Role of public support in sports fan formation processes: Approach by cultural transmission model[1]

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Abstract:

As described in this paper, we use a cultural transmission model to elucidate the formation process that transforms people into sports lovers. The cultural transmission model is useful for judging when people become sports lovers and how the sports lover share of a population changes because this model can analyze the process by which a naïve child receives various influences that form preferences. Moreover, we examine how public support of a sports team in one region affects the share of sports lovers and the sports team quality.

Keywords: 1. Sports fan, 2. Cultural transmission model, 3. Social preference
4. Public support

Introduction

The consumption activity of watching sports generates an experience value of "being moved." Through sports, people gain some non-monetary value, such as "being moved" and "being connected" with others, which cannot be replaced by viewing other phenomena. Numerous studies have used the contingent valuation method (CVM) to measure these non-monetary values of "being moved" and "making connections," which cannot be traded in the market. These studies, examining four North American major sports (hockey, basketball, football, and baseball) or European soccer teams as examples (Castellanos, Garcia & Sanchez, 2011), measured the non-monetary values that professional sports teams can generate. For instance, Johnson, Groothuis and Whitehead (2001) measured the annual intangible value that the Pittsburgh Penguins, an American professional ice hockey league, bestowed upon the local community, calculating its value: as much as 5.27 million dollars (on average, 5.57 USD per household). This value is significantly higher among Penguin fans than among those who are not. Fans not only receive experience values through watching their supported team's matches; they are also believed to receive other various benefits such as the existential value of the sports team, including the pride of living in the town that has

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the team.

Whether or not people become fans of their local sports teams is the result of various influential factors. The first factor is one's parents. If the parents are fans of the local professional sports team, and if someone has many opportunities for exposure to the team in question as a child (i.e., watching the matches by visiting the stadium or via media), then the probability that the person would naturally become a fan of that team is high. Fujimoto (2006) interviewed fans of the Osaka Kintetsu Buffaloes, a Japanese professional baseball team, in 2004, asking them what turned them into fans. The interviews revealed that the process of becoming a fan included having parents who were also team fans, and the strong influence of their own parents. However, some responses showed that the interview subjects became fans of the sports team through their friendships. In other words, a second factor is an effect from other people met via random matching. The personality, habits, and opinions of people around a person shape a person's personality. If the people one interacts with regularly are fans of a certain sports team, then it is likely that one would also become a fan of that sports team through that influence.

This paper presents the use of a cultural transmission model to examine the formation process that turns people into sports fans. Representative literature of the cultural transmission model includes a paper by Bisin and Verdier (2001). For the two culturally characteristic models, they constructed a model that determines which of the two culturally characteristic transmission processes a child will have: effects from within the household (family, direct vertical socialization) and effects from outside the household (i.e., society and peers, oblique socialization). After publication of this paper, the cultural transmission model has been applied in various fields. Such examples include Oliver, Thoenig, and Verdier (2008) who introduced the cultural transmission model into a trade model context (2008), and Gradstein and Justman (2005) who applied it to the field of education (for an outline of the cultural transmission model, refer to Bisin & Verdier, 2011). As described herein, the cultural transmission model will be applied to sports. The cultural model is effective when examining a process by which naïve (ignorant and undeveloped) children form their preferences based on various influences. Heretofore, no study has analyzed how people become fans of a particular sports team, or how the percentage of a teams' fans changes.

The role of a local sports team is extremely important during the fan formation process. A local sports team can improve the team quality through actions such as obtaining star players to expand the fan demographic, or by creating a new stadium. Improvement of the team's quality provides great benefits to fans. During this time, fans who are parents will harbor strong hopes that their children will form the same preference, and become a fan of the same team. Based on these points, the first objective of this study is to apply the cultural transmission model to the context of sports. Thereby, one can elucidate the fan base percentage effects that are produced by sports team investments designed to improve the team quality. In addition, this study examines parental and friend effects on the fan base percentage.

Public support from local municipalities for local sports teams also has an extremely strong effect on the fan base percentage. As described above, the existence of the sports team bestows various benefits to local residents through values derived from the viewing of matches. Particularly regarding intangible benefits (i.e., existential value), the local community does not pay a price through the market, thereby generating a positive externality. The existence of a local sports team having a public financial presence in the community can be regarded as theoretical evidence that the local municipalities provide public support for the sports team. In the U.S., public support of sport teams is common, with local municipalities providing support for the construction and management fees of the stadium. According to Leeds and Allmen (2012), of the 11.34 billion dollars used for constructing new stadiums for North American sports during 2000–2011, 6.1 billion dollars were funded through public expenditure. However, as Kobayashi (2015) reported, close scrutiny is given to tax funds that are used for sports in Japan. For that reason, some teams affiliated with Nippon Professional Baseball (NPB) must pay expensive stadium usage fees to the local municipality, and are thereby forced to manage their team affairs strictly. For instance, the Yokohama DeNA BayStars pay 25% of their entrance fee revenue to the local municipality as a stadium usage fee. In fact, this fee is the team's major expenditure, followed by the wages paid to the players. If the local municipalities would give public support to the team by reducing the stadium usage fee, then the team might use the money saved to enhance their service to fans or acquire star players to increase the team value. This use of funds will increase the sports fan percentage among the population. In light of that argument, the second point of this paper is to examine effects of public assistance for local sports teams on the sports fan percentage and the sports team value.

The structure of this paper is the following. First, a cultural transmission model will be constructed and applied to the sporting field to assess the educational investment behavior of individuals toward their own children. Next, this study shall assess investment effects on increasing the sports team value and the percentage of its fans. Based on those results, the effects of public support of a sports team on the percentage of sports fans and on the sports team value in a stationary state can be verified. Finally, this paper presents a summary of the study findings and a discussion of future tasks.

Cultural transmission model for sports preference

Sports fan utility and imperfect empathy

As described in this paper, we apply a theoretical study of cultural transmission of sports preferences from parents to children. Bisin and Verdier (2001) constructed a theoretical model of cultural transmission. It has since been applied to various fields. We consider application of the cultural transmission model by which affection for a local sports team is transmitted from parents to children.

