# ELASTIC EXPERIMENTAL ANALYSIS OF COLUMNS WITH SPANDREL AND WING WALLS

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( Received May 31, 1983)

The spandrel and wing walls in the reinforced concrete structure are of important structural elements in that they bear the shearing force. Since how to design these walls greatly affect the safety of the structure, it is necessary to estimate correctly the shearing force ratios of the walls before we design the structure. However, we haven't so far had a standardized method of determining the shearing force ratios for the spandrel and wing walls. So in many cases the design of the column with walls depends on the designers' own methods. Therefore the structures are designed at fairly various shearing force ratios. In this report the basic data on the shearing force ratios of the column with walls are obtained by the photo-elastic experiments with the specimens ——— the columns without walls and with walls of various length and various thicknesses.

## §1. Introduction

Although the spandrel and wing walls in the frame are generally neglected in structure analyses, it is often pointed out that these walls affect largely toward the rigidity, strength and fracture characteristics of building structures ( rahmen frame ). Presently a number of data on the column with spandrel walls have been reported[1),2)] and have been used for the practical structural calculation.

But there are few research reports and many obscure points about the column with wing walls. For example, it is not comprehended how shearing force ratios for the column with spandrel and wing walls should be determined.

In this paper we present it how to deal with the elastic stiffness of columns with spandrel and wing walls, and offer the basic data which can be used in practice for the structural calculations. The basic data are

i ) Shearing force ratios of the column with to without walls.

ii ) Shearing force ratios of the wall part to the column part in the column with walls.

The stress analysis in order to obtain the data was made through photo-elastic model, and the photo-elastic experiments gave fairly good agreement with the past reinforced concrete experimental results.

## §2. Specimens and Experiments

The specimen material used in this experiment is the epoxy resin plates and steel plates are fixed to make the upper and lower parts of the specimens stiffen as showen in Fig. 1. The col-

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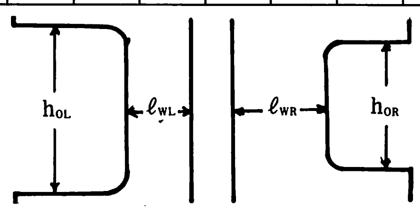
umn and walls are butted and jointed, and the corners of opening in the walls have a certain curvature in order to avoid the fracture due to the effect of stress concentration. The dimensions and shapes of specimens are shown in Fig. 1 and Table−1.

The specimens were set in the apparatus ( Fig. 2 ) and loaded at the half height ( $h/2$ ) of the specimens. Isoclinics in case of two kinds of wall thickness have been shown in Photos. − 2 ~ 3 and isochromatics have been shown in Photos. − 4 ~ 7 ( $b/t=6.0$ ) and in photos. − 8 ~11 ( $b/t=3.0$ ).

Table−1 Size of Specimen

| | Type I ( $h_R=h_L$ ) | | | | | Type II ( $h_R \neq h_L$ ) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $h_R$ | 16 | 32 | 48 | 64 | 80 | 16 | 32 | 48 | 64 | 80 |
| $h_L$ | | | | | | 32 | 48 | 64 | 84 | 100 |



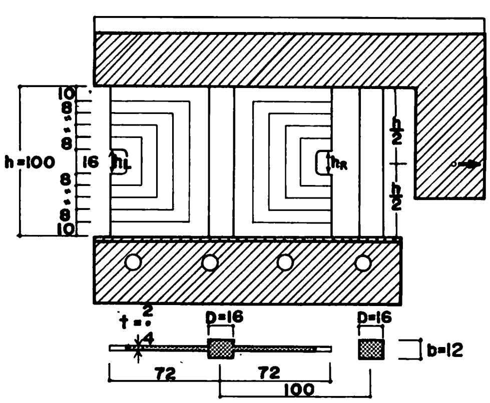




Fig. 1. Dimensinons and Shape of the Specimens
( scale unit : mm )

