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Periodic Anomalous Vertical Crustal Movement in Tectonic Regions and its Influences on Volcanic Activities －Sakurajima Volcano－

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Abstract

Recent observations (GSI, 1999) of secular changes in ellipsoidal and orthometric heights have discounted the approximation that the two are equal, which has to date been confirmed from continuous GPS and leveling observations. The approximation was made on the assumption that the effect of the temporal change in geoidal height is usually very small. Contrary to this theoretical assumption, recent observations reveal that the two terms do not coincide at all.

In order to solve this problem, a new practical equation for studying the time variation in orthometric height, which provides unstable modes of crustal movement, is presented.

By making some assumptions to solve this equation, strong instability modes were estimated to occur with periods of 16～17 years and a little under 50 years. These show that periodic anomalous crustal deformation modes fluctuate with temporal changes in the geopotential surface, which is understood to relate to the fundamental fluctuation modes of plate motion in the N-W part of the Philippine Sea plate, as pointed out by Mogi (1995).

1. Introduction

The precise determination of temporal changes in vertical crustal movement was made possible by recent geodetic observations obtained through GPS, leveling and gravity surveys. The relationship between the height components of crustal movement is given by the geodetic relation $\partial h = \partial H + \partial N$, where ∂h is the temporal change in ellipsoidal height as measured by GPS, ∂H is the temporal change in orthometric height as measured by leveling, and ∂N is the temporal change in geoidal height as measured by gravity surveys.

Nakagawa (1976) pointed out the possibility of detecting level changes by taking repeated gravity measurements. Hagiwara et al. (1980) showed that the relation between gravity changes and elevation is very useful in estimating subsurface density variations during seismic activity. They also showed that a comparison of gravity change data and volumetric strain data may enable us to reveal the cause of such gravity changes. Johnsen et al. (1980) suggested that the inflation and deflation of the floor of a caldera is caused entirely by the inflow and out flow of magma, as determined from comparisons of gravity and leveling data using a Bouguer type relationship.

In contrast, Heck and Mälzer (1983) pointed out the importance of time-varying geoidal height, based on an empirical formula for the relationship between gravity change and leveling height given by Heiskanen and Moritz (1967) and Hagiwara (1978). They pointed out that neglecting temporal changes in geoid undulations produces a relative error in the calculated vertical movement of the order of $3 \cdot 10^{-3}$ and was responsible for systematic errors in recently observed crustal movements, reaching the magnitude of the standard deviations of leveled height changes.

Arnold (1986) theoretically derived the empirical formula and pointed out that the global term in this formula

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is of second order and rather small compared with the space-time change in the local values of the gravity and the height

In volcanic areas, particularly in and around the Sakurajima volcano, the time varying geoidal height is detected by GPS observations (Tanaka et al., 1995 and Tanaka et al., 1996). Based on these observations, Tanaka (1997A) clarified that the spatial pattern of geoid undulation coincides with a volcanic depression in and around the crater, and that the fluctuations agree with the volcanic underground structure determined from gravity anomalies by Yokoyama and Ohkawa (1986). In addition, Tanaka (1997B) obtained the relation $\partial H \sim k_2 \ \partial N$, where ∂N is assumed to be proportional to $H_0 \ \partial \triangle g/g_0$ and $\triangle g$ is the directly measured gravity difference between observation points. Using this relation, the recent time-variation in geoidal height for the Sakurajima volcano was estimated and compared with that for the case of the 1914 eruption for the purpose of the long-term prediction of volcanic eruption.

From recent continuous GPS measurements between Kakegawa and Omaezaki in the Tokai region, the secular change in ellipsoidal height ∂h was estimated to be -6.0 mm/yr for a period of about 3.5 years from January 1995 to July 1999.

By contrast, the secular change in orthometric height ∂H obtained from repeated leveling surveys in the same district (GSI, 1999) was about -4.6 mm/yr. This result shows that ∂h does not coincide with the theoretical equation $\partial H + \partial N$, where ∂N is usually neglected because it is very small. A linear relationship of $\partial h \sim \partial H$ was expected to be established theoretically between GPS and continuous leveling observations, however, recent observations (GSI, 1999) show no coincidence at all between the two terms.

The theoretical treatment of time-varying geodetic heights has to date been developed on a global scale, for example, by Vaniček (1987), Vaniček and Christou (1994), Grafarend (1997). However, these theories are too complex to be directly applied to a time-varying local dynamic field because the temporal change in geoidal height ∂N is neglected, which is not the case in local leveled height, and ∂h does not coincide with ∂H. The theoretical equation of gravity change due to surface displacement was derived by Okubo (1991). From this result, it was made possible to determine the temporal change in vertical-crustal movement due to gravity change.

We consider that in order to resolve the temporal change of vertical crustal movements, it is necessary to derive a simple equation, similar to an equation of motion, to obtain a physical picture in a local temporal field. In this report, a practical equation for making prognoses of vertical crustal movement is presented, as derived from the relationship between temporal vertical changes and GPS, leveling and gravity survey data. Long term anomalous crustal deformation modes linked to large-scale instability are estimated from recently observed crustal deformation data and compared with past earthquakes in and around Japan.