We consider an individual who lives during two periods. The first term is assumed as a child period. The second term represents an adult period. An individual has one child. The regional population of a generation is normalized to 1. An individual has either of two preferences (L or N) during the adult period. An individual with preference L loves local professional sports team and obtains value from team's existence. An individual with preference N has no such preference and derives no value from a local professional sports team. The difference of these two preferences is shown in the utility function. The utility function of an individual with preference L is the following.

$$U_L = u(w - T - f) + v \tag{1}$$

The utility function of an individual with preference N is

$$U_N = u(w - T), \tag{2}$$

where w is an individual property and f is the expense for the sport such as the payment of the game watching and the fan club admission fee. T denotes a lump-sum tax. The revenue is used to support the professional sports team in the region. Therefore, $w - T - f$ expresses the amount of consumption of private goods that an individual with preference L can expend aside from sports. $u(\cdot)$ satisfies $u'(\cdot) > 0, u''(\cdot) < 0$. v is the benefit derived from the existence of the local professional sports team that only an individual with preference L can obtain, and which includes both an existential value that the existence of the team brings and an experience value such as an impression and a sense of belonging derived through game watching. The model of this paper closely resembles that of Bisin and Verdier (2000), who considered people with preferences of two types: one obtaining benefits from public goods, and another not.

The individual in the child period has no preference because the person is naïve. The preference prevailing the adult period is determined through parental education and social learning. Then a person has either utility function of (1) or (2). We assume an altruistic individual who conducts decision-making considering the utility that his child will obtain in the future. Therefore, the utility function during an adult period is a sum of the utility of (1) or (2) and the utility that the adult's child will obtain in the future.

We assume that an individual with preference L will recognize utility V^{LL} of the child in the future if the child has the same preference L as his own.

$$V^{LL} = u(w - T - f) + v \tag{3}$$

Moreover, we assume that an individual with preference L will recognize the utility V^{LN} of the child in the future if the child has preference N different from his own.

$$V^{LN} = u(w - T) \tag{4}$$

We also assume that the individual hopes the child has the same preference as his own in the future, i.e., the following are inferred from (3) and (4).

Assumption 1

$$u(w - T - f) + v > u(w - T) \tag{5}$$

However, we assume that an individual with preference N will recognize the utility V^{NN} of the child in the future if the child has the same preference N as his own.

$$V^{NN} = u(w - T) \tag{6}$$

Moreover, we assume that an individual with preference N will recognize the utility V^{NL} of the child in the future as follows if the child has preference L different from his own.

$$V^{NL} = u(w - T - f) \tag{7}$$

Actually, (7) shows that an individual with preference N cannot accurately predict the utility function of the child in the future if the child has preference L different from his own: he has *imperfect empathy* (Bisin & Verdier, 2001). Therefore, an individual with preference N cannot correctly ascertain the benefit v derived through the existence of the professional sports team. For that reason, he might abstract it from the utility function in his child's future, although originally the utility function of an individual with preference L is (1). It seems clear from (6) and (7) that an individual with preference N also hopes his child has the same preference in the future.

Cultural transmission and social learning

An individual educates his child to have the same preference as his own. Therefore, $\tau_i (i = L, N)$ represents the probability that the child will have the same preference as the parent in the future. The child has the same preference as the parent with probability τ_i through education by the parent, but education by the parent does not influence the child with probability $1 - \tau_i$. A child who is not influenced by the parent education is not influenced with probability $1 - \tau_i$ and derives a preference (L or N)

through social environment effects, i.e., social learning. When the share of individuals with preference L is denoted as q_t and that with preference N is denoted as $1-q_t$, the child uninfluenced by the parent education has preference L with probability q_t or preference N with probability $1-q_t$ by social learning (Figure 1).

Therefore, with probability p_t^{LL}, the child of an individual with preference L has the same preference L at period t. With probability p_t^{LN}, the child of an individual with preference L has a different preference N at period t. With probability p_t^{NN}, the child of an individual with preference N has the same preference N at period t. Also, with probability p_t^{NL}, the child of an individual with preference N has a different preference L at period t . The equations expressing those respective probabilities are shown below.

$$p_t^{LL} = \tau_L + (1-\tau_L)q_t \tag{8}$$

$$p_t^{LN} = (1-\tau_L)(1-q_t) \tag{9}$$

$$p_t^{NN} = \tau_N + (1-\tau_N)(1-q_t) \tag{10}$$

$$p_t^{NL} = (1-\tau_N)q_t \tag{11}$$

The share of individuals with preference L at period $t+1$, i.e., the share of sports team fans is the following, as inferred from (8) and (11).

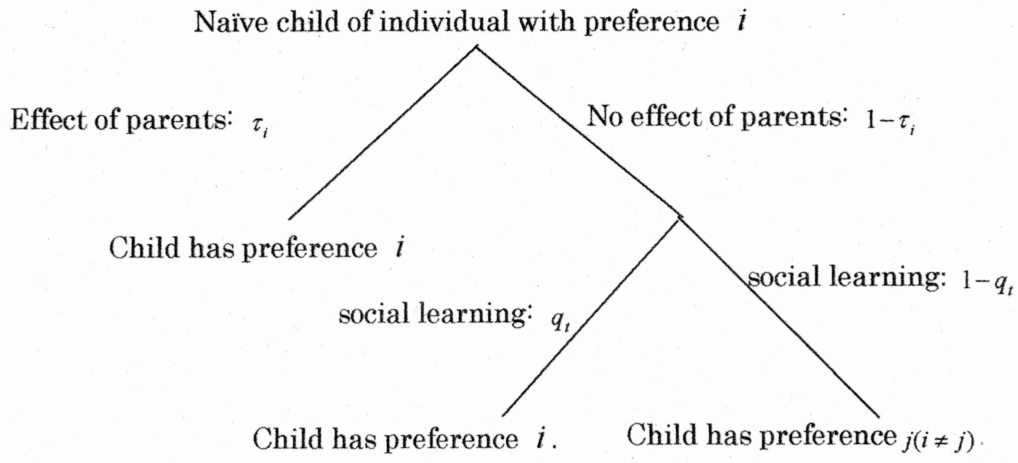
$$\begin{aligned} q_{t+1} &= q_t p_t^{LL} + (1-q_t)p_t^{NL} \\ &= q_t + q_t(1-q_t)(\tau_L - \tau_N) \end{aligned} \tag{12}$$

