On the Strict Treatment of the Dynamic Stability in Elasticity under Periodic Force

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Abstract

There are many papers which deal with problems of the dynamic stability in elasticity under periodic force. In these systems, the deformation before instability is symmetric deformation and the deformation after instability is asymmetric deformation.

In conventional treatment of the systems, the symmetric deformation modes are assumed to be solved under separate variables from the asymmetric deformation modes. Assuming separate variables, the symmetric deformation modes are solved with linear terms, then these solutions are substituted into the equations of motion corresponding to the asymmetric deformation modes. The derived Mathieu-Hill equations are treated as the basic equations for the dynamic stability under periodic force.

In this paper the approximate procedure is not adopted, but the nonlinear equations of motion where the symmetric and the asymmetric deformation modes are coupled are analyzed by applying the method of harmonic balance.

Introduction

There are many papers which deal with problems of the dynamic stability in elasticity under periodic force. In these systems, the deformation before instability is the symmetric deformation (axial deformation in rods, inplane deformation in plates, symmetric deformation in arch, ring and shells), and the deformation after instability is the asymmetric deformation (bending deformation in rods and plates, asymmetric deformation in arch, ring and shells). If the equations of motion are derived by considering the finite deformation theory in elasticity, the symmetric and the asymmetric deformation modes are coupled through nonlinear terms of spring. Then, applying periodic force with symmetric mode, we get the equations of motion which have external periodic force terms, and the symmetric and the asymmetric deformation modes coupled in nonlinear spring terms.

In conventional treatment of the systems, the symmetric deformation modes are assumed to be solved under separate variables from the asymmetric deformation modes. Assuming separate variables, the symmetric deformation modes are solved with linear terms, then these solutions are substituted into the equations of motion corresponding to the asymmetric deformation modes. The derived Mathieu-Hill equations are treated as the basic equations for the dynamic stability under periodic force.

In this paper the approximate procedure is not adopted, but the nonlinear equations of motion where the symmetric and the asymmetric deformation modes are coupled are analyzed by applying the method of harmonic balance. Then, the obtained results are compared with the results which are obtained by adopting the conventional approximate procedure.

In order to examine whether the obtained solutions are stable or unstable for disturbance, the complex eigenvalue problem is solved.

In the last, an approximate method to solve a sort of nonlinear equations of motion is proposed. By applying the method, some nonlinear vibrations are analyzed without solving nonlinear algebraic equations by iteration procedures such as the Newton-Raphson procedure.

1. The Equations of Motion with Two-degree-of-freedom

The elastic system under periodic force with symmetric deformation mode in which we consider the symmetric defomation mode (x_1) and the asymmtric mode (x_2) is governed by the following equations;

$$\ddot{x}_1+\omega_1^2 x_1+\alpha_1 x_2^2+2\alpha_2 x_1 x_2^2+\alpha_3 x_1^2+\alpha_4 x_1^3=f_1\cos\omega t, \qquad (1.1\text{-a})$$

$$\ddot{x}_2+\omega_2^2 x_2+2\alpha_1 x_1 x_2+2\alpha_2 x_1^2 x_2+\alpha_5 x_2^3=0. \qquad (1.1\text{-b})$$

The axial deformation mode (x_1) and the bending deformation mode (x_2) in rods, the inplane deformation mode (x_1) and the bending deformation mode (x_2) in plates, the symmetric deformation mode (x_1) and the asymmetric deformation mode (x_2) in arches[16)17)] and ring, and the deformation mode for $n=0$ (x_1) and the mode for $n=m(m\neq 0)(x_2)$ of shells of revolution[14)] (where n means harmonic number in circumferential direction of the shells) are all expressed by Eq. (1.1). So in this paper Eq. (1.1) are considered.

Assuming Eq. (1.1-a) is solved statically and linearly, normal mode $x_1{}^*$ is given by

$$x_1{}^*=\frac{f_1}{\omega_1^2}\cos\omega t. \qquad (1.2)$$

Substiuting Eq. (1.2) into Eq. (1.1-b), we obtain the following equation:

$$\ddot{x}_2+(\omega_2^2+2\alpha_1\frac{f_1}{\omega_1^2}\cos\omega t)x_2+\alpha_5 x_2^3=0 \qquad (1.3)$$

where the coefficient α_2 is assumed to be zero. In the conventional treatment of the dynamic stability in elasticity, the quadratic order terms of the deformation before instability are neglected. Now, assuming Eq. (1.1-a) is solved linearly, normal mode $\dot{x}_1{}^*$ is given by

$$x_1{}^*=\frac{f_1}{\omega_1^2-\omega^2}\cos\omega t. \qquad (1.4)$$

Substituting Eq. (1.4) into Eq. (1.1-b), we obtain the following equation corresponding to Eq. (1.3)

$$\ddot{x}_2+(\omega_2^2+2\alpha_1\frac{f_1}{\omega_1^2-\omega^2}\cos\omega t)x_2+\alpha_5 x_2^3=0. \qquad (1.5)$$

In the problems called the dynamic stability in elasticity under periodic force, the Mathieu-Hill equations expressed in Eqs. (1.3) and (1.5) are treated as basic equations of them. Many authors[5)~12)] who investigated the problems based on the Mathieu-Hill equations corresponding to Eq. (1.3), where not only the dynamic response before instability but also the influence of nonlinear spring terms are neglected. Bolotin[1] and Koval[9] investigated the equations corresponding to Eq. (1.5), which include the linear response before instability.