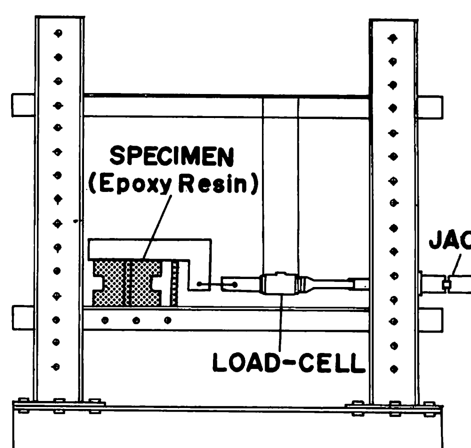



Fig. 2. Test Apparatus

## § 3. Photo-Elastic Analysis

The photo-elastic sensitivity $\alpha$ are obtained through the tension tests.

Columns (thickness =12 mm) ; $\alpha=0.95$ mm/kg

Walls (thickness =2 mm, 4 mm) ; $\alpha=0.93$ mm/kg

There are shown the isochromatics and bending moment diagram of columns without wall in

Photo. −1 and Fig. 3 respectively, in which bending moment values are obtained from eq. (1).

$$M=\frac{n}{\alpha b} \quad Z=\frac{nD^2}{6\alpha} \tag{1}$$

$$Q=\frac{M_{(top)}+M_{(bottom)}}{h} \tag{1'}$$

where M : bending moment

n : fringe order

Q : shearing force

$\alpha$ : photo-elastic sensitivity of the column

D : column depth

b : column thickness

$Z=\frac{bD^2}{6}$ : section modulus of the columns

h : height of column

The difference between the stress of the left and right columns is so little as to be neglected.

The shearing unit stresses $\tau_{xy}$ in the wall are obtained as follows :

$$\tau_{xy}=\frac{(\sigma_1-\sigma_2)}{2}\sin 2\theta \tag{2}$$

where

$$(\sigma_1-\sigma_2)=\frac{n}{\alpha t}$$

$\sigma_1$ , $\sigma_2$ : principal stresses

$\alpha$ : photo-elastic sensitivity of the wall

t : wall thickness

$\theta$ : angle of principal stress direction to datum line

The principal stress difference ( $\sigma_1-\sigma_2$ ) is obtained from the isochromatics, and the angle $\theta$ is 45 digrees on the middle horizontal section as shown in Photos. −2 and 3 , and therefore sin $2\theta$ in eq. (2) is equal to 1.0 .

The shearing forces on the middle height section of the wall are obtained by integrating the shearing unit stress $\tau_{xy}$ as follows.

$$\left.\begin{aligned} Q_{mL}&=t\int_0^{\ell_{WL}}\tau_{xy}dx=\int_0^{\ell_{WL}}\frac{n}{2\alpha}dx \\ Q_{WR}&=t\int_0^{\ell_{WR}}\tau_{xy}dx=\int_0^{\ell_{WR}}\frac{n}{2\alpha}dx \end{aligned}\right\} \tag{3}$$

where $Q_{WL}$ , $Q_{WR}$ : shearing force on the lefthand-side wall section and that on the righthand-side wall section

where $\ell_{WL}$ , $\ell_{WR}$ : length of the lefthand-side wall and that of the righthand-side wall

The stresses ( fringe orders ) on the column with the wall are very complicated, accordingly the shearing forcese ( $Q_c$ ) of the column with the wall were determined following equation (4).

$$Q_c=Q-(Q_0+Q_W) \tag{4}$$

where Q : total load subjected to specimen

$Q_0$ : shearing force of the column without wall

$Q_W = Q_{WL} + Q_{WR}$ : shearing force on the middle height section of the wall