2. Preliminary Estimation of Vertical Crustal Movement in and around Sakurajima Volcano

Let us estimate time-varying heights caused by geopotential changes and examine the validity of the estimation.

The upheaval of geoidal height as a result of excess mass at shallow depth is given by Hagiwara and Tomoda (1986),

$$\partial N = Gm/[g_0 (x^2 + D^2)^{1/2}] \quad (1)$$

where, G denotes the universal constant of gravitation, $m = (4/3)\pi R^3 \triangle \rho$ represents the excess mass relative to the surrounding mass distribution, R is the radius of the spherical body, $\triangle \rho$ is the density difference, g_0 is the mean gravity, D is the depth to the center of the spherical mass, and x is the distance from the point on the earth's surface just above the center of gravity ($\triangle g$) by the excess mass. The maximum geoidal height is given by

$$\partial N_{max} = Gm/(g_0 D),\ \partial \triangle g \sim Gm/D^2 \text{ and } \partial N_{max} \sim D\, \partial \triangle g/g_0 \quad (2)$$

At the time of the Aira caldera eruption (about 22,000 YBP), the total ejection of mass is estimated to be 5×10^{11} tons from gravity structure data (Yokoyama and Ohkawa,1986). At the time of eruption, $\triangle g$ is estimated to have been around -33 mgal, and $\partial N_{max}\sim$-34 cm, from the left hand side of equation (2) assuming that the mean depth of spherical cavity is almost the same depth as that at the time of the 1914 eruption of Sakurajima; D$\sim$10 km and R$\sim$3.4 km (Mogi, 1957).

At the time of the 1914 eruption of Sakurajima, ∂N_{max} is estimated to have around -2 mm and $\triangle g\sim$-0.2 mgal, assuming $m=3\times10^9$ tons ejected from D$\sim$10 km. This ejected value corresponds to the value of about 0.6% of the total ejection of mass (5×10^{11} tons) at the time of the Aira caldera eruption. This suggests that the intrinsic eruptive energy of Aira may be very high.

The ground deformation caused by the eruption of Sakurajima has to date been explained by two models based on the Mogi (1957) model; a tensile fault-deflation source model (depth: 0.5 km) by Hashimoto and Tada (1988), and a twin pressure source model (depth: 3 km and 10 km) by Eto and Nakamura (1985). Using D$\sim$3 km and $m=3\times10^9$ tons, $\triangle g$ is estimated to be about -2.2 mgal and $\partial N_{max}\sim$-6.8 mm. In this case, ∂N_{max} occurs in the region from-2 mm to -6.8 mm. Hashimoto and Tada (1988) estimated a crustal subsidence of 157 cm using their model.

In the recent eruption of Minamidake, in the southern part of the Sakurajima crater, $\triangle g$ is estimated to have been 300 μ gals and $\partial N_{max}\sim$0.9 mm using D$\sim$3 km if the total ejection mass is assumed to be 4×10^8 tons (Yokoyama, 1989).

Let us investigate the relationship between gravity change and secular orthometric height change, in a volcanic area with a large gravity gradient such as Sakurajima. In this case, we use the following gravimetric leveling equation for practical use by introducing a new dimensionless coefficient k_2:

$$\partial H\sim k_2 H_0\,\partial\triangle g/g_0\sim k_2\,\partial N \quad (3)$$

where ∂N is approximated as $H_0\,\partial\triangle g/g_0$. The coefficient of scale k_2 differs according to the observation site and is given by

$$k_2=(\partial\triangle g/\partial H)^{-1}(g_0/H_0) \quad (3)'$$

where $\partial\triangle g/\partial H$ is the temporal change in the gravity gradient.

Heck and Mälzer (1983) determined the temporal change in precise geoidal height ∂N using the empirical formula

$$\triangle_t g=a+b\,\triangle_t H, \qquad \triangle_t H=\triangle_t g/b-a/b \quad (4)$$

$$\zeta=\triangle_t H+\triangle_t N \quad (4)'$$

where $\triangle_t g$, ζ, $\triangle_t H$ and $\triangle_t N$ correspond to $\partial\triangle g$, ∂h, ∂H and ∂N, respectively, in this paper, and a and b are constants for which a/b cancels out in the sum of $\triangle_t H+\triangle_t N$ in equation (4)'. Equation (4) is derived by replacing $g_0/(H_0 k_2)$ by b and $\partial\triangle g$ by $\triangle_t g$ - a in equation (3). Arnold (1986) theoretically derived equation (4) and pointed out that the global term included in the coefficients(-b/a) is of second order and rather small.