In this paper Eq. (1.1) is solved by the method of harmonic balance, which makes clear the influence of nonlinear response before instability and the nonlinear vibration after instability[15]. The results are compared with the results which are obtained by applying the conventional treatment.

2. Investigation of Stability of Periodic Solutions

Assuming periodic solutions ($\boldsymbol{x}_0(t)$) corresponding to undisturbed motion are obtained, the variational equations for disturbed motion are expressed in the form

$$\ddot{\xi}_i+\omega_i^2\xi_i+\sum_j H_{ij}(\boldsymbol{x}_0)\xi_j=0\,(i=1\sim n) \tag{2.1}$$

where $\boldsymbol{x}_0=\{x_1^0 x_2^0\cdots\cdots x_n^0\}$

$x_i^0(t)=x_i^0(t+T)$: periodic solutions,

$\xi_i=\delta x_i$: variations,

$H_{ij}(\boldsymbol{x}_0)$: periodic functions.

Particular solutions of Eq. (2.1) are expressed in the form[1)2)]

$$\xi_i=e^{\mu t}\phi_i(t)\,(i=1,\cdots\cdots,n) \tag{2.2}$$

where μ is the characteristic exponent and $\phi_i(t)$ are periodic functions with period T. The periodic functions $\phi_i(t)$ are assumed in the form

$$\phi_i(t)=\bar{C}_{i0}/2+\sum_k(\bar{C}_{ik}\cos\frac{k\pi}{T}t+\bar{S}_{ik}\sin\frac{k\pi}{T}t). \tag{2.3}$$

Substituting Eqs. (2.2) and (2.3) into Eq. (2.1) and equating the coefficients of identical $e^{\mu t}\sin\omega t$, $e^{\mu t}\cos\omega t$ ($\omega=k\pi/T$), the following equations are obtained:

$$[\mu^2\boldsymbol{I}+\mu\bar{\boldsymbol{C}}+\bar{\boldsymbol{K}}]\{\boldsymbol{y}\}=0 \tag{2.4}$$

where $\{\boldsymbol{y}\}=\{C_{10}C_{11}S_{11}\cdots\cdots\}$.

In order for Eq. (2.4) to have nontrivial solutions, the following complex eigenvalue problem is obtained:

$$\det[\mu^2\boldsymbol{I}+\mu\bar{\boldsymbol{C}}+\bar{\boldsymbol{K}}]=0. \tag{2.5}$$

When the variational equation is expressed as the Mathieu-Hill equation with one-degree-of-freedom, Hayashi[2] approximately applied the following condition for stability of the undisturbed periodic solution:

$$\det\bar{\boldsymbol{K}}>0:\ \text{stable},\quad \det\bar{\boldsymbol{K}}<0:\ \text{unstable}. \tag{2.6}$$

But in this paper the equations of motion with multi-degree-of-freedom are investigated, so the complex eigenvalue problem defined in Eq. (2.5) is solved to examine the stability of the undisturbed periodic solutions. In this case the following condition is applied.

$Re\mu_i \leqq 0 \ (i=1, \cdots\cdots, 2n)$: stable,

$Re\mu_j > 0 \ j \in \{G|1, 2, \cdots\cdots, 2n\}$:unstable. (2.7)

3. Numerical Analysis

Shallow arch models are used as the system governed by Eq. (1.1), because the coefficients of Eq. (1.1) for the shallow arch[16)17)] are explicitly calculated

$$\omega_1^2 = 1+H^2/2, \alpha_1 = -H, \alpha_2 = 1/2,$$
$$\omega_2^2 = 16, \quad \alpha_3 = -3H/4, \quad \alpha_4 = 1/4, \alpha_5 = 4 \tag{3.1}$$

where normal modes x_1 and x_2 correspond to the first and the second mode of the arch, respectively, and H means the nondimensionalized arch rise.