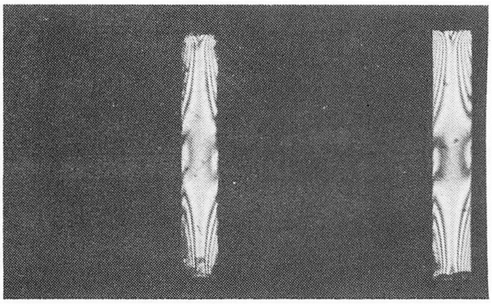


Photo.-1 Isochromatics of Columns without Wall

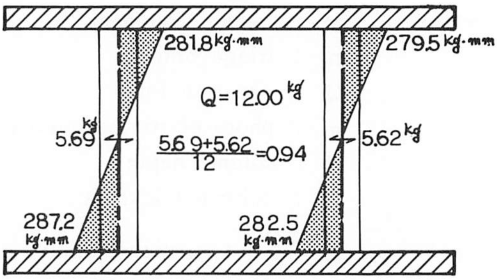



Fig. 3. Bending Moment Diagram in the Columns without Wall of Photo 1.

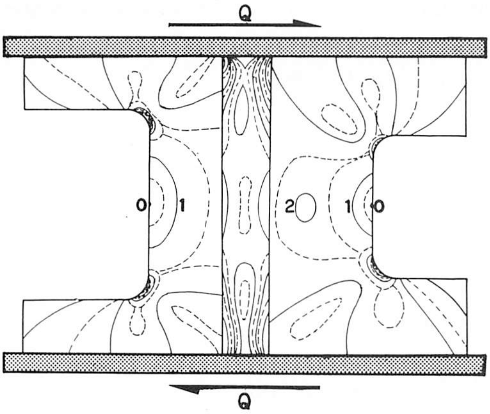



Fig. 4. Fringe Orders

Specimen $\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=0.64$ $\frac{h_{OR}}{h}=0.48$

$\frac{\ell_{WL}}{D}=1.53$ $\frac{\ell_{WR}}{D}=2.185$

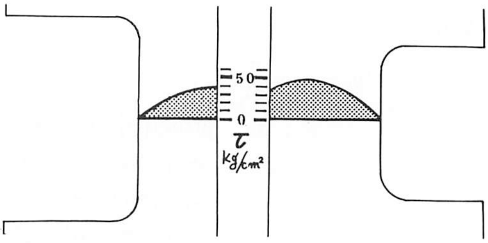



Fig. 5. Typical Shearing unit Stresses Distribution on the middle Height Section of the Wing Wall

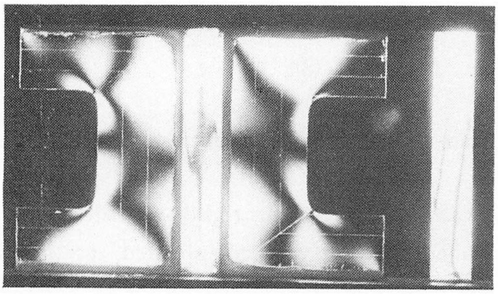


Photo.-2 Isoclinics

$\theta=45°$ $\frac{b}{t}=6.0$, $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.48$, $\frac{\ell_{WL}}{D}=\frac{\ell_{WL}}{D}=2.185$

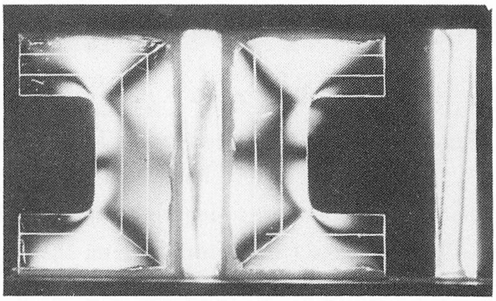


Photo.-3 Isoclinics

$\theta=45°$ $\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.48$ $\frac{\ell_{WL}}{D}=\frac{\ell_{WR}}{D}=2.185$

