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1 Introduction

The purpose of this paper is to analyze the effect of the privatization of the public sector on market equilibrium, environmental damage, and social welfare in a differentiated mixed oligopoly. De Fraja and Delbono (1989) developed a mixed oligopoly model to show that the privatization of a welfare-maximizing public firm may improve social welfare. That work was pioneering in that it considered a public firm that pursues social welfare rather than profit. Many researchers have since extended this mixed oligopoly market in various manners. Matsumura (1998) considered a public firm that maximizes not social welfare but the weighted average of social welfare and profit and showed that partial privatization is the optimal strategy to maximize social welfare.

De Fraja and Delbono (1989) and Matsumura (1998) each considered the mixed oligopoly in which a public firm and private firms produce homogeneous

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goods and exist in a Cournot competition market. Conversely, Fujiwara (2007) applied a quasi-linear utility function, which was also applied in Ottaviano, Tabuchi, and Thisse (2002), to a mixed oligopoly market and determined the number of firms endogenously in the long run.

Some researchers have introduced the environmental factor into the mixed oligopoly model. Beladi and Chao (2006) presented a model to examine whether privatization improves the environment. Ohori (2006) and Barcena-Ruiz and Garzon (2006) compared the optimal pollution tax level between mixed and pure oligopoly markets. Kato (2006) compared the effects of tradable and non-tradable emission permits in a mixed oligopoly. Naito and Ogawa (2009) analyzed the relationship between each environmental regulation and the privatization of the public firm and showed that direct environmental regulation is desirable.

In this work, we construct a model that synthesizes the models of Fujiwara (2007) and Naito and Ogawa (2009) and analyze the relationship between the optimal privatization and environmental regulation in a differentiated mixed oligopoly market. This paper is organized into the following sections. In the next section, we describe the basic model and equilibrium without any environmental regulation. In section 3, we analyze the situation under direct regulation in which the government requires all firms to make the optimal investment in terms of environmental improvement. In section 4, we consider the case in which the government implements an economic method such as emission taxation. Finally, we conclude by summarizing the results of our analysis and describing our future directions.

2 The model

2.1 Consumer

In our economy, we consider households with homogeneous preferences. Following Ottaviano, Tabuchi and Thisse (2002) (hereafter, the OTT model), we specify the quasi-linear utility function as follows.

$$u = \alpha \sum x_j - \frac{\beta - \gamma}{2} \sum x_j^2 - \frac{\gamma}{2} \left(\sum x_j \right)^2 + z, \quad \alpha, \beta, \gamma > 0, \; \beta > \gamma, \tag{1}$$

where $x_j (j = 0, \ldots, n)$ and z are the consumption of the ith differentiated good and the numeraire, respectively. Let $x_i (i = 1, \ldots, n)$ represent the supplied by the private firms. The parameter γ denotes the degree of product differentiation.[1] Maximizing the utility function under the budget constraint, we derive the inverse demand functions of the public firm and the private firms as follows. Here, let x_0 represent the demand of goods produced by the public firm.

$$p_0 = \alpha - (\beta - \gamma) x_0 - \gamma \sum x_j, \tag{2}$$

$$p_i = \alpha - (\beta - \gamma) x_i - \gamma \sum x_j, \tag{3}$$

Substituting (2) and (3) into (1), the consumer surplus is obtained as a function of outputs. Let CS represent the consumer surplus.

$$CS = x_0 \left(\frac{\beta}{2} x_0 + \gamma \sum x_i \right) + \frac{\beta - \gamma}{2} \sum x_i^2 + \frac{\gamma}{2} \left(\sum x_i \right)^2, \tag{4}$$

2.2 Production

Though one public firm and $i (i = 1, \ldots, n)$ private firms produce the differentiated goods, all firms share common production and pollution abatement

[1] See Ottaviano et al. (2002) for the definitions of each parameter.

technologies. Next, we define the profit functions of the public firm and the private firms as follows.

$$\pi_0 = \left(\alpha - \beta x_0 - \gamma \sum x_i\right) x_0 - \frac{x_0^2}{2} - \frac{k_0^2}{2}, \tag{5}$$

$$\pi_i = \left[\alpha - (\beta - \gamma)x_i - \gamma \left(x_0 + \sum x_i\right)\right] x_i - \frac{x_i^2}{2} - \frac{k_i^2}{2}, \tag{6}$$

where the second terms in (5) and (6) denote the firms production costs. The third term in each expression denotes the cost of the firm痴 pollution abatement investment because we describe $k_j (j = 0, \ldots, n)$ as the pollution abatement investment of firm j. We assume that each firm emits the pollution in its own production process. Next, we define $x_j - k_j (j = 0, \ldots, n)$ as the firm's emission function, which depends on the firm痴 production and abatement investment. Thus, the environmental damage caused by firms is as follows.

