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Fundamental Studies on the Thawing of Frozen Fish-III

Numerical Analysis for Thawing Process

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Abstract

Finite element analysis was used to simulate the temperature history of thawing · a frozen fish. The program used in this study includes two-dimensional simplex elements and equations for the estimation of the physical properties with phase-change. As an example of the application, comparisons of the experimental results with the computed results are made in case of the air thawing of a skipjack. Similar results were obtained, but for more agreement inhomogeneity of foods materials and boundary conditions have to be more carefully treated.

In the design of equipment for thawing on an industrial scale, frozen fish should be thawed without causing thermal damage, discolouration and other chemical or physical changes that will affect quality. Therefore thawing times and temperature distributions in fish being processed must be predicted with fair accuracy. There are many formulas[1)~9)] available for estimating transient state heat conduction in a solid body which undergoes a phase-chage. But those analytical solutions are limited to geometrically regular shapes and to very special boundary conditions. And also finite difference approximations are very difficult to apply when complicated boundaries are considered.

COMINI et al.[10)] suggested the finite element method as the most practical way to simulate the freezing processes. Accordingly, in this paper we describe applications of this method to the computation of the temperature distributions in frozen fish during thawing in a room temperature.

Physical Aspects

When foods are cooled below the freezing point (T_f °C), a ratio (ξ) of ice to total water at a temperature (T °C) is represented approximately by[11)]:

$$\xi=1-\frac{T_f}{T}. \tag{1}$$

The heat capacity (dq) required for 1 kg of water to rise the temperature (dT)

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during thawing is given as[7]:

$$dq = c_1(1-\xi)dT + c_2\,\xi\,dT - \lambda_w\,d\xi, \tag{2}$$

where c_1=specific heat of water in liquid, c_2=specific heat of ice and λ_w=latent heat of ice. From Eq. (1), Eq. (2) becomes:

$$\frac{dq}{dT} = c_2 + (c_1-c_2)\frac{T_f}{T} - \lambda_w\frac{T_f}{T^2}\ . \tag{3}$$

Finally, dq/dT is defined as the specific heat (c_f) of foods during thawing. Namely[7]:

$$c_f = c_2 + (c_1-c_2)\frac{T_f}{T} - \lambda_w\frac{T_f}{T^2} \tag{4}$$

As is seen from Eq. (4), c_f is expressed for a function of temperature (T). When the temperature is closer to the freezing point (T_f), the specific heat (c_f) increases. While, for homogeneous foods materials approximate values of physical properties above and below the freezing point are given as[12]:

$$c = c_w X + c_l Y + c_d(1-X-Y), \tag{5}$$

$$\rho = \rho_w X + \rho_l Y + \rho_d(1-X-Y), \tag{6}$$

$$k = \frac{k_l\{k_w X + k_d(1-X-Y)\}}{\{k_w X + k_d(1-X-Y)\}\ Y + k_l(1-Y)^2}, \tag{7}$$

where c, ρ and k represent specific heat, density and thermal conductivity respectively. Subscribed w, l and d represent water, lipid and dry-solid. And X, Y and (1-X-Y) are mass fractions of water, lipid and dry-solid in foods respectively. Provided $T \leqq T_f$, c_w, ρ_w and k_w in Eqs.(5) to (7) are replaced as follows:

$$c_w = c_f\ , \tag{8}$$

$$\rho_w = (1-\xi)\rho_1 + \xi\,\rho_2\ , \tag{9}$$

$$k_w = (1-\xi)k_1 + \xi\,k_2\ . \tag{10}$$

And provided $T > T_f$, c_w, ρ_w and k_w are equal to the values of each physical property of water in liquid.

Formulations

In this section we present the formulations of the finite element method for the two-dimensional heat conduction of our problem. Assuming the isotropic heat conduction coefficient, our problem is governed by the equation:

$$\rho c\frac{\partial T}{\partial t} - k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = 0\ , \tag{11}$$

with the boundary condition:

$$k\left(\frac{\partial T}{\partial x}n_x + \frac{\partial T}{\partial y}n_y\right) + \alpha(T-T_a) = 0 \tag{12}$$

(on the boundary Γ),

where n_x and n_y denote the direction cosines of the outward normal to the boundary surface, α is the convective heat transfer coefficient and T_a is the temperature of the atmosphere. In Eq.(12) we have neglected the radiative effects.