Let $H=10$, $f_1=30$ in Eq. (1.1), the solutions of Eq. (1.1) are assumed in the form

$$x_1 = C_{10}/2 + C_{11}\cos\omega t,$$
$$x_2 = C_{20}/2 + C_{2\frac{1}{2}}\cos\frac{1}{2}\omega t + S_{2\frac{1}{2}}\sin\frac{1}{2}\omega t + C_{21}\cos\omega t + S_{21}\sin\omega t. \tag{3.2}$$

Since x_2 in Eq. (1.1-b) has the trivial solution, Eq. (1.1) is solved under $x_2 \equiv 0$. The resonance curve in this case is shown in Fig. 1. On the resonance curve shown in Fig. 1, there are four branching points, and from these points nontrivial solutions of x_2 branch out. Then, for the case x_2 has nontrivial solution, the resonance curves of Eq. (1.1) are shown in Fig. 2. From the points P_1, P_2 (denoted in Fig. 2), the vibration component $C_{2\frac{1}{2}}$ branches out. The component S_{21} bifurcates from P_3, and the components C_{20} and C_{21} branch out from P_4.

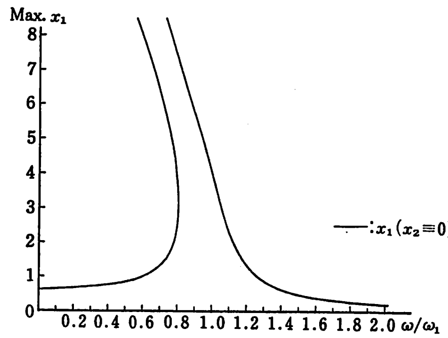

Fig. 1 Resonance Curve of the Symmetric Mode x_1, (Symmetric Force).

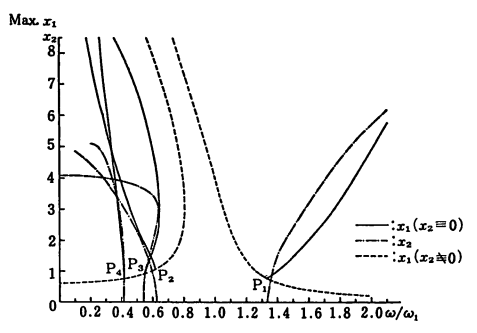

Fig. 2 Resonance Curves of the Asymmetric Mode x_2, (Symmetric Force).

Then, we will investigate the stability of the periodic solutions shown in Fig. 2. For the variational equations of Eq. (1.1), periodic solutions in Eq. (2.3) are assumed in the form

$$\phi_1 = \bar{C}_{10}/2 + \bar{C}_{11}\cos\omega t + \bar{S}_{11}\sin\omega t,$$
$$\phi_2 = \bar{C}_{20}/2 + \bar{C}_{2\frac{1}{2}}\cos\frac{1}{2}\omega t + \bar{S}_{2\frac{1}{2}}\sin\frac{1}{2}\omega t + \bar{C}_{21}\cos\omega t + \bar{S}_{21}\sin\omega t. \tag{3.3}$$

Substituting Eqs. (2.2) and (3.3) into the variational equations, the eigenvalue problem corresponding to Eq. (2.5) is obtained. The result of the stability of periodic solutions is shown in Fig. 3, where Det means det$\bar{K}$ in Eq. (2.5). The resonance curve for $x_1(x_2 \equiv 0)$ is stable in the region of ω/ω_1 for $(0, P_4)$, (P_3, P_2), (P_1, ∞). The branching resonance curve from P_1 is stable, and the other branching resonance curves are unstable. And the sign of det$\bar{K}$ in Eq. (2.5) are shown in Fig. 3.

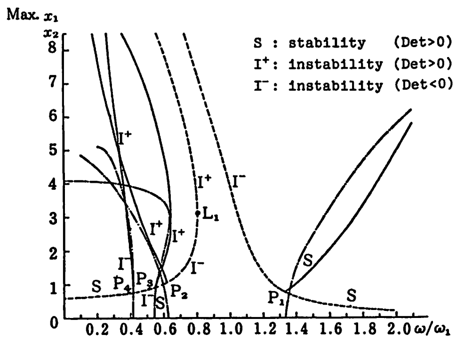

Fig. 3 Stability of the Periodic Solutions in Figs. 1 and 2.

4. Comparison of the results

In this section the results shown in the section 3 are compared with the results obtained by applying the conventional treatment. The main instability region is shown in Table 1. Table 1 shows that the dynamic response before instability should be considered to analyze the dynamic stability of the shallow arch model adopted in this paper. The difference between the results of Eqs. (1.5) and (1.1) is caused by the nonlinear response before instability and the influence of α_2 in Eq. (1.1-b).

Table 1

	ω/ω_1
Mathieu-Hill Equation (Eq. (1, 3))	0.891< <1.310
Mathieu-Hill Equation (Eq. (1, 5))	0.660< <1.349
present (1, 1))	0.630< <1.337

The Width of the Region of Main Instability, (The First-order Approximation).

Then, the behavior after instability is investigated. Bolotin[1] described two procedures to analyze the behavior.