$$ED = \frac{1}{2}\left[\sum_{j=0}^{n}(x_j - k_j)\right]^2 \tag{7}$$

Thus, we define the social welfare as follows.

$$W \equiv CS + \sum_{j=0}^{n} \pi_j - ED \tag{8}$$

Following Naito and Ogawa (2009), we assume that the public firm maximizes the weighted average of social welfare and profit. We follow this analytical framework and assume that the private sector owns share $\theta \in [0, 1]$ of public firm 0, and the government owns the remaining share $(1-\theta)$. The parameter θ expresses the extent of privatization. The public firm maximizes the following weighted average of social welfare and profit, V.

$$\begin{aligned} V &\equiv \pi_0 + (1-\theta)\left(CS + \sum \pi_i - \frac{(x_0 + \sum_{i=1}^{n} x_i - k_0 - \sum k_i)^2}{2}\right) \\ \theta &\in [0,1] \end{aligned} \tag{9}$$

Assuming the symmetry of private firms, all firms have a common production level, $x_i = x$, and pollution abatement investment level, $k_i = k$. Therefore, in the non-regulated case, the first order condition to maximize the profit of the public firm with respect to production is as follows.

$$\begin{aligned} \frac{\partial V}{\partial x_0} &= \frac{\partial \pi_0}{\partial x_0} + (1-\theta)\left(\frac{\partial CS}{\partial x_0} + \sum \frac{\partial \pi_i}{\partial x_0} - \left(x_0 + \sum x_i - k_0 - \sum k_i\right)\right) \\ &= \alpha + (-\beta - 2 - \beta\theta + \theta)x_0 + n(-\gamma - 1 + \theta)x + (1-\theta)(k_0 + nk) \\ &= 0, \end{aligned} \tag{10}$$

The first order condition to maximize the profit of the public firm with abatement investment under non-regulation is given by

$$\begin{aligned} \frac{\partial V}{\partial k_0} &= \frac{\partial \pi_0}{\partial k_0} + (1-\theta)\left(x_0 + \sum x_i - k_0 - \sum k_i\right) \\ &= (2-\theta)k_0 - (1-\theta)x_0 - n(1-\theta)x \\ &= 0. \end{aligned} \tag{11}$$

The first order conditions to maximize the profit of a private firm with respect to production and abatement investment under non-regulation are given by

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + (-2\beta + \gamma - 1 - \gamma n)x - \gamma x_0 = 0. \tag{12}$$

and

$$\frac{\partial \pi_i}{\partial k_i} = -k_i. \tag{13}$$

The pollution abatement investments of private firms are equal to zero, that is, $k_i = 0$. Solving (10), (11), (12), and (13), we derive the Cournot-Nash equilibrium without the environmental regulation of government as follows.

$$x_0^* = \frac{\alpha\left((2-\theta)(n-n\theta-2\beta+\gamma-1)-(1-\theta)^2 n\right)}{\Omega} \tag{14}$$

$$k_0^* = \frac{\alpha(1-\theta)(-2\beta+\gamma-1-n(\beta+\theta\beta+2-\gamma-1))}{\Omega} \tag{15}$$

$$x^* = \frac{\alpha\left(\gamma(2-\theta)+(1-\theta)^2-(2-\theta)(\beta-\theta+\theta\beta+2)\right)}{\Omega} \tag{16}$$

$$k^* = 0, \tag{17}$$

where Ω is defined as follows.

$$\begin{aligned} \Omega &\equiv (1-\theta)^2(2\beta-\gamma+1)-(2-\theta)\left((\beta-\theta+\theta\beta+2)(2\beta-\gamma+n\gamma+1)\right. \\ &- \left.\gamma n(\gamma-\theta+1)\right) \end{aligned} \tag{18}$$

Substituting (14), (15), (16), and (17) into (7), we derive the equilibrium environmental damage under non-regulation. Moreover, we describe the relationship between the equilibrium environmental damage and the privatization of the public firm in Figure 1. [2] ¿From Figure 1, we can observe that the environmental damage in equilibrium is an increasing function of privatization of the public firm. Assuming that the government never instituted any environmental regulation, the privatization of the public firm makes worsens the environmental damage. Thus, we derive the following lemma.