Following the usual formulations of the method of weighted residuals, we consider the functional:

$$\begin{aligned}\delta\chi &= \int_{\Omega} \delta T^{*}\left\{\rho c\frac{\partial T}{\partial t} - k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\right\} dA \\ &= \int_{\Omega} \left\{\delta T^{*}\rho c\frac{\partial T}{\partial t} + k\left(\frac{\partial(\delta T^{*})}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial(\delta T^{*})}{\partial y}\frac{\partial T}{\partial y}\right)\right\} dA \\ &\quad + \int_{\Gamma} \alpha\delta T^{*}(T - T_a) dL \end{aligned} \tag{13}$$

where dA and dL denote the two and the one dimensional integration and δT^{*} is the weighted function. By deviding our domain Ω into the finite number of the element regions, the functional $\delta\chi$ is written as:

$$\delta\chi = \sum \delta\chi^{e}, \tag{14}$$

where $\sum$ represents the summation over all elements. The element functional $\delta\chi^{e}$ has the same form as Eq.(13) replaced T and δT^{*} by their functions in element T^{e} and δT^{e*}. Adopting the Galerkin's method, T^{e} and δT^{e*} are given in the two-dimensional simplex element by:

$$\begin{aligned} T_e &= L_i T_i, \\ \delta T^{e*} &= L_i \delta T_i, \qquad (i = 1\sim 3), \end{aligned} \tag{15}$$

where L_i are the barycentric coordinates and T_i and δT_i represent the tempratures and their infinitesimal variations at the three vertices of the triangle. The time derivative of T^{e} is approximated by:

$$\frac{\partial T^{e}}{\partial t} = L_i \frac{T_i(t) - T_i(t - \Delta t)}{\Delta t}. \tag{16}$$

Then it follows:

$$\begin{aligned}\delta\chi^{e} = \delta T_i \Big\{ &\left[k(M_x + M_y) + \frac{\rho c}{\Delta t} M + \alpha B\right]_{ij} T_j(t) \\ &- \frac{\rho c}{\Delta t}(M)_{ij} T_j(t - \Delta t) + \alpha(B_a)_i \Big\}, \end{aligned} \tag{17}$$

where,

$$(M_x)_{ij} = \int_{\Omega^e} \frac{\partial L_i}{\partial x}\frac{\partial L_j}{\partial x} dA,$$

$$(M_y)_{ij} = \int_{\Omega^e} \frac{\partial L_i}{\partial y}\frac{\partial L_j}{\partial y} dA,$$

$$(M)_{ij} = \int_{\Omega^e} L_i L_j dA,$$

$$(B)_{ij} = \int_{\Gamma^e} L_i L_j dL,$$

$$(B_a)_i = \int_{\Gamma^e} L_i T_a dL. \tag{18}$$

The integrals of Eq.(18) are easily performed by using the following formulas[13]:

$$\int_{\Omega^e} L_1^a L_2^b L_3^c dA = \frac{a!\, b!\, c!}{(a+b+c+2)!} 2S,$$

$$\int_{\Gamma^e} L_1^a L_2^b dL = \frac{a!\, b!}{(a+b+1)!} l, \tag{19}$$

where S and l denote the area of the triangle element and the length of the boundary. Finally we can obtain the function $\delta\chi$ by adding $\delta\chi^e$ for all element. If $\delta\chi$ is required to be zero for any variations of δT_i ($i=1\sim N$, N is the number of the nodal points of our domain Ω), it follows a matrix equation to determine the temperature at all nodal points $T_i(t)$ if $T_i(t-\Delta t)$ are known.

Experimental and Computations

Thawing test

One sample used for this study was a round skipjack, which was 48.0 cm length and 2.56 kg weight, and another was a cut body with 5 cm length of skipjack, whose both sides were insulated. The samples were at first equilibrated in cold storage at -25°C. Then the sample of a round skipjack placed in a box was thawed in still air at 26～30°C, and another sample was thawed in forced air at about 2.0 m/sec.

Thermocouples (Cu.-Con, 0.3 mmϕ) were inserted in the body of a skipjack

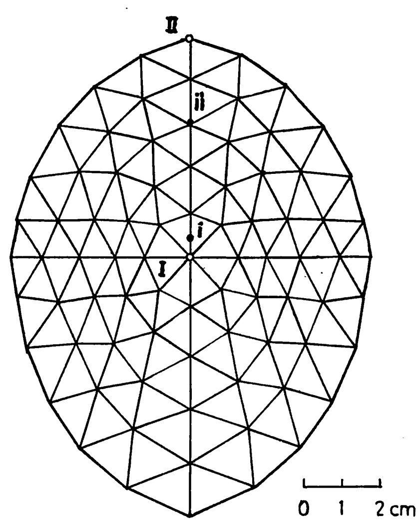

Fig. 1. Finite element mesh used for studying still-air thawing of a skipjack.

before freezing and were connected to ER-4036 automatic temperature recorder (YOKOGAWA ELECTRIC WORKS LTD.).