Ishihara et al.(1995) obtained vertical changes of -12.1 cm and gravity changes of 166 μ gal over the last 17 years at Sakurajima. The average secular gradient of level change is -7.1 mm/yr and the rate of gravity change $\alpha\sim10\,\mu$ gal/yr. A large gravity/level ratio of -13 μ gal/cm is estimated, which corresponds to $\triangle g\sim6\triangle g_B$ obtained from an assumed density of 2.2 g/cm^3, where $\triangle g_B$ is the Bouguer gravity anomaly. Using these values and taking values of $H_0\sim$408 m (Mt. Haruta) and $g_0\sim$980 gal, we obtain $k_2\sim-1.75\times10^3$ and $\partial N\sim6.91\times10^{-3}$cm. Then, $\partial N\,k_2\sim-12.1$ cm is obtained as the vertical height difference for the last 17 years. Using this coefficient k_2, we can obtain the crustal deformation ∂H at the time of the 1914 eruption of Sakurajima; $\partial H\sim k_2\,\partial N\sim1750\cdot0.9$ mm=158cm, assuming D$\sim$3km, $m=3\times10^9$ tons, $\triangle g\sim$-2.2 mgal and $H_0\sim$408 m. This coincides with the results given by Hashimoto and Tada (1988).

Thus , it may be appropriate to consider ∂N as negligible compared with the change in orthometric height ∂H. The approximation $\partial h\sim\partial H$ holds from continuous GPS and leveling observations because the time varying geoidal height ∂N is usually very small in the equation $\partial h=\partial H+\partial N$. However, the obtained results show that the temporal change in vertical crustal deformation caused by temporal changes in local geopotential field can not be neglected.

3. Detection of Temporal Vertical Crustal Movements by GPS, Leveling and Gravity Surveys

Two types of temporal changes in ellipsoidal and orthometric heights have been observed by continuous GPS measurements and repeated leveling surveys between Kakegawa city (BM140-1) and Omaezaki (B.M.2549) (Geographical Survey Institute, 1999). The orthometric height demonstrates the secular change accompanied by a 6~8 year-long fluctuation on a linear secular trend of subsidence in addition to annual seasonal change, whereas the ellipsoidal height demonstrates a linear secular subsidence only for the last three years, as seen in Figs.1 (A) and (B). Simultaneous observations by GPS and leveling surveys at almost similar locations have been carried out since 1995 (GSI, 1999) and the difference between the rates of secular changes in orthometric and ellipsoidal heights is very large compared with the expected theoretical values.

For the purpose of clarifying these phenomena, the following equation is presented. That is, we assume that $\partial h \sim k_1 \, \partial H$, where the coefficient k_1 is the ratio of ∂h to ∂H and is determined from GPS and leveling observations. The observed k_1 is very large compared with the expected value, although it is nearly equal to 1. The following equations can thus be assumed, provided that; the temporal change in orthometric height ∂H is divided into a linear secular part ∂H_s and a temporal variational part ∂H_v, ∂H_v is superposed on ∂H_s, the relation $\partial h \sim c$ holds from GPS observations, as shown in Fig.1 (B) and the time average of the temporal variational part ∂H_v over a long period is expected to be nearly equal to 0.

$$\partial H = \partial H_s + \partial H_v \tag{5}$$

Then, the following relation is re-assumed,

$$\partial h \sim k_1 \partial H_s \sim c \tag{6}$$

The coefficient k_1 can be expressed in the following form from equations (2) and (3)',

$$k_1 \sim 1 + \triangle k + \cdots = [\partial h / \partial H]$$
$$\sim [(\partial H + \partial N_{max}) / \partial H] \sim 1 + (1/k_2)(D/H_0) \tag{7}$$

The term $\triangle k$ is almost equal to $\triangle k \sim (1/k_2)(D/H_0)$ when $\triangle k \ll 1$. From equation (7), we can obtain equation (3) as,

$$\partial H = (1/\triangle k) \, \partial N_{max} \sim k_2 (H_0/D) \, \partial N_{max} \sim k_2 H_0 \, \partial \triangle g / g_0 = k_2 \, \partial N \tag{8}$$

From this equation, we find that ∂H can be obtained from precise micro-gravity and leveling surveys. Then, we get the following equation, using equations (6), (5) and (3):

$$\partial h = -k_1 \, \partial H_v + k_1 k_2 (H_0/D) \, \partial N_{max} \tag{9}$$

The first term on the left-hand side of equation (9) represents the secular change of ellipsoidal height, which is given by GPS observations related to a conventional first order leveling survey by coefficient k_1. The first term on the right-hand side, ∂H_v, represents the temporal variational term to be solved below. The second term on the right-hand side represents the temporal variational term of the secular change in height caused by the time-varying geopotential field, and controls the other heights.

Now, in order to solve this equation, equation (9) is arranged as,

$$\partial H_v - k_2 H_0 \, \partial \triangle g_v / g_0 = -\partial h / k_1 \sim -c/k_1 \tag{9'}$$

This is derived from equations (4) and (4)' given by Heck and Mälzer (1983), and equations (5) and (6), using the relations $\triangle_t g = \partial \triangle g_v + \partial \triangle g_s = \partial \triangle g_v + a$, and $\triangle_t H = \partial H_s + \partial H_v$, and neglecting $\triangle_t N$. The variational term $\partial \triangle g_v$ shows the relative change to the secular change term a. From the right side of this equation, we obtain that $h = h_0 - ct$ from $\partial h \sim -c$.

