

Global Solution to Nonholonomic System with Stochastic Feedbacks Based on Non-Smooth Stochastic Lyapunov Function

Taiga Uto¹ and Yûki Nishimura¹

Abstract

Nonholonomic systems are mechanical systems with constraints on their velocity that are not derivable from position constraints [1]. Many control systems such as cars, underwater vehicles, snake robots, and space robots are included in a class of nonholonomic systems. Hence, control problems of nonholonomic systems are important for the basic development of control engineering. However, controlling nonholonomic systems are difficult because many of them have no continuous state-feedback stabilizer [2]. Therefore, discontinuous state-feedback laws [3] or time varying state-feedback laws [4] are considered, while the designs of them are generally complicated.

Recently, the effective use of stabilization by noise is proposed for deriving a simple approach to design stabilizers for nonholonomic systems [5]. This strategy provides state-feedback laws with Gaussian white noises such that the states of the target system converges to the origin with probability one. Because disturbance terms exist, the resulting stochastic systems are represented by stochastic differential equations. The stability of the systems are analyzed via stochastic Lyapunov theory. The characteristic feature of the analysis is to deal with the systems like time-independent systems, while they are, in fact, time-varying.

Using the above strategy of stabilization by noise, we tried to stabilize a chained system by state-feedback laws with a one-dimensional Wiener process [6]. This provides stochastic control laws simpler than ones in [5] and [7]. While we designed a non-smooth stochastic Lyapunov function (SLF) for ensuring the stabilization, the proof is under construction due to the lacks of ensuring global solutions to the stochastic differential equation and analyzing the behavior of the states in the region that the SLF is non-smooth. In this paper, we show the existence of a global solution to the closed-loop system.

References

1. A. Bloch, J.E. Marsden and D.V. Zenkov, *Nonholonomic Dynamics*, pp. 1-2, 2005.
2. R.W. Brockett, Asymptotic stability and feedback stabilization, *Differential Geometric Control Theory*, pp. 181-193, 1983.
3. A. Astolfi, Discontinuous control of nonholonomic systems, *Systems & Control Letters*, Vol. 27, No. 1, pp. 37-45, 1996.
4. O.J. Sordalen and O. Egeland, Exponential stabilization of nonholonomic chained systems, *IEEE Transactions of Automatic Control*, Vol. 40, No. 1, pp. 35-49, 1995.
5. Y.Nishimura, Stabilization by artificial Wiener processes, *IEEE Transactions of Automatic Control*, Vol. 61, No. 11, pp. 3574-3579, 2016.
6. T. Uto and Y. Nishimura, Stabilization of Nonholonomic Systems by One-Dimensional Artificial Wiener Processes, *Proceedings of SICE Annual Conference 2016*, 2016.
7. Y. Nishimura, Stabilization of Brockett integrator using Sussmann-type artificial Wiener processes, *Proceedings of 52nd IEEE Conference on Decision and Control*, 2013.

¹ Graduate School of Science and Engineering, Kagoshima University, 890-0065, Kagoshima, Japan