02 | 0.47026078E-03 | 0.18508551E-04 | 0.39661180E-06 | |
| 1.20 | 0.28604471E-01 | 0.19798883E-02 | 0.29187342E-03 | -0.11470774E-04 | -0.98713770E-06 | |
| 1.40 | 0.17876634E-01 | 0.48454299E-02 | -0.37376289E-04 | -0.17023671E-04 | -0.11892518E-06 | |
| 1.60 | 0.89087949E-02 | 0.54868036E-02 | -0.3348972E-03 | -0.4243071E-05 | 0.68576909E-06 | |
| 1.80 | 0.24453784E-02 | 0.48569264E-02 | -0.26904412E-03 | 0.80837569E-05 | 0.47290808E-06 | |
| 2.00 | -0.18610519E-02 | 0.37378659E-02 | -0.20054760E-03 | 0.12089359E-04 | -0.13634817E-06 | |
| 3.00 | -0.61541141E-02 | -0.52565860E-03 | 0.14495378E-03 | -0.78203727E-05 | 0.33090811E-06 | |
| 4.00 | -0.34034071E-02 | -0.10388390E-02 | 0.55936223E-04 | 0.18042878E-05 | -0.15413018E-06 | |
| 6.00 | -0.59301439E-03 | -0.30310235E-03 | 0.33420199E-04 | 0.10967287E-05 | 0.14350723E-06 | |
| 8.00 | -0.85343434E-04 | -0.52874517E-04 | -0.12453198E-04 | -0.1186441E-05 | 0.23344689E-07 | |
| 10.00 | -0.11621065E-04 | -0.79954764E-05 | -0.26557900E-05 | -0.50918482E-06 | -0.43869989E-07 | |

Table with columns: ZEROS OF K1a(x) (S = 0.4 to S = 3.0), ZEROS OF M1B(X) (S = 0.4 to S = 3.0). Each section contains a list of 25 numbered rows of numerical values.