The time variation of the geopotential field does not exhibit periodicity as expected due to the earthquake cycle or the eruption cycle at present. Therefore, $\partial \triangle g_v$ in equation (9)' is assumed , for convenience, to have the simplest periodic function in order to clarify the physical picture of long term crustal deformation, written as

$$\partial \triangle g \sim \partial \triangle g_v \sin(\Omega t - \beta) \tag{10}$$

where Ω and β are the characteristic angular frequency and phase difference produced by velocity differences between plate motions, respectively. Substituting this into equation (9)', on the assumption that k_2 times geoidal

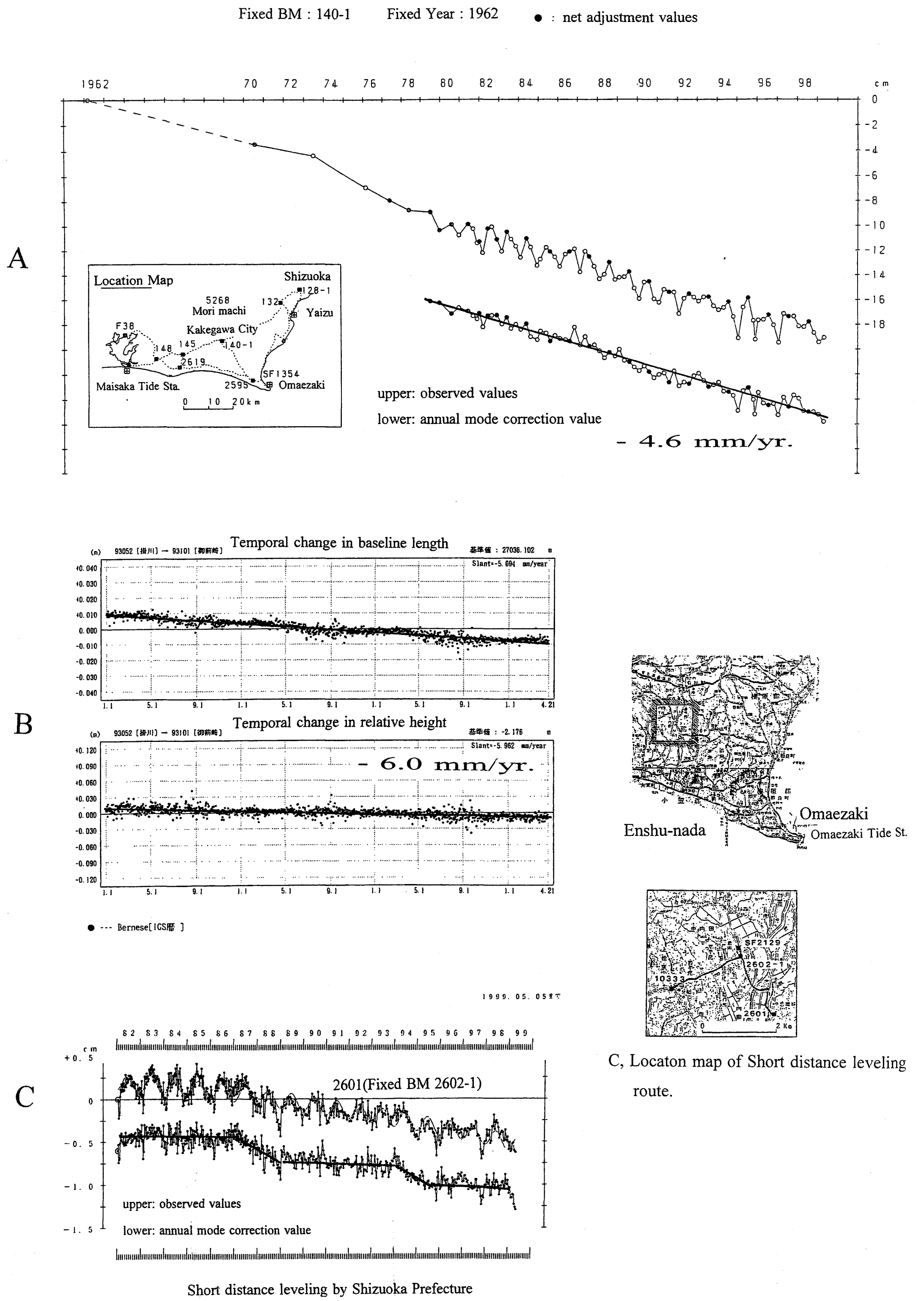

Fig. 1 Comparison of temporal changes in orthometric and ellipsoidal heights between Omaezaki (BM2595) and Kakegawa (BM140-1) (GSI,1999).

A: Temporal variations in orthometric height of Omaezaki relative to Kakegawa (−4.6 mm/yr). Annual seasonal changes in height superposed on linear decreasing secular change with period 7~8 years are observed. Peaks in secular variation at about 1985 and 1991 are seen, coinciding with peaks in volcanic ash ejected by Sakurajima eruptions.

B: Temporal variations of baseline length (upper) and relative height (lower) around Omaezaki relative to Kakegawa by continuous GPS observations (−6.0 mm/yr).

