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# On the Calculations of the Coefficients of the Cyclotomic Polynomials II

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## Abstract

We report some results on the calculations of the  $n$ -th cyclotomic polynomial  $\Phi_n(x)$  for  $n = 111546435 = 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$ . In the coefficients of this polynomial, the maximum value and the minimum value are  $\geq 4071770387$  and  $\leq -4248451085$ , respectively from the point of view of our calculations.

Key words: cyclotomic polynomial, coefficients, supercomputing.

## 1 Grytczuk-Tropak's Formula

Let  $\Phi_n(x) = \sum_{j=0}^{\phi(n)} a_j(n)x^j$  be the  $n$ -th cyclotomic polynomial, where  $\phi(n)$  is the Euler function. The coefficients of the polynomial  $\Phi_n(x)$  are given as follows (A.Grytczuk & B.Tropak[4]):

$$a_j(n) = -\frac{1}{j} \cdot \mu(n) \cdot \sum_{m=0}^{j-1} a_m(n) \mu((n, j-m)) \phi((n, j-m))$$

where  $\mu(n)$  is the Möbius function and  $(n, j-m)$  is the greatest common divisor of integers  $n$  and  $j-m$ .

This formula is deducible both from the recurrence expression representing of the roots of the algebraic equations by use of the elementary expressions (van der Waerden[5, 102page, Aufgaben.1]) and from the Hölder equation for the Ramanujan sums (Hölder[6, Theorem 272]).

Thanks to Professor B.Tropak's suggestions, the running program for the calculations of the cyclotomic polynomial is rewritten to this formula. To obtain the

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complete list of the coefficients of polynomial  $\Phi_n(x)$ , it is sufficient for degree term  $[\phi(n)/2] + 1$ , since  $\Phi_n(x)$  is a symmetric polynomial. We put  $n = 111546435 = 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$ . In this case,  $\phi(n) = 36495360$ . Due to the time limit of our supercomputing, we could not obtain the complete list of  $a_j(n)$ 's. But the first term with degree  $\leq \phi(n)/10 = 3649536$  was outputted.

### Main Output Results

The maximum and minimum values of the coefficients in the first term with degree  $\leq \phi(n)/10 = 3649536$  are 4071770387 and -4248451085 respectively. That is,

$$\Phi_{3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23}(x) = \underbrace{x^{36495360} + \dots + \bigcirc x^{32845824}}_{\text{maximum}=4071770387, \text{minimum}=-4248451085} + \dots + 1.$$

## 2 The program

The followings are the Fortran90 program by the algorithm of the above Grytczuk-Tropak's Formula. The greater part of the consumptions of the time in our program runnings is due to the label 400 of do loop.

---

```
!A.Grytczuk and B.Tropak
  parameter (isize=36495360)
  integer*8 T(isize)
  integer*8 a(0:isize)
  integer*8 prime(20)
  integer*8 euler,r,s,n,sum,myu,const
!      read(5,*)n
      n=3*5*7*11*13*17*19*23
!      n=3*5*7
      call check(n,prime,s)
      myu=(-1)**s
!
      euler=1
      do 100 j=1,s
```

```

        euler=euler*(prime(j)-1)
100  continue
!
        do 500 r=1,euler
            T(r)=1
500  continue
!
        do 300 j=1,s
            do 300 r=1,euler/prime(j)
                T(prime(j)*r)=T(prime(j)*r)*(1-prime(j))
300  continue
!
        a(0)=1
        const=euler/10
!        do 400 j=1,euler
            do 400 j=1,const
!        do 400 j=1,10000000
            sum=0
            do 600 m=0,j-1
                sum=sum+a(m)*T(j-m)
600  continue
            a(j)=-myu*(sum/j)
400  continue
        write(6,*) 'max=',MAXVAL(a), 'min=',MINVAL(a)
        end

```

---

The file containing subroutine check is omitted as it is a simple and short program.

### 3 Running Informations

Our calculations have been done by the NEC supercomputer SX-4. The followings are compiler and system messages. This lists show that User Time(sec) is 27018.020478 sec=7 hours 30 minutes. The integers 4071770387 and -4248451085 must have overflow as 32-bits integer data.

```
cyc_Grytczuk_Tropak_64.f90:

f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 14:
           Vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 18:
           Vectorized loop.
f90: vec(2): cyc_Grytczuk_Tropak_64.f90, line 22:
           Partially vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 23:
           Vectorized loop.
f90: vec(1): cyc_Grytczuk_Tropak_64.f90, line 32:
           Vectorized loop.
f90: vec(4): cyc_Grytczuk_Tropak_64.f90, line 37:
           Vectorized array expression.
f90: vec(4): cyc_Grytczuk_Tropak_64.f90, line 37:
           Vectorized array expression.
f90: cyc_Grytczuk_Tropak_64.f90, _MAIN: There are 7 diagnoses.
check_64.f90:

f90: vec(1): check_64.f90, line 14: Vectorized loop.
f90: check_64.f90, check: There is 1 diagnosis.
moebius_64.f90:

f90: vec(3): moebius_64.f90, line 8: Unvectorized loop.
f90: moebius_64.f90, moebius: There is 1 diagnosis.
start time=Tue Nov 23 18:00:11 JST 1999
max= 4071770387 min= -4248451085
```

```
***** Program Information *****
Real Time(sec)      : 28949.717421
User Time(sec)     : 27018.020478
Sys Time(sec)      : 64.466699
Vector Time(sec)   : 27015.028925
```

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|                 |   |                  |
|-----------------|---|------------------|
| Inst.Count      | : | 1561253529530.   |
| V.Inst.Count    | : | 416268499499.    |
| V.Element Count | : | 106561013327958. |
| FLOP Count      | : | 7300322.         |
| MOPS            | : | 3986.450393      |
| MFLOPS          | : | 0.000270         |
| VLEN            | : | 255.991057       |
| V.Op.Ratio (%)  | : | 98.936935        |
| Memory Size(MB) | : | 561.031250       |
| MIPS            | : | 57.785637        |
| I-Cache(sec)    | : | 2.754349         |
| O-Cache(sec)    | : | 1.222460         |
| Bank(sec)       | : | 0.692612         |

end time=Wed Nov 24 02:02:40 JST 1999

logout

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## References

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