

Form-Finding Analysis for Membrane Structure with Cable Using Geometric Energy Minimization

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Abstract

In the membrane structure, a structural stiffness is given by introducing the geometric rigidity. Therefore, the form-finding analysis is necessary to obtain the structural stiffness effectively. In the form-finding process of the membrane structure, the minimal surface is used for a design prefiguration shape. Because, the shape of the minimal surface is equivalent to the curved surface of the uniform stress field. The discretization procedures with finite element technique are generally adopted and are used for the minimal surface analysis (as shown in reference [1]). However, in some ill-conditioned cases, these procedures are difficult to obtain a solution of the minimal surface without the special approach such as reduction of variables.

In other hands, discrete differential geometry is enthusiastically researched in mathematics for applied digital geometry processing and physical solution. The Willmore energy is one of the functional of discrete differential geometry. This energy of a surface is defined as the mean and Gaussian curvature (as shown in reference [2]). It is shown that the minimal surface exists at the critical point of the Willmore energy. In addition, the Willmore energy can be described as a discrete curved surface preserving invariant of a continuum curved surface. In the computation using this energy, a numeric steady approximate solution is obtained with simple algorithm.

In this paper, at first, we introduced the Willmore energy by the continuum and discrete formulation, and evaluate this energy in comparison with the area functional. This functional is computed on the discretized catenoid model. Furthermore, we show the form-finding analysis method of the membrane structure with cable, and verify accuracy of solutions and effectiveness of this method.

References

1. T. Suzuki and Y. Hangai, Shape Analysis of Minimal Surface by the Finite Element Method, Spatial Structures at the Turn of the Millennium, Vol.2, Structural Form, 1991.
2. Alexander I. Bobenko and P. Schroder, Discrete Willmore Flow, Eurographics Symposium on Geometry Processing, 2005.