Lemma 1 ***Assuming that the government never instituted any environmental regulation, the privatization of the public firm worsens the environmental damage.***

[2] We adopt the following simulation parameters: $\alpha = 1, \beta = 2, \gamma = 0.5$, and $n = 10$

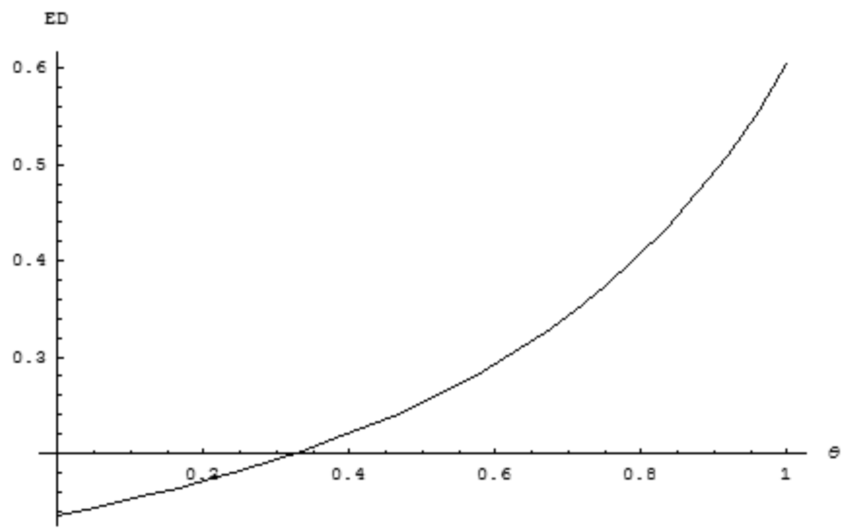

Figure 1: The relationship between the environmental damage and the privatization

Substituting (14), (15), (16), and (17) into (8), the equilibrium social welfare is described as a function of θ.

$$W(\theta) = CS^* + \pi_0^* + \sum \pi_i^* - \frac{(x_0^* + \sum x_i^* - k_0^*)^2}{2}. \tag{19}$$

Next, we describe the relationship between social welfare and privatization in Figure 2. As we can observe in Figure 2, the privatization of the public firm decreases social welfare under non-regulation. Thus, the optimal privatization is $\theta = 0$, implying that the full nationalization of the public firm is optimal. Here, we derive the following proposition.

3 Direct environmental regulation

We analyzed the effect of the privatization of the public firm on environmental damage and social welfare under non-regulation. We now assume that the

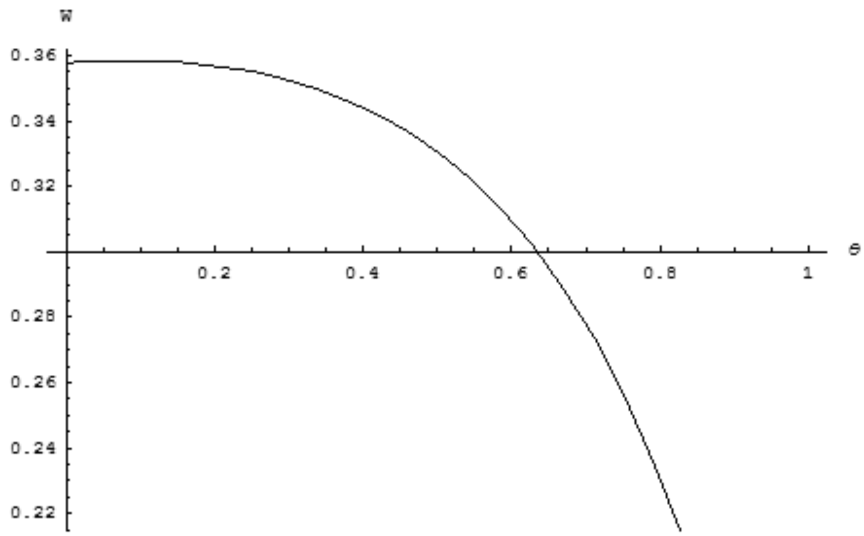

Figure 2: The relationship between social welfare and privatization

government institutes direct environmental regulation of each firm, such that each firm is required to make a pollution abatement investment. Thus, we consider the degree of this investment a policy variable in this section. In the previous section, each firm determines its pollution abatement investment to maximize its own objective function. In this section, the government determines this abatement investment with respect to social welfare. Thus, each firm is unable to determine this investment and selects only its production level. The timing of game in this section is as follows. First, the government determines the privatization level of the public firm. Next, it directly imposes the pollution abatement investment on each firm. Finally, each firm chooses its production level under the privatization level of the public firm and the pollution abatement investment as given. Let k represent the pollution abatement investment determined by the government. Because each firm determines only its own production level, each firm's first order conditions to its objective function are as follows.