Education effort for a child by the parent

It requires effort for the parent to educate a child to have the same future preferences. For example, an individual with preference L educates a child to love the sports team by increasing the opportunity to be exposed to sports and making his child join a sports club. Let $H(\tau_L)$ $(H'(\cdot)>0, H''(\cdot)>0)$ represent the education effort cost of individuals with preference L, i.e., it increases as the level τ_L of educational effort by individual with preference L increases. Therefore, the decision problem related to the level of educational effort by an individual with preference L at period t is

$$\underset{\tau_L}{Max} \quad u(w-f)+v+\delta\left[p_t^{LL}V^{LL} + p_t^{LN}V^{LN}\right] - H(\tau_L).$$

Figure 1: Cultural transmission of an individual with preference i.



Here, the discount rate is denoted as δ $(0<\delta<1)$. Clauses 1 and 2 represent the utility of an individual at this period and the expected utility of the child in the subsequent period. This expected utility signifies that the child of individual with preference L has preference L with probability p_t^{LL} and obtains utility V^{LL} and he has preference N with probability p_t^{LN} and obtains utility V^{LN}. Clause 3 represents the educational effort cost. An individual decides τ_L to maximize this object function. Therefore, the first-order condition of maximization is the following.

$$\delta(1-q_t)\left[u(w-T-f)+v-u(w-T)\right]=H'(\tau_L) \tag{13}$$

Similarly, an individual with preference N educates his child to love anything except sports (e.g. learning, music, and art) by learning lessons other than sports. When the education effort cost of an individual with preference N is denoted as $G(\tau_N)\,(G'(\cdot)>0, G''(\cdot)>0)$, then the decision problem related to the level of educational effort by individual with preference N at period t is the following.

$$\underset{\tau_N}{Max} \quad u(w)+\delta\left[p_t^{NN}V^{NN}+p_t^{NL}V^{NL}\right]-G(\tau_N)$$

Solving the maximization problem above, the first-order condition is

$$\delta q_t\left[u(w-T)-u(w-T-f)\right]=G'(\tau_N). \tag{14}$$

From (13) and (14), the following Lemma 1 holds (see Appendix A1 for proof).

Lemma 1

(1) Educational effort of individual with preference L decreases and educational effort of individual with preference N increases as the expense of sports increases.

(2) Educational effort of individual with preference L increases and educational effort of individual with preference N decreases as benefit derived through the sports team existence increases.

(3) Educational effort of individual with preference L increases and educational effort of individual with preference N decreases as individual property increases.

(4) Educational effort of individual with preference L decreases and educational effort of individual with preference N increases as the share of individuals with preference L increases.

(5) Educational effort of individual with preference L decreases and educational effort of individual with preference N increases as the lump-sum tax rate increases.

The growth of expenses for sports decreases the benefit derived from becoming a home team fan. Therefore, (1) of Lemma 1 holds. However, because the growth of benefit derived through the sports team existence increases the fan benefit, (2) of Lemma 1 holds. Next, because the amount of consumption allocated to non-sports consumption increases as an individual's initial property increases, (3) of Lemma 1 holds. (4) of Lemma 1 is approved from a substitute relation between learning by education from the parent and social learning. When the share of individuals with preference L increases, an individual with preference L lowers the level of educational effort because the probability that the child has the same preference rises, but an individual with preference N raises the level of educational effort because the probability that the child has a different preference increases through social learning. (5) of Lemma 1 is contrary to (3) of Lemma 1: Because the amount of consumption that can be spent except sports consumption decreases if the rate of lump-sum tax increases, (5) of Lemma 1 holds.

Object function of the sports team

A professional sports team exists in a region. The sports team can increase profits by increasing the number of home town fans and displaying many games to many fans in the stadium, on television, and on the internet. The professional sports team manager must increase the number of fans in the short term and over the long term. Therefore, the professional sports team manager should devote consideration to profits now and in the future. The professional sports team profit function for t period is

$$\pi_t = q_t R + \delta q_{t+1}^e R - C(v,T). \tag{15}$$

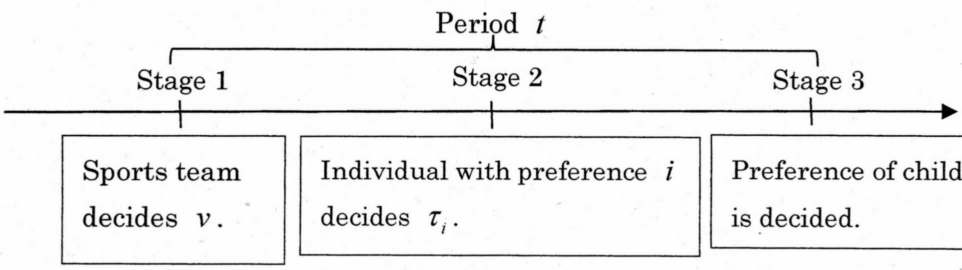
Therein, R stands for revenue (sum totals of the ticket income, the goods income, the advertising revenue, and the broadcasting right fee, etc.) of the professional sports team per fan in each period. For simplicity, R is the same level in each period. Because q_t is the number of fans in period t, $q_t R$ is the revenue of sports team in period t. q_{t+1}^e is the expected value of the number of fans in period $t+1$. Consequently, $\delta q_{t+1}^e R$ is the future revenue at period $t+1$. $C(v,T)$ represents the investment by the sports team to improve the team quality. To improve the team quality, it is necessary to acquire a star player, to enhance fan service, and to repair the stadium: $C(v,T)$ satisfies $C_v(v,T) > 0, C_{vv}(v,T) > 0$. Here, $C_v(v,T) = dC/dv, C_{vv}(v,T) = d^2C/dv^2$. Moreover, because the revenue derived from the lump-sum tax (in other words, amount of public support) is used to allay stadium repair costs etc., i.e., it has the effect of depressing the investment cost of the sports team: $C_T(v,T) < 0$ holds. Here, $C_T(v,T) = dC/dT$. In addition, we assume that $C_{vT}(v,T) < 0$.