C: Temporal variations in height by short distance leveling of BM2601 relative to BM 2129 (upper) and BM2602-1 (lower) carried out by Shizuoka Prefecture. Step-like Change (about 2 years from 1987 to 1988 and 1994 to 1995) with period of 7 years is observed. Temporal changes by leveling and GPS height differ from theoretically expected values.

fluctuation has a sinusoidal wave form, yields

$$\partial H_v - k_2 H_0/g_0 \partial \triangle g_v \sin(\Omega t - \beta) = -c/k_1 \quad (11)$$

We can solve this equation simply as follows:

$$H_v = h_0 - (c/k_1)\, t + k_2 N_0 \sin(\Omega t - \beta) \quad (12)$$

where h_0 is the initial value and c is the rate of secular change in ellipsoidal height. In this case, $N_0 = (H_0/g_0) \triangle g_0$ is independent of time and is an arbitrary constant value characterized by the local geoidal field. When $H_v = 0$ and $t = 0$, we obtain the initial state $h_0 = k_2 N_0 \sin(\beta)$.

Here, we consider a special case in which this system approaches the maximum unstable mode assuming that $\triangle g_v$ in the right side of equation (10) is a function of time. This means the detection of resonance mode of parametric excitation type between crustal deformations caused by plate motion and geoidal change. When t approaches $(\pi/2 + \beta)/\Omega$, that is, $\sin(\Omega t - \beta)$ approaches 1, the following expression is assumed to hold by setting $k_2 (H_0/g_0) \triangle g_v^{max} \sim k_2 N^{max} \sim H_v^m$:

$$H_v^m \sim k_2 N^{max} \sim k_2 (N_v + \triangle N) \sim k_2 N_v \mu \sim H_v \mu \quad (13)$$

where $\mu = (1 + \triangle N/N_v)$, H_v^m is the maximum height at the maximum geoidal height N^{max}. Then, we can obtain the following relation in the region of this maximum value using equations (10) and (13), i.e.,

$$H_v^m \sim \mu H_v \sin(\Omega t - \beta) \quad (13)'$$

Substituting equation (13)' into equation (9'), the following equation in the process approaching the maximum unstable mode is obtained:

$$\partial H_v - \partial (H_v \mu \sin(\Omega t - \beta)) = -c/k_1 \quad (14)$$

Transposing and arranging the second term on the left-hand side of this expression, a secular change in height having the form of a periodic function alone is obtained as

$$H_v = (h^0 - F(t))/(1 - \mu \sin(\Omega t - \beta)) \quad (15)$$

where h^0 is a constant and $F(t) = (c/k_1)\, t$ is given by GPS and leveling observations.

In equation (15), when the denominator is 0, it is found that ∂h becomes 0 in equation (9)'. From this result, we find that H_v in equation (15) changes with a step-like variation at the marginal maximum state according to the event cycle because $\triangle N$ is nearly equal to 0. This indicates that if the elevation increases at the maximum level, it returns from the high unstable level to the initial linear stable level $\partial h \sim 0$, similar to gravity-induced bubble decay instability in space plasma physics.

Applying this process to inland type earthquakes, decay instability, that is, the chaotic fracture mode of crustal deformation, is expected to occur in a marginal state at almost the maximum height via increasing local geoidal undulations so as to stabilize the geopotential in the area and to regulate further crustal upheaval (Tanaka, 2000).

This cyclic process is compared with the pattern of earthquake cycle presented by Scholz et al. (1973) and Fujita and Fujii (1973). The detection of this decay mode is very important for short term earthquake prediction, including the detection of gravity-induced electrokinetic waves through tectono-magnetism.

4. Application of This Theory to Tectonic Regions in Japan

Now, let us make an estimate of the fluctuating modes given by equation (15).

The numerator F(t) in (15) represents a uniform secular subsidence with annual seasonal variation eliminated. This has been given by the Geographical Survey Institute for predicting the possible occurrence of a Tokai earthquake. A periodic fluctuation in H_v at the intermediate stage is expected to appear when the sine term in the denominator approaches zero. This indicates the existence of temporal fluctuations in the geopotential field. However, the most important problem is to obtain the necessary information and to reveal a strong unstable decay mode preceding an earthquake cycle. We consider a strong gravity-induced decay instability mode for the case when the denominator becomes zero in equation (15), that is, $\sin(\Omega t - \beta) \rightarrow 1$, as follows: Locations are shown in Fig.2.