Computation

In order to satisfy the descrete type maximum value principle in the case of the consistent mass scheme, we have devided our domain into strictly acute type triangles as is shown in Fig. 1 and have carefully chosen the finite difference of time $\Delta t=20$ minutes. The computation was carried out by FACOM M-190 in Kyushu University.

Results and Discussion

It is important to understand the physical factors which govern the thawing processes. Physical properties of foods with phase-change were evaluated from Eqs.(4) to (10) assuming the physical constants of water, ice, lipid and dry-solid in foods as were shown in Table 1. The mass fraction of water and lipid in the dorsal muscle of a skipjack were estimated as $X=0.74$, $Y=0.01$[14]. While the freezing or the melting point (T_f) of a skipjack was assumed as -2.0°C. The sample was considered to be homogeneous materials of the dorsal muscle, and thermal properties of the bone, the ventral muscle, internal organs and the skin were not considered in this paper.

Table 1. Approximate values of physical constants for water & ice, lipid and dry-solid near at 0°C, 760 mmHg[12].

constituent	Content (kg/kg)	Density (kg/m³)	Specific heat (kcal/kg °C)	Latent heat (kcal/kg)	Thermal conductivity (kcal/mh °C)
Water liquid	X	$\rho_1=1000$	$c_1=1.0$	$\lambda_w=80$	$k_1=0.5$
Water ice		$\rho_2=920$	$c_2=0.5$		$k_2=2.0$
Lipid	Y	$\rho_l=920$	$c_l=0.5$		$k_l=$ 0.1 (oil), 0.15(fat*)
Solid	1-X-Y	$\rho_d=$ 130 (plant), 1300(animal*)	$c_d=0.35$		$k_d=2.0$

*: Used in the present paper

A cross section at the middle of a body of a skipjack was used for the finite element analysis, and 136 triangular elements and 81 nodal points were used in the mesh utilized as was shown in Fig. 1. The isotherm fields after 8 hours during thawing are shown for an example of the computation in Fig. 2. Time-temperature curves concerning representative points of the domain, such as I and II in Fig. 1, are shown in Fig. 3. Experimental thawing curves concerning the representative points *i* and *ii* in Fig. 1 are shown in Fig. 4. In the case of the cut body of the skipjack shown in Fig. 5, computed and experimental results are presented in Figs. 6 and 7 respectively.

The computed results are in approximate agreement with the experimental

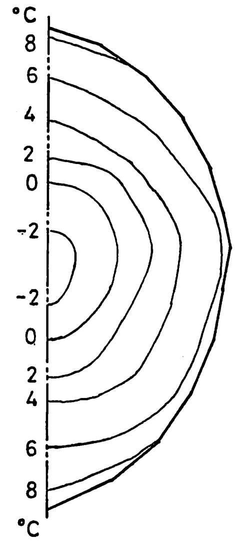

Fig. 2. Isotherm fields during still-air thawing of a skipjack after 8 hours.

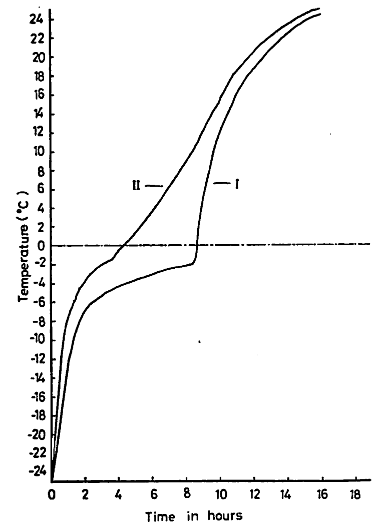

Fig. 3. Computed time-temperature curves during still-air thawing of a skipjack for representative points I and II in Fig. 1. (T_a=27°C, α=8 kcal/m^{2}h °C)

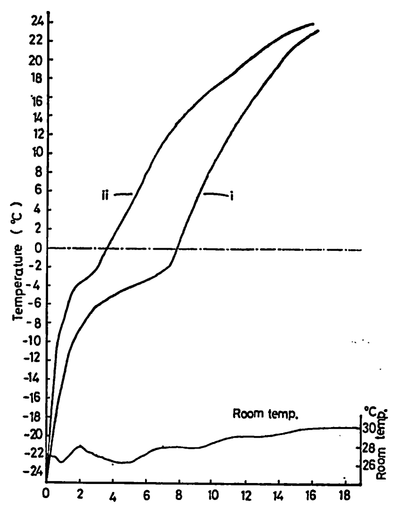

Fig. 4. Experimental time-temperature curves during still-air thawing of a skipjack for representative points i and ii in Fig. 1.