$$\frac{\partial V}{\partial x_0} = \alpha + (-\beta - 2 - \beta\theta + \theta)x_0 + n(-\gamma - 1 + \theta)x + (1-\theta)(n+1)k = 0, \quad (20)$$

and

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + (-2\beta + \gamma - 1 - \gamma n)x - \gamma x_0 = 0. \quad (21)$$

Under direct environmental regulation, the following matrix is given by (20) and (21).

$$\begin{pmatrix} (1+\theta)\beta + 2 - \theta & n(1-\theta+\gamma) \\ \gamma & 2\beta + (n-1)\gamma + 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x \end{pmatrix} = \begin{pmatrix} \alpha + (1-\theta)(n+1)k \\ \alpha \end{pmatrix} \quad (22)$$

Here, we define Δ as the determinant of the coefficient matrix in (22).

$$\Delta = 5\beta - \theta - 2\gamma + n\gamma + 2\beta^2 - \theta\beta + \theta\gamma - \beta\gamma - n\gamma^2 + 2\theta\beta^2 + n\beta\gamma - \theta\beta\gamma + n\theta\beta\gamma + 2 \quad (23)$$

Using Cramer's formula, x_0 and x are derived as follows.

$$x_0 = \frac{(2\beta + (n-1)\gamma + 1)(\alpha + (1-\theta)(n+1)k) - \alpha n(1-\theta+\gamma)}{\Delta} \quad (24)$$

and

$$x = \frac{\alpha((1+\theta)\beta + 2 - \theta) - \gamma(\alpha + (1-\theta)(n+1)k)}{\Delta} \quad (25)$$

Because the government solves the pollution abatement investment via backward induction, it determines the direct pollution abatement investment that maximizes social welfare by considering (24) and (25). Thus, the optimal pollution abatement investment is derived as a function of privatization.

$$
\begin{aligned}
k^* &= \frac{1}{G}\Big\{A((\alpha\gamma - D)\,n^2 + (E - D + (\gamma - C + F\gamma)\,\alpha)\,n + (E - C\alpha(1+F))) \\
&\quad + F\alpha\,(2+\beta)\,C^2 + (-E(2+\beta) + (nD - 2n\alpha\gamma)(1+\gamma))\,CF \\
&\quad + (Fn\gamma E(1+\gamma) - 2BF\gamma(D - \alpha\gamma))\Big\}, \qquad (26)
\end{aligned}
$$

where A, B, C, D, E, F, and G are defined as follows, respectively.

$$A \equiv ((1+\theta)\beta + 2 - \theta)\,(2\beta + (n-1)\gamma + 1) - \gamma n(1 - \theta + \gamma),$$

$$B \equiv \frac{1}{2}n + \frac{1}{2}n\beta - \frac{1}{2}n\gamma + \frac{1}{2}n^2\gamma + \frac{1}{2}n^2,$$

$$C \equiv 2\beta + (n-1)\gamma + 1$$

$$D \equiv \alpha\,((1+\theta)\beta + 2 - \theta)$$

$$E \equiv \alpha n(1 - \theta + \gamma)$$

$$F \equiv (1-\theta)(n+1)$$

$$
\begin{aligned}
G &\equiv -\,(n+2)\,(n+1)\,A^2 - 2F\,(n+1)\,(C - n\gamma)\,A \\
&\quad + F^2\left((\beta+2)\,C^2 - 2n\gamma\,(\gamma+1)\,C + 2B\gamma^2\right)
\end{aligned}
$$

Figure 3 describes the effect of privatization on environmental damage under direct environmental regulation. We observe that the privatization of the public firm increases the environmental damage after first decreasing that damage. Thus, we observe that the nationalization of the public firm does not necessarily reduce the environmental damage. When the government does