Optimal investment of a professional sports team

Timeline

Here, we explain the game (Figure 2). In the first stage, a sports team chooses v to maximize (15) in period t. At the second stage, an individual with preference $i(=L,N)$ decides level $\tau_i (i = L,N)$ of educational effort for the child after observing v. At the third stage, the child preference is decided depending on the process in Figure 1.

Figure 2: Timeline.



Optimal investment of professional sports team

At the first stage, the sports team chooses the quality v of the team considering the reaction function $\tau_i(v)$ on the educational effort level chosen by an individual with preference $i(=L,N)$ at the second stage. $\tau_i(v)$ influences proportion q_{t+1} of preference L (number of fans) in the next period from (12), which influences the sports team profit through q_{t+1}. Substituting $\tau_L(v)$ and τ_N obtained by (13) and (14) for (12), a sports team can predict the number q_{t+1} of fans in period $t+1$ (here, τ_N is independent of v). Substituting q_{t+1} for (15), one can solve the following profit maximization problem.

$$\underset{v}{Max} \quad \pi_t = q_t R + \delta\left[q_t + q_t(1-q_t)\left(\tau_L(v) - \tau_N\right)\right] R - C(v,T).$$

The following holds from the first-order condition of maximization.

$$\frac{d\pi_t}{dv} = \delta R q_t (1-q_t)\tau_L{}'(v) = C_v(v,T) \tag{16}$$

We were able to obtain the following by arranging (16) from (13).

$$\delta^2 R q_t (1-q_t)^2 H'' \circ H'^{-1}\left(\delta(1-q_t)(u(w-f)+v-u(w))\right) = C_v(v,T) \tag{17}$$

From (17), the following Lemma 2 holds (see Appendix A2 for proof).

Lemma 2 If it is assumed that the educational effort cost of individual with preference L is $H(\tau_L) = \frac{a}{2}\tau_L{}^2$, then the following hold.

(1) The level of the sports team investment increases when the sports team revenue per fan increases.

(2) The level of sports team investment does not change when individual property increases.

(3) The level of sports team investment increases when the discount rate increases. The sports team emphasizes future revenues.

(4) The level of sports team investment decreases when the number of fans increases if the share of individuals with preference L (the number of fans) in period t is greater than one-third. However, the opposite holds if the share of individuals with preference L in period t is less than one-third.

(5) The level of sports team investment increases when the amount of public support increases.

Actually, (1) of Lemma 2 means that an increase in revenue per fan expands the number of fans; thereby, it promotes investment. Moreover, (2) of Lemma 2 means that because an increase in an individual initial property does not influence the marginal educational effort $\tau_L'(v)$, it does not influence the level of investment. Also, (3) of Lemma 2 means that if the discount rates rise, i.e., the profit in the future is emphasized. The sports team increases the level of investment to increase the number of fans in the subsequent period. In addition, (4) of Lemma 2 shows that if the number of fans is larger, the number of fans increases by virtue of education by individuals with preference L and social learning, i.e., the sports team is not actively making investments. However, if the number of fans is less, then the sports team will actively invest and try to improve future profits as the number of fans increases. Finally, (5) of Lemma 2 shows that the investment cost decreases because of an increase in the amount of public support of the sports team. Therefore, the sports team invests actively.

Preference Dynamics

Here, we assume the educational effort cost function $H(\tau_L)$ of individuals with preference L and the educational effort cost function $G(\tau_N)$ of individuals with preference N for simplicity as follows.

Assumption 2
$$H(\tau) = G(\tau) = \frac{a}{2}\tau^2 \quad \text{for any} \quad \tau(=\tau_L = \tau_N) \tag{18}$$

Moreover, the investment cost function to improve quality v of the sports team is assumed.

Assumption 3
$$C(v,T) = \frac{b(T)}{2}v^2 \tag{19}$$

$b(T)$ is a parameter related to the sports team investment amount; $b'(T) < 0$ is assumed. Using (18) and (19), sports team quality $v(q_t)$ is obtainable depending on q_t, as follows from (17).

$$v(q_t) = \frac{\delta^2}{ab(T)} R q_t (1-q_t)^2 . \tag{20}$$

Substituting $v(q_t)$ of (20) for (13), the educational effort $\tau_L(q_t)$ of an individual with preference L obtained using (18) depends on q_t as follows.

$$\tau_L(q_t) = \frac{\delta}{a}(1-q_t)\left[u(w-T-f) + \frac{\delta^2}{ab(T)} R q_t (1-q_t)^2 - u(w-T)\right] \tag{21}$$

Moreover, using (18) the educational effort $\tau_N(q_t)$ of individual with preference N

obtained from (14) is depending on q_t as shown below.

$$\tau_N(q_t) = \frac{\delta}{a} q_t \left[u(w-T) - u(w-T-f) \right] \tag{22}$$

Substituting $\tau_L(q_t)$ of (21) and $\tau_N(q_t)$ of (22) for (12), one can rewrite the preference dynamics as shown below.

$$q_{t+1} - q_t = q_t(1-q_t)\frac{\delta}{a}\left[q_t(1-q_t)^3 \frac{\delta^2}{ab(T)} R - \left(u(w-T) - u(w-T-f) \right) \right] \tag{23}$$