1. Procedure 1

In this procedure Eq. (1.1) are dealt in the form

$$\ddot{x}_1+\omega_1^2 x_1+\alpha_1 x_2^2=f_1 \cos \omega t,$$
$$\ddot{x}_2+\omega_2^2 x_2+2\alpha_1 x_1 x_2=0 \qquad (4.1)$$

The solutions of Eq. (4.1) are assumed in the form

$$x_1 = C_{10}/2 + C_{11}\cos\omega t,$$

$$x_2 = A_2\cos\frac{1}{2}\omega t, \quad or \quad A_2\sin\frac{1}{2}\omega t. \tag{4.2}$$

Substituting Eq. (4.2) in to Eq. (4.1) and applying the condition $A_2 \fallingdotseq 0$, we have the following equation:

$$A_2^2 = \frac{\omega_1^4(4\omega_2^2/\omega_1^2 - \eta^2)(1-\eta^2) \pm 4\alpha_1 f_1}{2(1-2\eta^2)} \tag{4.3}$$

where $\eta = \omega/\omega_1$.

Let $H=10$, $f_1=30$, the value A_2 calculated by Eq. (4.3) is shown in Fig. 4 by the solid lines, where the dotted lines are corresponding to Fig. 2.

2. Procedure 2

Bolotin[1)] also showed the procedure to determine the steady-state amplitude after instability. In this case the following Mathieu-Hill equation was dealt:

$$\ddot{x}_2 + \Omega_2^2(1-2\mu\cos\omega t)x_2 + \alpha_5 x_2^3 = 0. \tag{4.4}$$

The solution x_2 to Eq. (4.4) was assumed in the form

$$x_2 = a\sin\frac{1}{2}\omega t + b\cos\frac{1}{2}\omega t. \tag{4.5}$$

Substituting Eq. (4.5) into Eq. (4.4) and assuming $ab \fallingdotseq 0$, the following equation was obtained

$$A_2 = \frac{2\Omega_2}{\sqrt{3}\,\alpha_5}\sqrt{\frac{\omega^2}{4\Omega_2^2} - 1 \pm \mu} \tag{4.6}$$

where $A_2 = \sqrt{a^2+b^2}$.

Instead of Eq. (1.4), x_1 is assumed as $x_1^* = C_{10}^*/2 + C_{11}^*\cos\omega t$, then coefficients in Eq. (4.4) corresponding to Eq. (1.5) are obtained

$$\Omega_2^2 = \omega_2^2 + \alpha_1 C_{10}^*,$$

$$\mu = -\alpha_1 C_{11}^*/\Omega_2^2. \tag{4.7}$$

As the values C_{10}^* and C_{11}^*, what are obtained at branching points P_1 and P_2 in Fig. 2

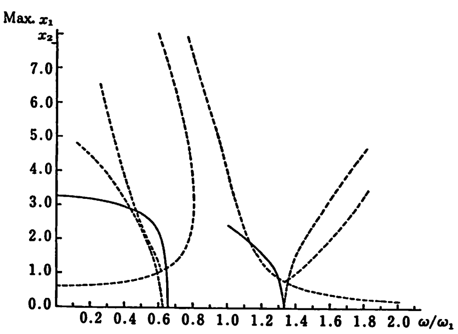

Fig. 4 Resonance Curves after Instability Obtained by the Conventional Treatment 1.

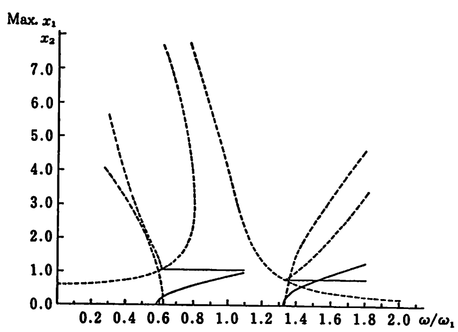

Fig. 5 Resonance Curves after Instability Obtained by the Conventional Treatment 2.

are used. Then A_2 in Eq. (4.6) is calculated and shown in Fig. 5 by the solid lines.

Then, the stability of the solution x_1 shown in Fig. 1 is investigated by assuming the following periodic functions corresponding to Eq. (3.3):

$$\phi_1=\bar{C}_{10}/2+\bar{C}_{11}\cos\omega t+\bar{S}_{11}\sin\omega t,$$
$$\phi_2\equiv 0. \quad (4.8)$$

In this case, it is well-known that the sign of det $\bar{\boldsymbol{K}}$ is expressed as shown in Fig. 6.