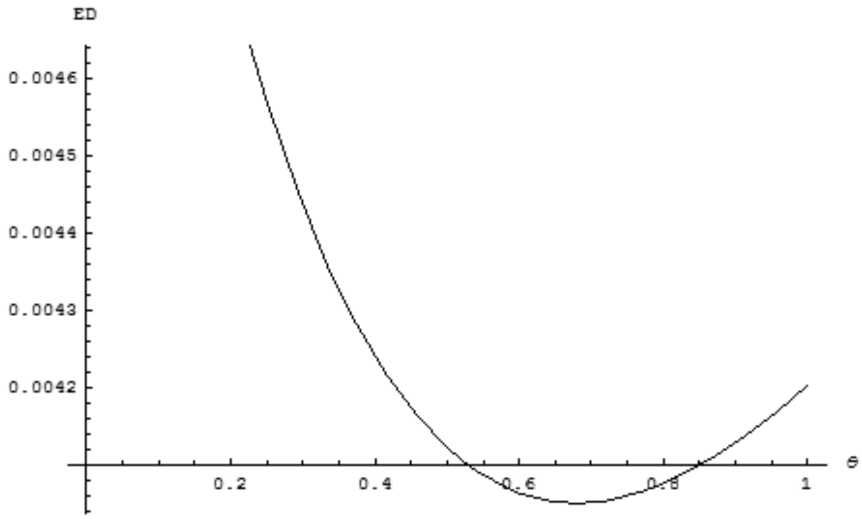

Figure 3: The relationship between environmental damage and privatization under direct environmental regulation

not impose any environmental regulation on each firm (non-regulation), privatization worsens the environmental damage. However, assuming that the government imposes the direct environmental regulation on firms, neither full nationalization nor full privatization minimizes the environmental damage. Moreover, we consider the effect of privatization on social welfare. Figure 4 describes the relationship between the privatization of the public firm and social welfare under direct environmental regulation. As we observe in Figure 4, the optimal privatization level is partial privatization, that is, when $theta$ is approximately 0.4. This optimal privatization level under direct environmental regulation is not identical to the privatization level under environmental non-regulation. [3] We know that it is necessary to consider environmental

[3]We adopt the following parameters in Figure 3 and Figure 4: $\alpha = 1, \beta = 2, \gamma = 0.5$, and $n = 10$

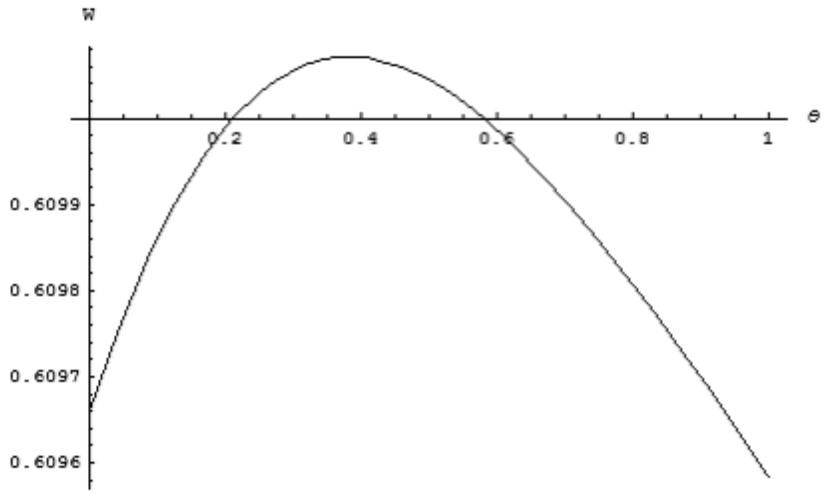

Figure 4: The relationship between the privatization of the public firm and social welfare under direct regulation

regulation when the government increases the privatization of a public firm that emits any environmental pollution. Therefore, we derive the following proposition.

Proposition 1 ***Assuming that the government imposes direct environmental regulation on firms, neither full nationalization nor full privatization minimizes the environmental damage. Moreover, the privatization level to maximize social welfare is partial privatization.***

4 Indirect regulation (Emission tax)

We analyzed the effect of direct environmental regulation on equilibrium and social welfare in the previous section. In the previous section, the government requires each firm to invest in pollution reduction directly; we now consider the case in which the government imposes an emission tax on each firm and controls the firms economically. Let τ represent the emission tax rate per unit of pollution on a firm. Imposing the emission tax on each firm, the government can receive the tax revenue. Thus, we must define the social welfare function as follows.

$$W_i \equiv CS + \pi_0 + \sum_{i=1}^{n} \pi_i + T - ED, \tag{27}$$

where T is defined as follows.