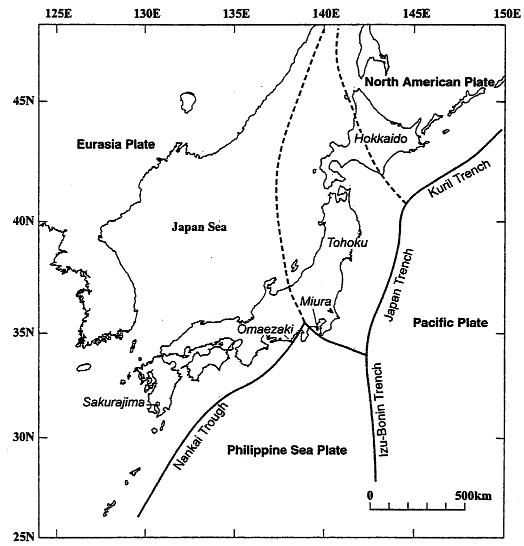

Fig. 2 Location map associated with tectonically active areas in and around Japan. In Hokkaido-Tohoku, subsidence rates are 8～9 mm /yr in south-eastern part of Hokkaido and 3.5～8 mm/yr in eastern part of Tohoku. In Miura peninsula, subsidence rate is 3.7 mm /yr in Aburatubo tide station. In Tokai district, subsidence rate is 7 mm/yr in Omaezaki tide station and 5 mm/yr by leveling as shown in Fig.1(A). Part of the difference (～1 mm/yr) is suggested to reflect sea level rise as well as in Aburatubo (Tanaka and Gomi, 1989). In Sakurajima Volcano, crustal deformation has changed from long-term deflation phase to inflation phase after transition in 1993～1995.

Examples of possible gravity-induced decay instability modes:

(1) Unstable fluctuation mode and strong instability mode near the Miura peninsula

The coefficients of earthquake occurrence period Ω and phase difference β are assumed to be $2\pi/150$ yr and $2\pi/20$yr, respectively (standard event time (SET) is assumed to be 1923) (Tanaka and Gomi, 1989).

- The sine term becomes zero at t=7～8 years. This is observed as the phase difference.
- The strong instability mode occurs at t～about 45 years. This may occur in and around Zenisu ridge or the south Kanto region (Mizoue,1995). The inferred event time (IET) is about 1968-70 (there was large crustal deformation and silent earthquake in the south Kanto region) and the next event will occur around 2013.

(2) Unstable fluctuation mode and strong instability mode at Omaezaki peninsula

The earthquake occurrence period is assumed to be $\Omega=2\pi/120$ yr (Rikitake, 1982) and the phase difference $\sim\beta=2\pi/7.5$ yr (SET is assumed to be about 1944-46).

- The sine term becomes zero at t～16 yr. The sporadic height variations in 1993～1994 may be regarded as a 16-7 yr periodic event.
- The strong instability mode occurs at t～46 yr. IET is 1990-1992, the next is around 2036～2038, related to the precursory large earthquake preceding the main shock of the forthcoming Tokai earthquake.

(3) Hokkaido and Tohoku regions along North-Eastern Japan Trench

The coefficients Ω and β are assumed to be $\Omega\sim2\pi/100$ yr (Rikitake, 1982) and $\beta\sim2\pi/5$ yr (Tohoku Univ., 1998).

- The sine term becomes zero at t～20 yr, the recent repetitive earthquake period is a little under 20 years.
- The strong instability mode occurs at t～45 yr. This is a repetitive strong fluctuation mode. The synthesized superposing mode is very important.

(4) Variation in Sakurajima

The coefficients of Ω and β are assumed to be $\Omega\sim2\pi/120$ yr. and $\beta\sim2\pi/8$ yr. SET is assumed to be 1914 or 1944-46.

- The sine term becomes zero at t～16.3 yr. This mode is seen in the leveling results, (1944-46, 1961～63, 1978～80, 1995～97). Next IET is 2013～15.

· The strong instability mode occurs at t~50 yr. The last IET was 1994-6.

These values were calculated from estimated values, which are naturally subject to probability errors. Crustal deformations therefore will not necessarily occur at the time of strong instability. These modes simply represent a series of precursors of the crustal activity cycle. These results will give us some of a basis for the prediction of long-term earthquake precursors. For example, the fundamental mode of crustal activity in the N-W part of the Philippine Sea plate has a period of a little under 50 years, as pointed out by Mogi (1995).

5. Discussion and Conclusion

It was pointed out that the secular change in orthometric height ∂H can be divided into two parts; the conventional linear orthometric height ∂H_s , which is nearly equal to $\partial h/k_1$, and a variational part given by ∂H_v. These are controlled by $k_2 (H_0/D) \partial N_{max}$, where ∂N_{max} is given by equation (2) and k_2, the dimensionless coefficient defined by equation (3)', assuming a characteristic time. Coefficient k_2 was about 1.75×10^3 in the case of the Sakurajima volcano, whereas in Omaezaki, k_2 is estimated to be about 1.73×10^4, assuming that $g_0\sim10^3$ gals, $H_0\sim1.3\times10^4$ cm, $\partial H\sim1.8$ cm in 3 years (GSI, 1999), and $\partial(\triangle g)\sim8\,\mu$ gals in 3 years (GSI and ERI, 1999). As a matter of course, we can obtain an estimate of 6.1 mm/yr, using 2.67 μ gals/yr. In the Omaezaki district, a small change less than 1 mm in the secular trend of orthometric height, with a period of about 7 years, has been detected from repeated leveling by the Geographical Survey Institute and Shizuoka Prefecture.

Tajima et al. (1984) presented an elastic buoyancy deformation model for the annual seasonal mode, explaining both the phase and amplitude. Their elevation data is not in agreement with recent tidal phases as far as phase difference is concerned. Inouchi and Hosono (1987) found a strong phase correlation between changes in elevation and day time tidal level, although the problem of amplitude remains unsolved.