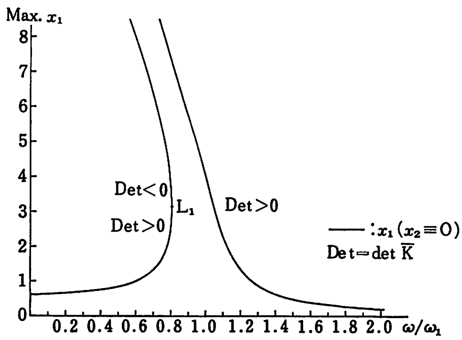

Fig. 6 Stability of the Periodic Solution of the Symmetric Mode x_1.

Comparing the results of Figs. 4, 5 with Fig. 2, there is substantial difference in the steady-state amplitude after instability for the same elastic systems. Then, it is clear that we may consider the equation (such as Eq. (1.1)) where the symmetric and the asymmetric modes are coupled in elasticity instead of the Mathieu-Hill equations (such as Eqs. (1.3), (1.5)), in order to determine the steady-state amplitude after instability. Especially, the elastic models where prebuckling deformation may have great influence on the buckling load of the systems in static analysis, should be investigated with the coupled equations.

Comparing the results shown in Fig. 3 with Fig. 6, we have the following conclusion. The stability of the periodic solution cannot be investigated by the procedure determining the sign of det$\bar{\boldsymbol{K}}$, and has to be concluded by solving the complex eigenvalue problem given by Eq. (2.7).

5. Proposal of an Approximate Procedure to Solve Nonlinear Vibrations

When we write resonance curves of a nonlinear system, we generally have to solve nonlinear algebraic equations with iteration methods. Here an approximate method to solve nonlinear equations of motion without iteration method is proposed.
Considering normal mode x_1 in Eq. (1.1), the following equation is obtained:

$$\ddot{x}_1+\omega_1^2x_1+\alpha_3x_1^2+\alpha_4x_1^3=f_1\cos\omega t. \quad (5.1)$$

The solution of Eq. (5.1) is approximately assumed in the form

$$x_1=C_{10}/2+C_{11}\cos\omega t. \quad (5.2)$$

Substiuting Eq. (5.2) into Eq. (5.1) and equating each harmonics, the following nonlinear algebraic equations are derived:

$$\omega_1^2 C_{10} + \frac{\alpha_3}{2}(C_{10}^2 + C_{11}^2) + \frac{\alpha_4}{2}(2C_{10}^3 + 3C_{10}C_{11}^2) = 0, \tag{5.3-a}$$

$$(\omega_2^2 - \omega^2)C_{11} + \alpha_3 C_{10}C_{11} + \frac{3}{4}\alpha_4(C_{10}^2 C_{11} + C_{11}^3) = f_1. \tag{5.3-b}$$

If ω is selected as a parameter, C_{10} and C_{11} are not expressed by the parameter. But selecting C_{10} as a parameter, C_{11}^2 is solved from Eq. (5.3-a) in the form

$$C_{11}^2 = -\frac{4\omega_1^2 C_{10} + 2\alpha_3 C_{10}^2 + \alpha_4 C_{10}^3}{2(2\alpha_3 + 3\alpha_4 C_{10})}. \tag{5.4}$$

From Eq. (5.3-b), the frequency ω^2 is derived

$$\omega^2 = \omega_1^2 + \alpha_3 C_{10} + \frac{3}{4}\alpha_4(C_{10}^2 + C_{11}^2) - \frac{f_1}{C_{11}}. \tag{5.5}$$

Then, C_{11} and ω^2 are expressed by a parameter C_{10}. But in Eqs. (5.4) and (5.5), $C_{11}=0$ and $C_{11}=\infty$ have to be excepted, so the proposed approximate method cannot be applied in the case

$$\begin{aligned} &4\omega_1^2 C_{10} + 2\alpha_3 C_{10}^2 + \alpha_4 C_{10}^3 = 0, \\ &2\alpha_3 + 3\alpha_4 C_{10} = 0. \end{aligned} \tag{5.6}$$

When Eq. (5.6) is satisfied, the potential curve which is defined in the following equation

$$V(x_1) = \int_0^{x_1} (\omega_1^2 \xi + \alpha_3 \xi^2 + \alpha_4 \xi^3)\, d\xi$$

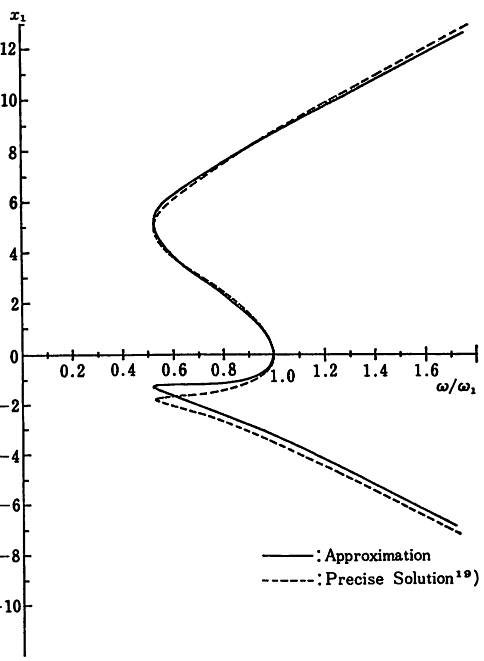

Fig. 7 Backbone Curves (H=3).