$$T \equiv \sum_{j=0}^{n} \tau(x_j - k_j) \tag{28}$$

Because the government imposes the emission tax on firms, the profit function of each firm is revised as follows.

$$\pi_0 = \left(\alpha - \beta x_0 - \gamma \sum x_i\right) x_0 - \tau(x_0 - k_0) - \frac{x_0^2}{2} - \frac{k_0^2}{2}, \tag{29}$$

$$\pi_i = \left[\alpha - (\beta - \gamma)x_i - \gamma\left(x_0 + \sum x_i\right)\right] x_i - \tau(x_i - k_i) - \frac{x_i^2}{2} - \frac{k_i^2}{2}, \tag{30}$$

Because the public firm maximizes the following weighted average of social welfare and profit as in the previous setting, the objective function of the public firm under the indirect environmental regulation is given by

$$V \equiv \pi_0 + (1 - \theta)\left[CS + \sum_{i=1}^{n} \pi_i + T - ED\right] \tag{31}$$

Here, τ denotes the emission tax per unit of pollution. First, the government determines the privatization level θ. Next, it determines the emission tax rate τ. The public firm and private firms choose their production levels and abatement investments. Because the purpose of the public firm is to maximize

(31), the first order conditions of the public firm with respect to x_0 and k_0 are as follows.

$$\begin{aligned}\frac{\partial V}{\partial x_0} &= (\theta - \beta - \theta\beta - 2)\,x_0 + (n\theta - n - n\gamma)\,x + (1-\theta)\,k_0 + (n - n\theta)\,k \\ &+ (\alpha - \theta\tau) = 0 \qquad (32)\end{aligned}$$

$$\frac{\partial V}{\partial k_0} = (1-\theta)\,x_0 + (n - n\theta)\,x + (\theta - 2)\,k_0 + (n\theta - n)\,k + \theta\tau = 0 \qquad (33)$$

Because the private firms maximize their profit functions (30) with respect to production and abatement investment, the first order conditions for profit maximization of private firms are as follows.

$$\frac{\partial \pi_i}{\partial x_i} = -\gamma x_0 + (\gamma - 2\beta - n\gamma - 1)\,x + \alpha - \tau = 0, \quad (i = 1, \cdots, n) \qquad (34)$$

$$\frac{\partial \pi_i}{\partial k_i} = -k_i + \tau = 0 \qquad (35)$$

The equilibrium under the indirect regulation is determined by four systems: (32), (33), (34), and (35). Thus, we describe this system with the following matrix.

$$\begin{pmatrix} \theta - \beta - \theta\beta - 2 & n\theta - n - n\gamma & 1-\theta & n - n\theta \\ -\gamma & \gamma - 2\beta - n\gamma - 1 & 0 & 0 \\ 1-\theta & n - n\theta & \theta - 2 & n\theta - n \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_0 \\ x \\ k_0 \\ k \end{pmatrix} = \begin{pmatrix} -\alpha + \theta\tau \\ -\alpha + \tau \\ -\theta\tau \\ -\tau \end{pmatrix} \qquad (36)$$

Here, we define Γ as the determinant of coefficient matrix of (36). The parameter Γ is derived as follows.

$$\begin{aligned}\Gamma &= 2(\theta+1)(2-\theta)\,\beta^2 + ((\theta+1)(2-\theta)(n-1)\,\gamma + \left(8 - 3\theta - \theta^2\right))\beta \\ &+ n(\theta-2)\,\gamma^2 + ((2-\theta)\,n + (2\theta - 3))\,\gamma + (3 - 2\theta) \qquad (37)\end{aligned}$$

Let $x_0^{**}, k_0^{**}, x^{**}$, and k^{**} represent the equilibrium values under the indirect environmental regulation. Using Cramer's formula, $x_0^{**}, k_0^{**}, x^{**}$, and $k^{*}*$ are derived as follows.

$$x_0^{**} = \frac{1}{\Gamma}\Big(\gamma(1-\theta)n^2 + (2\beta+\gamma+2)(1-\theta)n - \theta(2\beta-\gamma+1))\tau + \alpha\{(4-2\theta)\beta + (\theta-2)\gamma + (\theta-1)n + (2-\theta)\}\Big), \tag{38}$$

$$x^{**} = \frac{1}{\Gamma}\Big([(\theta+1)(\theta-2)\beta + ((\theta-1)n+\theta)\gamma + (2\theta-3)]\tau + \alpha[(2-\theta)((\theta+1)\beta-\gamma) + (3-2\theta)]\Big), \tag{39}$$