However, taking the above values of $\triangle_t g\sim2.7\,\mu$ gals/yr (GSI and ERI, 1999), $k_2\sim1.73\times10^4$, $g_0\sim10^3$ gals and $H_0\sim1.3\times10^4$ cm, ∂H then becomes 6 mm/yr. In the relation between $\triangle_t H$ and $\triangle_t g/b$, given by equation (4), ∂H becomes 8.7~14 mm/yr assuming $b=(0.2\sim0.31)\times10^{-5}$ s^{-2} (Heck and Mälzer, 1983) and 2.7 μ gals/yr. Recently, El-Fiky and Kato (2000) obtained a maximum subsidence rate of 7.7 mm/yr. From this, b is estimated to be about 0.35×10^{-5} s^{-2}. Using these results, the problem of amplitude and phase in the annual mode, which has remained unsolved since 1980, may be solved, although this value may be a little over estimated. Then, the fluctuation mode observed from the vertical secular change in Omaezaki may be produced by short periodic local geopotential changes, although there is a problem of how to determine the coefficient k_2 because it is very different in Omaezaki. This result yields the very important problem of short periodic crustal resonance oscillations caused by temporal geopotential changes and plate motion.

The space-time distribution of intermediate-depth earthquakes ($h\geqq60$ km, $M\geqq3$) in northeastern Japan shows a periodic fluctuation of about 6-7 years due to Pacific plate motion (Tohoku Univ., 1996).

In Sakurajima, $H_0\Delta g/g_0$ is estimated to be 0.12mm from recent observations as previously mentioned. From the characteristics of the recent eruption, Ω was assumed to have a period of about 130 years when we consider the recent eruption cycle in 1779 and 1914, and also the crustal activity in the Kyushu district, which has a characteristic 120 year cycle affected by the earthquake cycle on the Nankai Trough. Moreover, a fluctuation having 6~8 year cycle is observed in crustal deformation obtained from tide difference variations (GSI, 1995) and secular changes in volcanic ash (Ishihara, 2000). From these results, we assume that $\Omega\sim2\pi/120$ yr, $\beta\sim2\pi/6\sim8$ yr. Using these values, the sine term becomes zero at 15~20 years, and the 2nd term becomes zero at 45~50 years. In the vertical crustal movement of Sakurajima, a periodic fluctuation of 15~17 years has been observed in the period from 1955 to 1996 (Eto et al., 1998). It is pointed out that this mode may be produced by the fluctuation effect of plate motion.

A long-term forecast equation of secular change in orthometric height expressed by a periodic function was derived from the special case when the time varying geoidal height approaches an extreme state, i.e., $\sin(\Omega t-\beta)\rightarrow 1$. There may be a problem of transformation if we assume that $N_{max}\sim(N+\triangle N)\sim N\mu$ in equation (13). However, H_v^m is nearly equal to k_2N_{max} because $\triangle N$ is very small and almost invariable. Then, H_v^m is expected to exhibit a step-like variation for linear secular change according to an event cycle such as the seasonal change in height, as shown in Fig.1 (C). For detecting the anomalous event under these assumptions, the precise detection of superposed modes among various fluctuations is considered to be very important, because this dilatant pattern of crustal movement is compared with the earthquake cycle model presented by Scholz et al. (1973) and Fujita and Fujii (1973). The coefficient μ is not necessarily always 1 from field survey results (Tanaka, 2000), as is the case in rock breaking tests.

The 7～8 year periodic fluctuations of crustal deformation in the South Kyushu district, including volcanic activity, are considered to be produced by fluctuations in Philippine Sea plate motion because crustal activity in the Tokai region (Mogi, 1996) occurs almost simultaneously. Fig.3 shows the relationship of secular change between vertical crustal deformation in Omaezaki and the amount of volcanic ash from the Sakurajima volcano. Table 1 shows comparison of various crustal deformation modes between standard event time (SET) and inferred event time (IET) in and around Sakurajima volcano. This shows that regional tectonic motions are very important for judging anomalous crustal activities.

A summary of the results is shown below.

The following practical equation of the temporal change in elevation caused by temporal changes in the