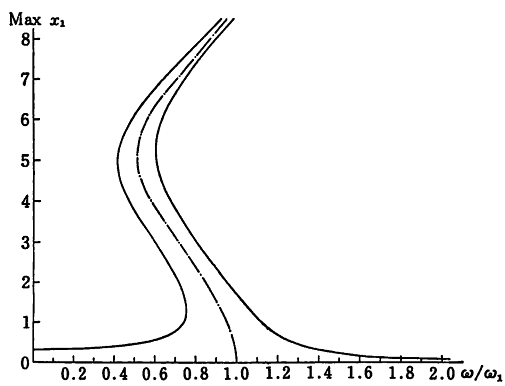

Fig. 8 Resonance Curves of the Symmetric Mode x_1.

has a symmetric axis.[19]

Let's apply the approximation method to some cases where Eq. (5.6) is not satisfied.

1) Free vibration ($f_1=0$ in Eq. (5.1))
Backbone curve of the shallow arch with $H=3$ is shown in Fig. 7 by the solid line. The dotted line in Fig. 7 is the precise solution[19] which is solved by assuming

$$x_1=C_{10}/2+\sum_{k=1}^{5} C_{1k}\cos k\omega t. \tag{5.7}$$

2) Forced vibration
Applying the approximation method to the shallow arch with $H=3$, $f_1=1.68$, the resonance curve is obtained and shown in Fig. 8. And the resonance curve shown in Fig. 1 for the model $H=10$, $f_1=30$ is obtained without iteration method.

3) The boundary region of instability
The solution x_1 solved from Eqs. (5.1), (5.2) is denoted as x_1^*, then Eq. (1.1-b) is given by

$$x_2+(\omega_2^2+2\alpha_1 x_1^*+2\alpha_2 x_1^{*2})x_2+\alpha_5 x_2^3=0 \tag{5.8}$$

Here, Eq. (5.8) is rewritten in the form

$$\ddot{x}_2+\Omega_2^2(1-2\mu_1\cos\omega t-2\mu_2\cos 2\omega t)x_2+\alpha_5 x_2^3=0 \tag{5.9}$$

where $\Omega_2^2=\omega_2^2+\alpha_1 C_{10}+\alpha_2 C_{10}{}^2/2+\alpha_2 C_{11}{}^2$,
$\mu_1=-C_{11}(\alpha_1+\alpha_2 C_{10})/\Omega_2^2$,
$\mu_2=-\alpha_2 C_{11}{}^2/(4\Omega_2^2)$.

The odd regions of instability of Eq. (5.9) satisfy the following equation;
the first approximation;

$$Det_1=(1+\mu_1-\frac{\omega^2}{4\Omega_2^2})(1-\mu_1-\frac{\omega^2}{4\Omega_2^2})=0. \tag{5.10-a}$$

the second approximation;

$$Det_{13}=\{(1+\mu_1-\frac{\omega^2}{4\Omega_2^2})(1-\frac{9\omega^2}{4\Omega_2^2})-(\mu_1+\mu_2)^2\}$$
$$\times\{(1-\mu_1-\frac{\omega^2}{4\Omega_2^2})(1-\frac{9\omega^2}{4\Omega_2^2})-(\mu_1-\mu_2)^2\}=0. \tag{5.10-b}$$

For the even regions of instability of Eq. (5.9), the following equation is obtained as the first approximation;

$$Det_2=(1+\mu_2-\frac{\omega^2}{\Omega_2^2})(1-\mu_2-\frac{\omega^2}{\Omega_2^2}-2\mu_1^2)=0. \tag{5.11}$$

Let $H=10$, $f_1=30$ for the shallow arch, $x_1^*(C_{10}, C_{11}$ and $\omega)$ is determined by Eqs. (5.4) and (5.5), then Det_1, Det_{13} in Eq. (5.10) and Det_2 in Eq. (5.11) are calculated and shown in Figs. 9, 10 and 11. The point which satisfies $\mathrm{Det}_1=0$ in Fig. 9 has the same frequency as the point P_1 in Fig. 2.
The point satisfies $\mathrm{Det}_{13}=0$ in Figs. 9, 10 is corresponding to the branching point which is obtained by assuming the following solution in Eq. (1.1):