A steady state is $q_{t+1} = q_t$. Therefore, the following is satisfied by setting the number of individuals with preference L at steady state as q^*.

$$q^*(1-q^*)\left[\frac{\delta^2}{ab(T)} R q^* (1-q^*)^3 - \left(u(w-T) - u(w-T-f) \right) \right] = 0 \tag{24}$$

From (24), the number q^* of individuals with preference L in the steady state is $q^* = 0$ or $q^* = 1$, i.e., all individuals is homogeneous. Alternatively, if $q^* \neq 0$ and $q^* \neq 1$ hold, then q^* satisfying the following is a stationary solution, i.e., all individuals are heterogeneous.

$$\frac{\delta^2}{ab(T)} R q^* (1-q^*)^3 = \left(u(w-T) - u(w-T-f) \right). \tag{25}$$

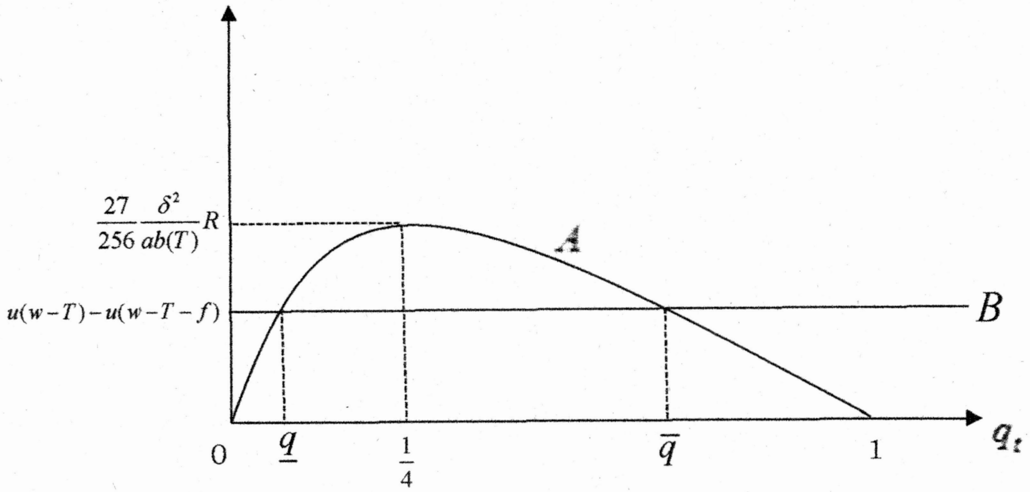
Here, when the left side of (25) equals A and the right side of (25) equals B, it is shown by Figure 3 that q^* satisfying (25) exists. The following Proposition 1 holds from Figure 3 (see Appendix A3 for proof).

Proposition 1 Under Assumption 2 and Assumption 3, if $q^* \neq 0$ and $q^* \neq 1$ hold, then the number q^* of individual with preference L at steady state satisfies the following.

(1) If $\frac{27}{256}\frac{\delta^2}{ab(T)} R > u(w-T) - u(w-T-f)$ holds, then two stationary solutions $q^* = \underline{q}, \overline{q}$ satisfying $\frac{\delta^2}{ab(T)} R q^* (1-q^*)^3 = u(w-T) - u(w-T-f)$ exist. They satisfy $0 < \underline{q} < \frac{1}{4}$ and $\frac{1}{4} < \overline{q} < 1$.

(2) If $\frac{27}{256}\frac{\delta^2}{ab(T)} R < u(w-T) - u(w-T-f)$ holds, then a stationary solution does not exist.

Figure 3: Stationary solutions except for $q^* \neq 0$ and $q^* \neq 1$.



Proposition 1 means that if both discount rate δ and the revenue R per fan are larger and the increment of utility when a child of non-sports-fans having the same preference N as his own is less, then two stationary interior solutions exist as $0 < \underline{q}, \overline{q} < 0$, i.e., all individuals are heterogeneous. However, if the reverse case is examined, then no stationary interior solution exists, i.e., all individuals are homogeneous.

Next, we will analyze the stability of stationary solutions. When (23) is shown in the figure, it is depicted as in Figure 4 and Figure 5. Therefore, the following Proposition 2 holds (see Appendix A4 for proof).

Proposition 2 The following hold for the number of fans q^* at steady state stability

(1) If $\frac{27}{256}\frac{\delta^2}{ab(T)}R > u(w-T)-u(w-T-f)$ holds, then four stationary solutions exist for which all individuals are fans of sports team ($q^* = 1$), all individuals are non-fans of sports team ($q^* = 0$), and fans exist along with non-fans ($q^* = \underline{q}, \overline{q}$). $q^* = 0, \overline{q}$ are stable stationary solutions and $q^* = \underline{q}, 1$ are unstable stationary solutions (Figure 4).

(2) If $\frac{27}{256}\frac{\delta^2}{ab(T)}R < u(w-T)-u(w-T-f)$ holds, then two stationary solutions exist for which all individuals are fans of sports team and all individuals are non-fans of sports team ($q^* = 0, 1$). $q^* = 0$ is a stable stationary solution and $q^* = 1$ is an unstable stationary solution (Figure 5).

Figure 4: Stability of four stationary solutions.