$$k_0^{**} = \frac{1}{\Gamma}\Big(2(\theta+1)(-n+\theta+n\theta)\beta^2 + (\big(\theta^2-1\big)n^2 + (\theta+1)n - \big(\theta^2+\theta\big))\beta\gamma + (2(\theta+2)(\theta-1)n + \theta(\theta+3))\beta + n((1-\theta)n-\theta)\gamma^2 + ((\theta-1)n^2 + (2-\theta)n - \theta)\gamma + (\theta-2n+2n\theta)\Big), \tag{40}$$

and

$$k^{**} = \tau. \tag{41}$$

Substituting (38), (39), (40), and (41) into (7), we derive the equilibrium environmental damage under the indirect regulation as follows. [4]

$$ED_i = \frac{1}{2}\left(x_0^{**} - k_0^{**} + n(x^{**} - k^{**})\right)^2 \tag{42}$$

Substituting (38), (39), (40), and (41) into (27), we obtain the social welfare $W_i(\tau, \theta)$ as a function of τ and θ. Next, we consider the optimal tax rate

[4] We adopt the following parameters in Figure 5: $\alpha = 1, \beta = 2, \gamma = 0.5$, and $n = 5$

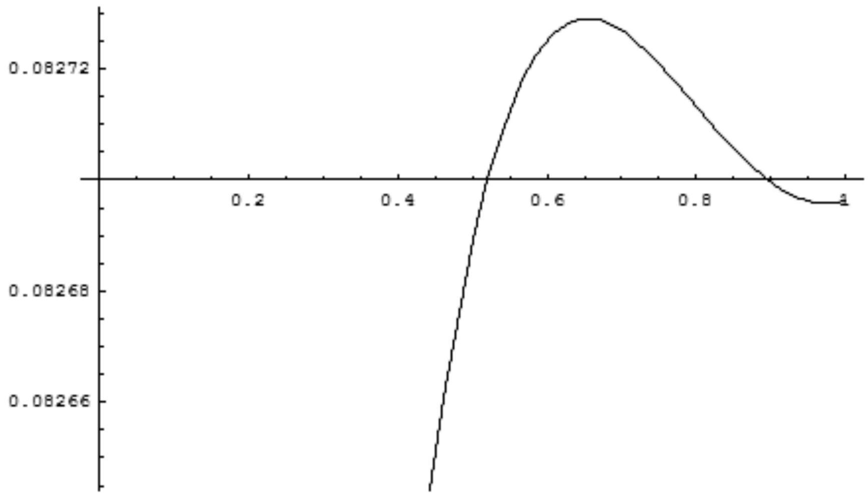

Figure 5: The relationship between optimal tax rate and privatization

to maximize social welfare under direct environmental regulation. Differentiating $W_i(\tau,\theta)$ with respect to τ, we derive the optimal tax rate as a function of θ.[5] We describe the relationship between optimal tax rate and privatization in Figure 5. ¿From Figure 6, we observe that the optimal tax rate depends on θ. We understand that the progress of the privatization at first to shift from nationalization to privatization diminishes the optimal tax rate to maximize the social welfare. However, privatization does not necessarily decrease the optimal tax rate. Thus, the effect of the privatization level on the optimal tax rate is ambiguous. Next, we consider the environmental damage under the indirect environmental regulation.

Because this equilibrium environmental damage is a function of the priva-

[5]By differentiating $W_i(\tau,\theta)$ with respect to τ, it is possible to derive the optimal tax rate explicitly. However, we analyze the property of the optimal tax rate with simulation because of its complication.

