

Stabilization of Fourth-Order Chained System by Rough Signals

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Abstract

Nonholonomic systems are difficult to control because they have the numbers of control inputs less than the degrees of freedom [1]. The feature generally makes the linear approximate systems uncontrollable. To stabilize nonholonomic systems, we have to design nonlinear control laws [2]. However, robust and systematic control design problems are still challenging due to the complexity of the nonlinear control laws.

To simplify control problems of nonholonomic systems, approximation algorithms [3] are effective. If we apply the algorithms to nonholonomic systems without drift terms, the “hidden control inputs” appears in the resulting system models. The hidden inputs make the models holonomic without decreasing the degrees of freedom. Furthermore, we can make the origin of the models exponentially stable by applying homogeneous approximation to the algorithms and the redesigning pre-inputs [4].

While the approximation algorithms simplify control problems of nonholonomic systems, there are at least two issues. The one is that, the maximum amplitudes of control inputs become larger. The other is that using homogeneous approximations makes control designing complicated. To solve these issues, we need further analysis of the behaviors of the states and the inputs in infinitesimally small time.

Recently, the analysis is developed [5,6] via a stabilizing method using rough path analysis [7]. A stabilizing method of [5] considers the stabilization of a third-order nonlinear system using second-order rough paths. While the method is considered to solve the above-mentioned issues, the literature does not consider them. Furthermore, the literature considers just second-order rough paths. This implies that a concrete designing method of higher-order rough paths has not been proposed yet.

In this paper, we show that the above-mentioned issues are solved via rough path analysis by designing a stabilizer of a fourth-order chained system using third-order rough paths and calculating numerical simulation of the resulting rough systems.

References

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