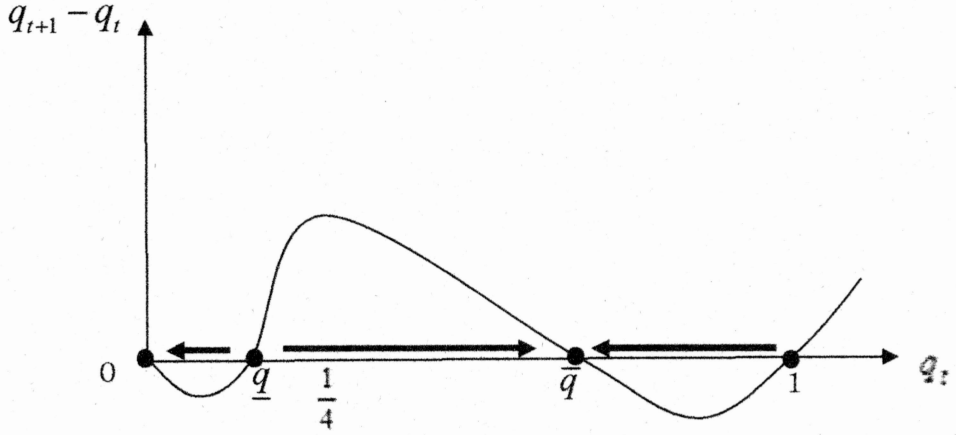
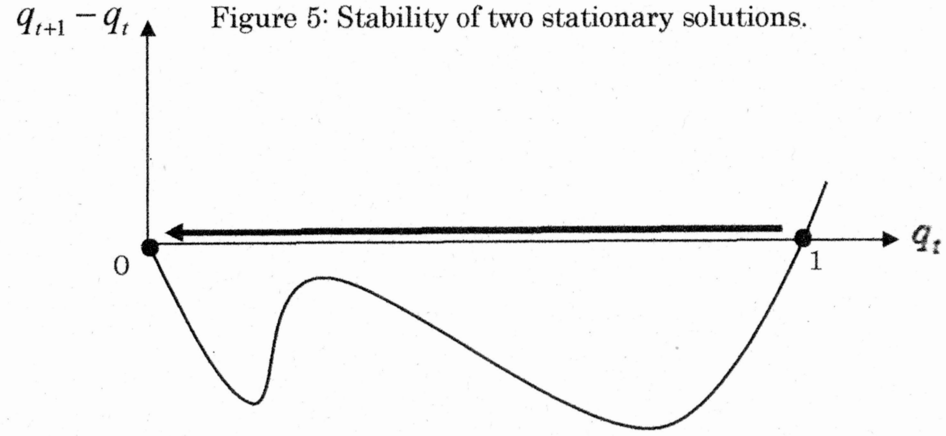


Figure 5: Stability of two stationary solutions.



(1) of Proposition 2 means that if the initial value q_0 of the number of fans is less than $\underline{q}(<1/4)$, then the number of fans becomes $q^*=0$, i.e., all individuals are non-fans. However, if the initial value q_0 is greater than $\underline{q}(<1/4)$, then the number of fans becomes $q^*=\overline{q}$, i.e., no fans of the sports team become $1-\overline{q}$, which indicates that when fans of the sports team are slightly in the minority ($\underline{q}<q_0<\overline{q}$), an individual with preference L actively undertakes educational investment to make the child have the same preference L. Also, the sports team invests actively to improve future profits. Therefore, the number of fans increases to $\overline{q}$. Additionally, this result suggests that when fans of the sports team are in the majority ($\overline{q}<q_0<1$), an individual with preference L does not actively invest to given one's own child the same preference L; moreover, the sports team does not actively invest to improve future profits.

Consequently, the number of fans decreases to $\overline{q}$. However, when fans of the sports team are in the minority ($0 < q_0 < \underline{q}$), the effects of educational investment by an individual with preference L and investment by the sports team are less than the effect of educational investment by an individual with preference N. The number of fans therefore becomes 0.

Actually, (2) of Proposition 2 indicates that all individuals become non-fans, irrespective of the initial value. Therefore, the benefit of becoming fans of sports team is less, irrespective of the number of fans. Not all individuals become fans.

Comparative statics

Finally, we examine how the number $q^* = \underline{q}, \overline{q}$ of sports team fans in a steady state changes for parameters δ, f, R, w, T. The following Lemma 3 holds for the result of comparative statics of stationary solutions $q^* = \underline{q}, \overline{q}$.

Lemma 3

(1) $\underline{q}$ decreases and $\overline{q}$ increases, so sports team fans increase when the discount rate increases. The sports team emphasizes future revenues.

(2) $\underline{q}$ decreases and $\overline{q}$ increases, so sports team fans increase when the sports team revenue per fan increases.

(3) $\underline{q}$ decreases and $\overline{q}$ increases, so sports team fans increase when individual property increases.

(4) $\underline{q}$ increases and $\overline{q}$ decreases, so sports team fans decrease when the expense for sports increases.

(5) Effects of increasing the number of fans by increasing investment in sports teams and enhancing the sports team quality and the effect of decreasing the number of fans by reducing the benefit that a non-fan child derives from becoming a fan occur when public support increases. If the former is larger than the latter, then $\underline{q}$ decreases and $\overline{q}$ increases. Therefore, the sports team fans increase.

From (5) of Lemma 3, we infer that the rise of the amount of public support to sports team can increase the number of fans $\overline{q}$ in a steady state. However, from (4) of Lemma 2, when the number of fans $\overline{q}$ is greater than one-third, the rise of $\overline{q}$ decreases the level of investment of the sports team, i.e., the sports team quality. Therefore, the following Proposition 3 holds from these two observations.

Proposition 3 Increasing the amount of public support of sports team increases the number of fans, but it might decrease the sports team quality.

Proposition 3 shows that increasing public support of sports teams increases the number of fans and lowers the investment cost of the sports team and consquently improves the level of investment, but increasing the number of fans decreases the sports team quality. That is to say, increasing the number of fans and enhancing the sports team quality share a tradeoff relation. Results show that one should judge whether to carry out public support until after the effects are verified carefully.

Concluding remarks

As described in this paper, we used a cultural transmission model to analyze the sports lover formation process. Moreover, we examined how public support of a sports team affects the number of fans and the sports team quality.

Results show the following. First, if both discount rate and the revenue per fan are larger and the increment of utility when a child of a non-sports fan has the same preference as a parent's, then two stationary interior solutions $0 < \underline{q}, \overline{q} < 0$ exist, i.e., all individuals are heterogeneous. Second, results demonstrated that if the initial value of the number of fans is less than $\underline{q}(<1/4)$, then the number of fans becomes 0. If the initial value is greater than $\underline{q}(<1/4)$, then the number of fans becomes $\overline{q}$. Third, results show that increasing public support of sports team increases the number of fans, although it might decrease the sports team quality.