results. We consider the slight discrepancies will be explained by the differences in the representations of the physical properties in each parts of fish. BONACINA et al.[15] proposed a procedure for the estimation of heat capacity and the themal conductivity in freezing problems, and COMINI et al.[10] have given formulae to estimate approximate values of the thermal properties above and below the freezing. REBELLATO et al.[16] applied their estimation of the thermal properties to the finite element analysis with parabolic elements for the computation of the temperature distribution in air-blast freezing of a lam

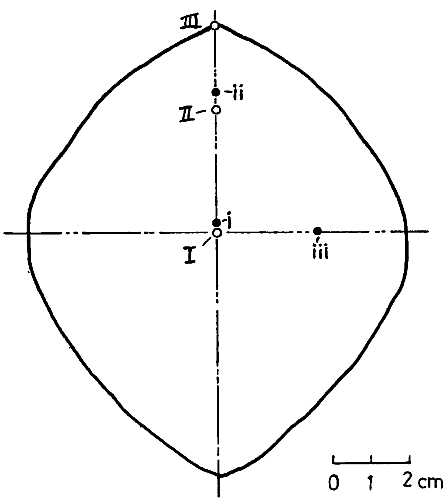

Fig. 5. Cross section of the cut body of a skipjack.

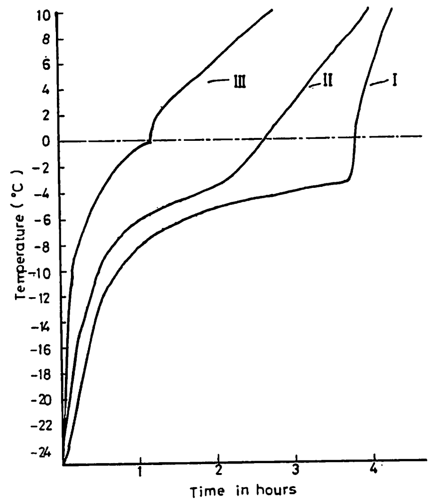

Fig. 6. Computed time-temperature curves during forced-air thawing of a skipjack for representative points I, II and III in Fig. 5. (T_a=23°C, α=25 kcal/m²h °C)

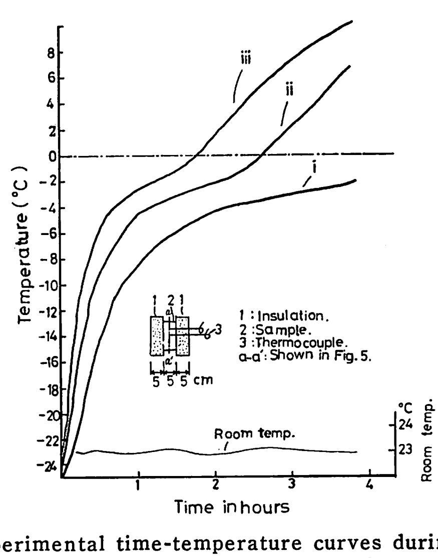

Fig. 7. Experimental time-temperature curves during forced-air thawing of a skipjack for representative points i, ii and iii in Fig. 5.

carcass and a beef side. We did not use their estimation of the thermal properties, because satisfactory results were not obtained in our case.

To obtain more agreement between the computed results and the experimental results in freezing and thawing problems, radiative corrections and the inhomogeneity of the materials have to be taken into account in addition to the more accurate physical properties.

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Notations

c =Specific heat (kcal/kg °C)
n_x, n_y=Direction cosines of the outward normal to the boundary surface
l =length of the boundary of one element
N=Number of the nodal points
q =Heat capacity (kcal/kg)
S =Area of the triangle elements (m^2)
T =Temperature (°C)
t =Time (sec)
X=Mass fraction of water
Y =Mass fraction of lipid
x, y=Cartesian coordinates (m)
α =Convective heat transfer coefficient (kcal/m^{2}h °C)
Γ =Boundary surface (m^2)
λ =Latent heat (kcal/kg)
ξ =Ratio of ice to total water in foods
ρ =Density (kg/m^3)
χ =Functional of the variational principle
Ω =Domain of definition (m^3)

Subscripts and superscripts

a =ambient
d =Dry solid
e =Element
f =Freezing point
l =Lipid
w =Water
x, y=In the x, y direction
1 =Water in liquid
2 =ice

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