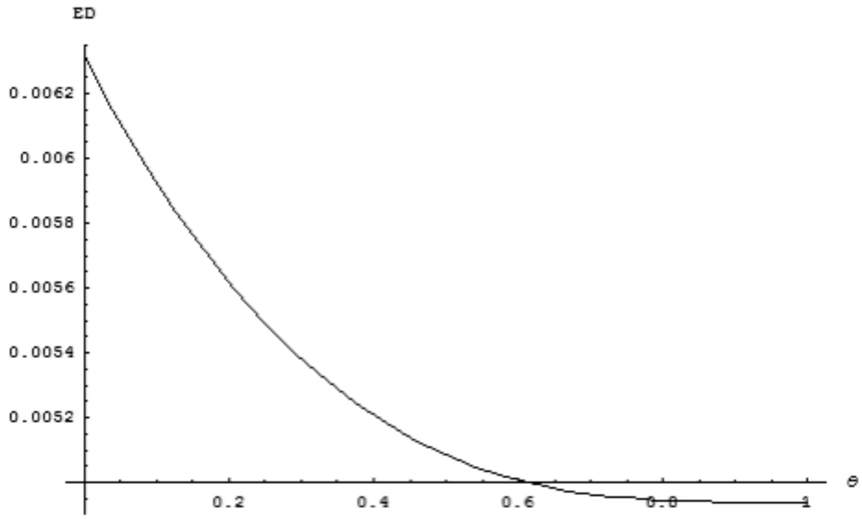

Figure 6: The environmental damage under indirect environmental regulation

tization level θ, the relationship between the environmental damage and the privatization level is described in Figure 6. [6] As in the derivation of the optimal tax rate, we substitute the optimal tax rate, which is derived above, into $W_i(\tau, \theta)$. Moreover, by differentiating the social welfare function $W_i(\tau, \theta)$ with respect to θ, we can observe the relationship between social welfare and the privatization level in Figure 7. In Figure 7, the optimal privatization level to maximize the social welfare is approximately 0.2, that is, $\theta \cong 0.2$. Similar to the direct environmental regulation case, the partial privatization is also optimal under the indirect environmental regulation. Comparing Figure 4 with Figure 7, the optimal privatization level under indirect environmental regulation is smaller than that under direct environmental regulation. Note that nationalization is optimal under no environmental regulation. Thus, these re-

[6] We adopt the following parameters in Figure 6 and Figure 7: $\alpha = 1, \beta = 2, \gamma = 0.5$, and $n = 10$

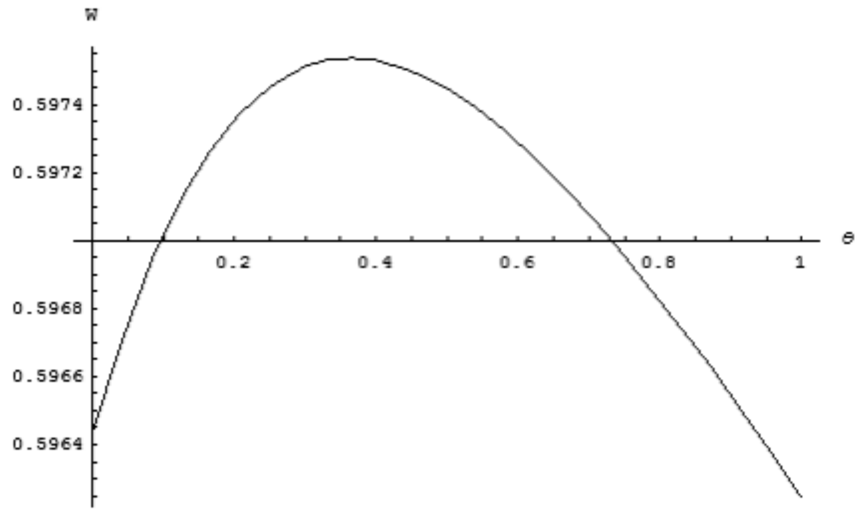

Figure 7: The relationship between social welfare and privatization under indirect environmental regulation

sults imply that the optimal privatization level of the public firm depends on the type of environmental regulation. In summary, we obtain the following proposition.

Proposition 2 ***Partial privatization is optimal under the indirect environmental regulation, although the privatization of the public firm decreases the environmental damage.***

5 Concluding remarks

We analyze a mixed oligopoly that produces differentiated products, in which one public firm and n private firms produce these products and emit pollution during the production processes. Moreover, we consider three types of environmental regulation regimes and firm behavior, environmental damage, and the optimal privatization level of the public firm under each regime. The results of our analysis imply the following economic considerations. First, if the government does not impose any environmental regulations on each firm, the full nationalization of the public firm is optimal to maximize social welfare. However, if the government imposes some environmental regulations on firms, the full nationalization is no longer optimal and partial privatization is optimal. As for environmental damage, the environmental damage under non-regulation is an increasing function of the privatization level θ. However, the environmental damage under indirect environmental regulation is a decreasing function of θ. Moreover, privatization of the public firm under direct regulation does not necessarily decrease the environmental damage. Thus, we must consider environmental regulation when we allow the privatization of a public firm that emits pollution. Finally, we assume some points to simplify our model. One such assumption is that we accept the number of private firms as given. Fujiwara [5] determines this number endogenously via the zero profit condition. Thus, we seek to apply a similar method in future work.

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