The remaining issues in this paper are the following. The first issue is that the sports team revenue has been treated as a constant. A better model might incorporate consideration of how the sports team quality and the educational level of individuals affect the ticket price for watching a game by introducing a sports spectator market. A second issue is that our analysis assumed that only one sports team exists in a region. Introducing a new variable to represent competitive balance among teams can extend the model such that plural sports teams exist in a region. Consequently, one might examine this study by comparison to previous studies that analyzed competitive balance.

Appendix A1

Proof of Lemma 1

Using implicit function theorem, the following hold from $u'(\cdot)>0, u''(\cdot)<0, H'(\cdot)>0, H''(\cdot)>0$.

$$\frac{d\tau_L}{df}=-\frac{\delta(1-q_t)u'(w-T-f)}{H''(\tau_L)}<0,\quad \frac{d\tau_L}{dv}=\frac{\delta(1-q_t)}{H''(\tau_L)}>0,$$

$$\frac{d\tau_L}{dw}=\frac{\delta(1-q_t)\left[u'(w-T-f)-u'(w-T)\right]}{H''(\tau_L)}>0,$$

$$\frac{d\tau_L}{dq_t}=-\frac{\delta\left[u(w-T-f)+v-u(w-T)\right]}{H''(\tau_L)}<0,$$

$$\frac{d\tau_L}{dT}=-\frac{\delta(1-q_t)\left[u'(w-T-f)-u'(w-T)\right]}{H''(\tau_L)}<0$$

Additionally, in the same way, using the implicit function theorem, the following hold from $G'(\cdot)>0, G''(\cdot)>0$.

$$\frac{d\tau_N}{df}=\frac{\delta q_t u'(w-T-f)}{G''(\tau_N)}>0,\quad \frac{d\tau_N}{dv}=0,$$

$$\frac{d\tau_N}{dw}=\frac{\delta q_t\left[u'(w-T)-u'(w-T-f)\right]}{G''(\tau_N)}<0,\quad \frac{d\tau_L}{dq_t}=\frac{\delta\left[u(w-T)-u(w-T-f)\right]}{G''(\tau_N)}>0,$$

$$\frac{d\tau_N}{dT}=-\frac{\delta q_t\left[u'(w-T)-u'(w-T-f)\right]}{G''(\tau_N)}>0$$

Appendix A2

Proof of Lemma 2

We assume the educational effort cost of individual with preference L $H(\tau_L)$ ($H'(\cdot)>0, H''(\cdot)>0$) to be the following quadratic function.

$$H(\tau_L)=\frac{a}{2}\tau_L{}^2 \tag{26}$$

Using (26), the educational effort cost of individual with preference L is the following from (13).

$$\tau_L(v)=\frac{\delta}{a}(1-q_t)\left[u(w-T-f)+v-u(w-T)\right] \tag{27}$$

Differentiating (27) by v, the following holds.

$$\tau_L'(v) = \frac{\delta}{a}(1-q_t) \tag{28}$$

Substituting (28) for (16), the first-order condition of sports team is expressed as shown below.

$$\frac{d\pi}{dv} = \frac{\delta^2}{a} Rq_t(1-q_t)^2 = C_v(v,T) \tag{29}$$

Using implicit function theorem for (29), the following holds from $C_v(\cdot) > 0, C_{vv}(\cdot) > 0, C_{vT}(\cdot) < 0$.

$$\frac{dv}{dR} = \frac{\frac{\delta^2}{a} q_t(1-q_t)^2}{C_{vv}(v,T)} > 0,\ \frac{dv}{dw} = 0,\ \frac{dv}{d\delta} = \frac{\frac{2\delta}{a} Rq_t(1-q_t)^2}{C_{vv}(v,T)} > 0,$$

$$\frac{dv}{dq_t} = \frac{\frac{\delta^2}{a} R(1-q_t)(1-3q_t)}{C_{vv}(v,T)}, \tag{30}$$

$$\frac{dv}{dT} = \frac{\frac{\delta^2}{a} Rq_t(1-q_t)^2 - C_{vT}(v,T)}{C_{vv}(v,T)} > 0.$$

From (30), the following are obtained:

$\frac{dv}{dq_t} < 0$ holds if $q_t > \frac{1}{3}$ holds; alternatively,

$\frac{dv}{dq_t} > 0$ holds if $q_t < \frac{1}{3}$ holds.

Appendix A3

Proof of Proposition 1

Differentiating $A = \frac{\delta^2}{ab(T)} Rq^*(1-q^*)^3$ of (25) for q^*, then

$$\frac{dA}{dq^*} = \frac{\delta^2}{ab(T)} R(1-4q^*)(1-q^*)^2 = 0.$$

From this equation, $1-q^* \neq 0$ holds; therefore, $q^* = \frac{1}{4}$ holds. Moreover, differentiating twice, the following holds.

$$\frac{dA^2}{d^2q^*} = -6\frac{\delta^2}{ab(T)} R(1-q^*) < 0$$

A becomes a concave function that $q^* = \dfrac{1}{4}$ is peak on q^*. From Figure 3, if the maximum value $\dfrac{27}{256}\dfrac{\delta^2}{ab(T)}R$ of the left side obtained by substituting A for $q^* = \dfrac{1}{4}$ is larger than $u(w-T)-u(w-T-f)$ of the right side, then q^* that satisfy (25) is $\underline{q}$ and $\overline{q}$. However, if the former is less than the latter, then q^* that satisfies (25) does not exist.