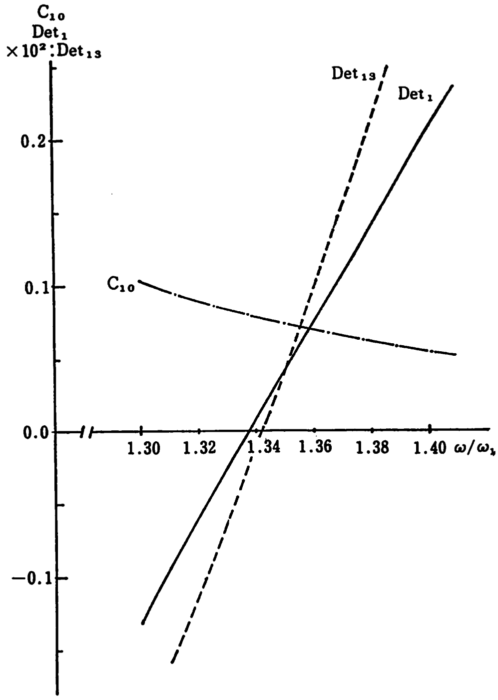

Fig. 9 Det_1 and Det_{13} (1).

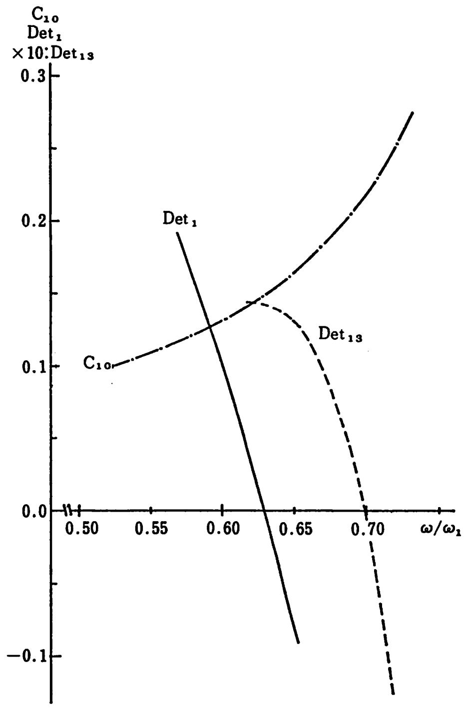

Fig. 10 Det_1 and Det_{13}, (2)

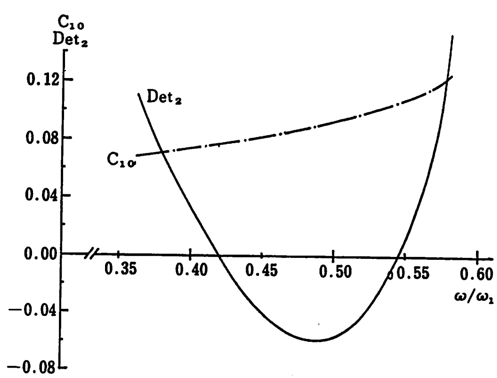

Fig. 11 Det_2.

$$x_1 = C_{10}/2 + C_{11}\cos\omega t,$$

$$x_2 = C_{20}/2 + C_{1\frac{1}{2}}\cos\frac{1}{2}\omega t + S_{1\frac{1}{2}}\sin\frac{1}{2}\omega t + C_{21}\cos\omega t$$

$$+ S_{21}\sin\omega t + C_{2\frac{3}{2}}\cos\frac{3}{2}\omega t + S_{2\frac{3}{2}}\sin\frac{3}{2}\omega t. \qquad (5.12)$$

And the difference of $Det_1=0$ and $Det_{13}=0$ is caused by the influence of higher harmonics of x_2.

The points which satisfy $Det_2=0$ in Fig. 11 have the same frequencies as the points

P_3 and P_4 in Fig. 2. Then, if the proposed approximate method is applied to Eq. (1.1), the branching points for x_2 are determined withoht iteration method.

4) The branching point for subharmonic oscillations

In order to determine the branching points for subharmonics in Eq. (5.1), the following approximate method may be applied.

Let's consider the branching points for 1/2-subharmonics, and the solution x_1 of Eq. (5.1) is assumed in the form

$$x_1 = C_{10}/2 + C_{1\frac{1}{2}} \cos\frac{1}{2}\omega t + S_{1\frac{1}{2}} \sin\frac{1}{2}\omega t + C_{11}\cos\omega t. \tag{5.13}$$

The nonlinear algebraic equations obtained by substituting Eq. (5.13) into Eq. (5.1) are solved on the condition $C_{2\frac{1}{2}} \equiv S_{1\frac{1}{2}} \equiv 0$. The conditions which correspond to the branching points for $C_{1\frac{1}{2}}$ or $S_{1\frac{1}{2}}$ are expressed in the form

$$\omega_1^2 - \frac{\omega^2}{4} + \alpha_3(C_{10} \pm C_{11}) + \frac{3}{4}\alpha_4(C_{10}^2 \pm 2C_{10}C_{11} + 2C_{11}^2) = 0. \tag{5.14}$$