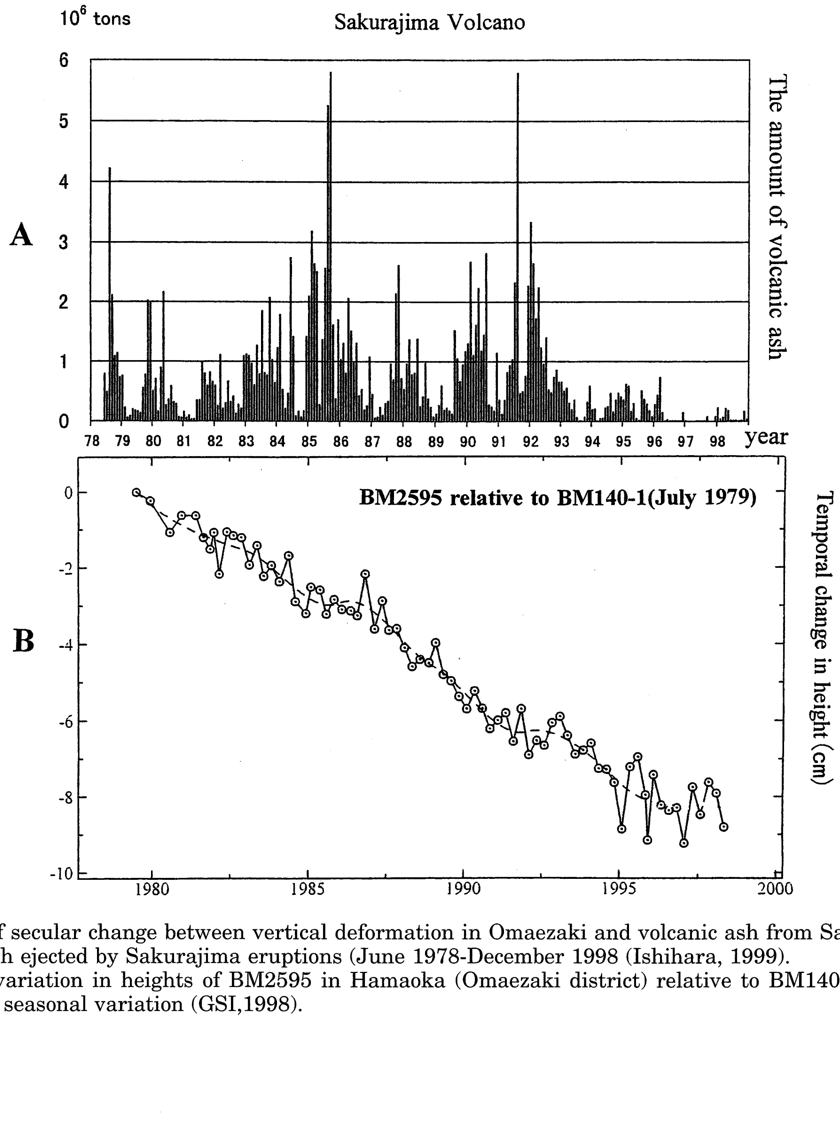

Fig. 3 Comparison of secular change between vertical deformation in Omaezaki and volcanic ash from Sakurajima volcano.
A: Volcanic ash ejected by Sakurajima eruptions (June 1978-December 1998 (Ishihara, 1999).
B: Temporal variation in heights of BM2595 in Hamaoka (Omaezaki district) relative to BM140-1 in Kakegawa after removal of seasonal variation (GSI,1998).

Table 1. Comparison of various crustal deformation modes between standard event time (SET) and inferred event time (IET) in and around Sakurajima Volcano. Data in Remarks were referred to Science Chronological Table 2000.

Event time	IET			
	50yrs mode	16~7yrs mode	7 ~ 8 yrs mode	Remarks
SET 1914		1914	1914	Sakurajima great erup. M7.1
			1922	1922Chijiwa Bay M6.9,6.5,1923 Kirishima S. erup.
		1930~31	1930	1929,31, Hyuganada M6.9,7.1
			1938	1939,41, Hyuganada M6.5,7.2
SET 1944~6	1944~6	1946 setting	1946 setting	Sakurajima erup. 10^8 m^3 ejected
			1954	1955 Sakurajima middle class erup.
		1962-3	1962	1961 Hyuganada M7.0
			1970-71	1968 Hyuganada M7.5, 1970 do. M6.7
		1978-80	1978 (Omaezaki)	Sakurajima max. uplift (high activities in Izu P.)
			1986 (Omaezaki)	1984 Hyuganada M7.1, 1987 do. M6.6
	1994-96	1994-97	1992 (Omaezaki)	1994 Hyuganada M6.4~1997 N.W.Kagoshima, Many E.Q. M6.5,Sakurajima subsiding → uplift
			2000-2(?)	
		2010-14		

geopotential field can be used to determine precise vertical crustal movements from survey results:

$$\partial h = -k_1 \partial H_v + k_1 k_2 \partial N$$

where ∂h and ∂H_v are the linear temporal change in ellipsoidal height and the secular variation in orthometric height ∂H, respectively, k_1 is a coefficient determined from $\partial h = k_1 \partial H$, k_2 is determined from repeated microgravity and leveling observations taken at the same locations, H_0 is the initial orthometric height, ∂N is the temporal change in geoidal height.

By making a few assumptions to solve this equation, strong instability modes, fluctuating with periods of 16~17 years and a little under 50 years, were detected in tectonic regions along trench and troughs. It is suggested that these represent anomalous crustal activity modes related to the fundamental fluctuation modes of plate motion, particularly, in the N-W part of the Philippine Sea plate. Short periodic height variations caused by local geoidal fluctuations were suggested to be triggering signals of seismic activity, and will be very important in making prognoses of anomalous crustal movement.

Application of this theory to other fields, for examples, fluctuating marginal instability of galactic boundary and market and product prices due to the investment difference between supply and demand, are considered possible.

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