Appendix A4

Proof of Proposition 2

First, we consider the case of $\dfrac{27}{256}\dfrac{\delta^2}{ab(T)}R > u(w-T)-u(w-T-f)$. In this case, there are four stationary solutions $(0, \underline{q}, \overline{q}, 1)$ from Proposition 1. Here, we set the right side of (23) as equal to the following.

$$E = q_t(1-q_t)\left(\frac{\delta^2}{ab(T)}Rq_t(1-q_t)^3 - \big(u(w-T)-u(w-T-f)\big)\right) \tag{31}$$

Differentiating (31) for q_t yields the following expression.

$$\frac{dE}{dq_t} = (1-2q_t)\left(\frac{\delta^2}{ab(T)}Rq_t(1-q_t)^3 - \big(u(w-T)-u(w-T-f)\big)\right) + q_t(1-q_t)\frac{\delta^2}{ab(T)}R(1-4q_t)(1-q_t)^2 \tag{32}$$

Substituting $q_t = 0$ for (32), the following holds.

$$\left.\frac{dE}{dq_t}\right|_{q_t=0} = -\big(u(w-T)-u(w-T-f)\big) < 0 \tag{33}$$

Substituting $q_t = 1$ for (32), the following holds.

$$\left.\frac{dE}{dq_t}\right|_{q_t=1} = u(w-T)-u(w-T-f) > 0 \tag{34}$$

Substituting $q_t = \underline{q}$ for (32), clause 1 of (32) is 0. The following holds from $0 < \underline{q} < 1/4$.

$$\left.\frac{dE}{dq_t}\right|_{q_t=\underline{q}} = \underline{q}(1-\underline{q})\frac{\delta^2}{ab(T)}R(1-4\underline{q})(1-\underline{q})^2 > 0 \tag{35}$$

Moreover, $q_t = \overline{q}$ for (32), similarly clause 1 of (32) is 0. The following holds from $1/4 < \overline{q} < 1$.

$$\left.\frac{dE}{dq_t}\right|_{q_t=\bar{q}} = \bar{q}(1-\bar{q})\frac{\delta^2}{ab(T)}R(1-4\bar{q})(1-\bar{q})^2 < 0 \tag{36}$$

Actually, (23) is drawn as Figure 4 from (33), (34), (35), and (36). From Figure 4, (1) of Proposition 2 holds.

Second, we consider the case of $\frac{27}{256}\frac{\delta^2}{ab(T)}R < u(w-T) - u(w-T-f)$. In this case, there are two stationary solutions (0,1) from Proposition 1. Here, (23) is drawn as Figure 5 from (33) and (34). From Figure 5, (2) of Proposition 2 holds.

Appendix A5

Proof of Lemma 3

To satisfy stationary solutions $q^* = \underline{q}, \bar{q}$ from (24), the following must hold.

$$\frac{\delta^2}{ab(T)}Rq^*(1-q^*)^3 - \left(u(w-T) - u(w-T-f)\right) = 0 \tag{37}$$

Using implicit function theorem for (37), $0 < \underline{q} < 1/4$ and $1/4 < \bar{q} < 1$, $u'(\cdot) > 0, u''(\cdot) < 0, b'(\cdot) < 0$ hold. Therefore, the following hold.

$$\left.\frac{dq^*}{d\delta}\right|_{q^*=\underline{q}} = -\frac{2\underline{q}(1-\underline{q})}{\delta(1-4\underline{q})} < 0, \quad \left.\frac{dq^*}{d\delta}\right|_{q^*=\bar{q}} = -\frac{2\bar{q}(1-\bar{q})}{\delta(1-4\bar{q})} > 0 \tag{38}$$

$$\left.\frac{dq^*}{dR}\right|_{q^*=\underline{q}} = -\frac{\underline{q}(1-\underline{q})}{R(1-4\underline{q})} < 0, \quad \left.\frac{dq^*}{dR}\right|_{q^*=\bar{q}} = -\frac{\bar{q}(1-\bar{q})}{R(1-4\bar{q})} > 0 \tag{39}$$

$$\left.\frac{dq^*}{dw}\right|_{q^*=\underline{q}} = \frac{u'(w) - u'(w-f)}{\frac{\delta^2}{ab}R(1-\underline{q})(1-4\underline{q})} < 0, \quad \left.\frac{dq^*}{dw}\right|_{q^*=\bar{q}} = \frac{u'(w) - u'(w-f)}{\frac{\delta^2}{ab}R(1-\bar{q})(1-4\bar{q})} > 0 \tag{40}$$

$$\left.\frac{dq^*}{df}\right|_{q^*=\underline{q}} = \frac{u'(w-f)}{\frac{\delta^2}{ab}R(1-\underline{q})(1-4\underline{q})} > 0, \quad \left.\frac{dq^*}{df}\right|_{q^*=\bar{q}} = \frac{u'(w-f)}{\frac{\delta^2}{ab}R(1-\bar{q})(1-4\bar{q})} < 0 \tag{41}$$

$$\left.\frac{dq^*}{dT}\right|_{q^*=\underline{q}} = -\frac{-\frac{\delta^2 R\underline{q}(1-\underline{q})^3 ab'(T)}{a^2\left(b(T)\right)^2} + \left(u'(w-T) - u'(w-T-f)\right)}{\frac{\delta^2}{ab}R(1-\underline{q})(1-4\underline{q})} \tag{42}$$

$$\left.\frac{dq^*}{dT}\right|_{q^*=\overline{q}} = -\frac{-\dfrac{\delta^2 R\overline{q}(1-\overline{q})^3 ab'(T)}{a^2\left(b(T)\right)^2} + \left(u'(w-T) - u'(w-T-f)\right)}{\dfrac{\delta^2}{ab} R(1-\overline{q})(1-4\overline{q})} \tag{43}$$

Clause 1 of the numerator of (42) and (43) represents an effect in increase of the number of fans by the rise of amount of public support. However, clause 2 of the numerator represents an effect in decrease of the number of fans by the rise of amount of public support. If the former is larger than the latter, then the sign of (42) is negative and the sign of (43) is positive. Alternatively, if the former is less than the latter, then the sign of (42) is positive and the sign of (43) is negative.

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