From Eq. (5.14), the regions of instability (branching points) for 1/2-subharmonic oscillations satisfy the following equation:

$$Det_{\frac{1}{2}} = \{\omega_1^2 - \frac{\omega^2}{4} + \alpha_3(C_{10}+C_{11}) + \frac{3}{4}\alpha_4(C_{10}^2 + 2C_{10}C_{11} + 2C_{11}^2\}$$

$$\times\{\omega_1^2 - \frac{\omega^2}{4} + \alpha_3(C_{10}-C_{11}) + \frac{3}{4}\alpha_4(C_{10}^2 - 2C_{10}C_{11} + 2C_{11}^2\} = 0. \tag{5.15}$$

Let $H=3$, $f_1=1.68$ for the shallow arch, $\mathrm{Det}_{\frac{1}{2}}$ is plotted in Fig. 12.

Meanwhile the branching resonance curves which are obtained from Eq. (5.1) by assuming the solution of Eq. (5.13) for the same shallow arch are shown in Fig. 13.[19] From the points P_1 and P_2, the vibration components $C_{1\frac{1}{2}}$ and $S_{1\frac{1}{2}}$ are respectively branching out. The proposed method can be applied to determine the instability region of any other subharmonics of Eq. (5.1).

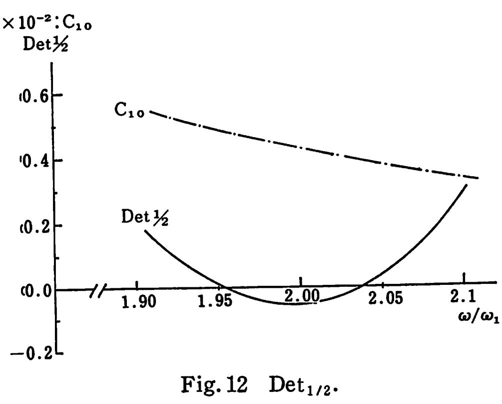

Fig. 12 $\mathrm{Det}_{1/2}$.

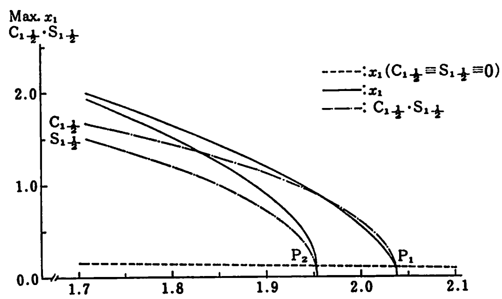

Fig. 13 Resonance Curves of 1/2-subharmonic Oscillation.

References

1) V.V. Bolotin: The Dynamic Stability of Elastic Systems, Holden-Day, 1964.
2) C. Hayashi: Nonlinear Oscillations in Physical Systems, McGraw-Hill, 1964.
3) V.E. Mettler and F. Weidenhammer: Ingenieur-Archiv, p. 421, 1962.
4) C.S. Hsu: Applied Mech., p. 367, 1963.
5) V.E. Mettler: Dynamic Stability of Structures, Ed. G. Herrman Pergamon Press, p. 169, 1965.
6) A. Vijayaraghavan, and R.M. Evan-Iwanowski: Appl. Mech., p. 985, 196.
7) T. Yamamoto and A. Saito: Memoirs of the Faculty of Eng. Nagoya Univ., vol. 22, 1970.
8) J. Tani: Int. J. Solids and Structures p. 169, 1974.
9) L.R. Koval: J. Acoust. Soc. Am., vol. 55, p. 91, 1974.
10) C.S. Hsu: Thin-Shell Structure, Ed. Y.C. Fung and E.E. Sechler, Prentice Hall, 1974.
11) N. Yamaki and K. Nagai: Mem. Inst. High Speed Mech., Tohoku Univ., vol. 36, No. 351, 1975.
12) N. Yamaki and K. Nagai: Mem. Inst. High Speed Mech., Tohoku Univ., vol. 37, No. 355, 1976.
13) D.G. Ashwell and A.P. Chauhan: J. Sound and Vibration, p. 313, 1973.
14) Y. Minakawa and Y. Hangai: Proc. 25th Japan National Congress for Appl. Mech., 1975.
15) Y. Minakawa: Doctoral Dissertation, Univ. of Tokyo, 1976.
16) G.A. Hegemier and F. Tzung: AIAA J., p. 1494, 1969.
17) N.J. Hoff and V.G. Bruce: J. of Mathmatics and Physics, p. 276, 1954.
18) Y. Minakawa: Proc. 27th Japan National Congress for Appl. Mech., 1977.
19) Y. Minakawa: Transactions of The Architectural Inst. of Japan, No. 276, 277, 278, 279 and 280, 1979.