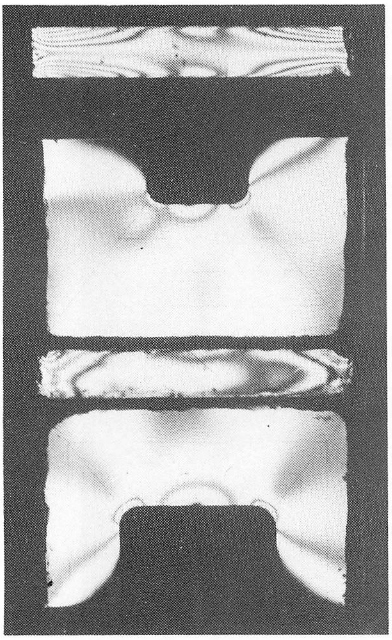


Photo.-4 Isochromatics ( dark field )

$\frac{b}{t}=6.0$ $\frac{h_{OL}}{h}=0.48$ $\frac{h_{OR}}{h}=0.32$ $\frac{\ell_{WL}}{D}=2.185$ $\frac{\ell_{WR}}{D}=2.850$

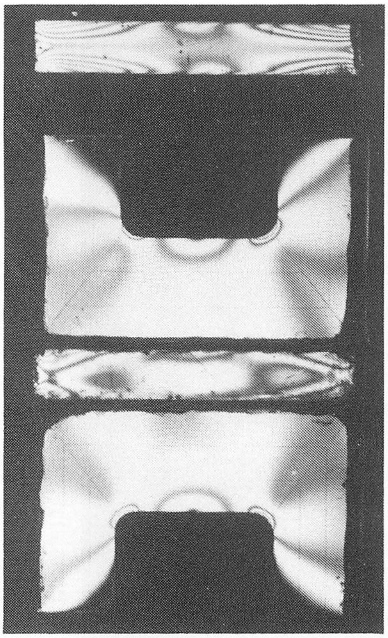


Photo.-5 Isochromatics ( dark field )

$\frac{b}{t}=6.0$ $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.48$ $\frac{\ell_{WL}}{D}=\frac{\ell_{WR}}{D}=2.185$

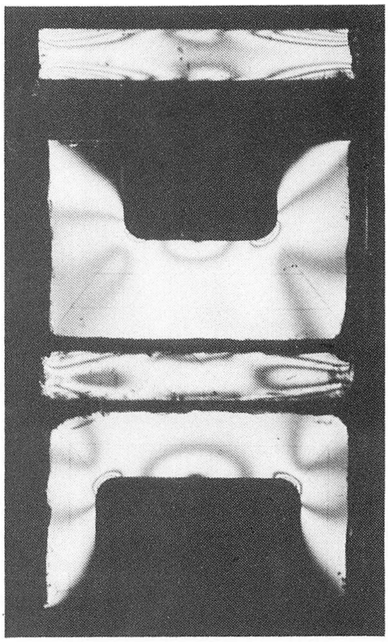


Photo.-6 Isochromatics ( dark field )

$\frac{b}{t}=6.0$ $\frac{h_{OL}}{h}=0.64$ $\frac{h_{OR}}{h}=0.48$ $\frac{\ell_{WL}}{D}=1.53$ $\frac{\ell_{WR}}{D}=2.185$

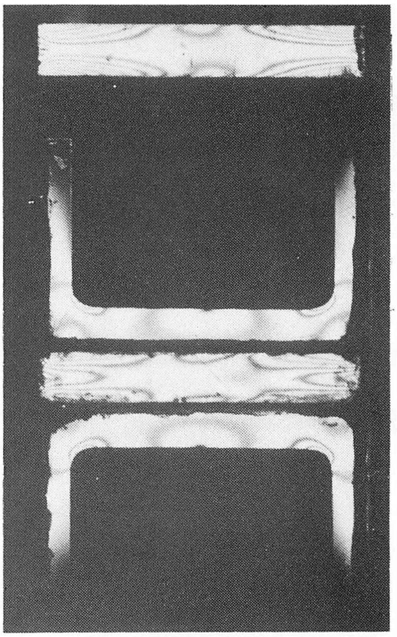


Photo.-7 Isochromatics ( dark field )

$\frac{b}{t}=6.0$ $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.80$ $\frac{\ell_{WL}}{D}=\frac{\ell_{WR}}{D}=0.875$

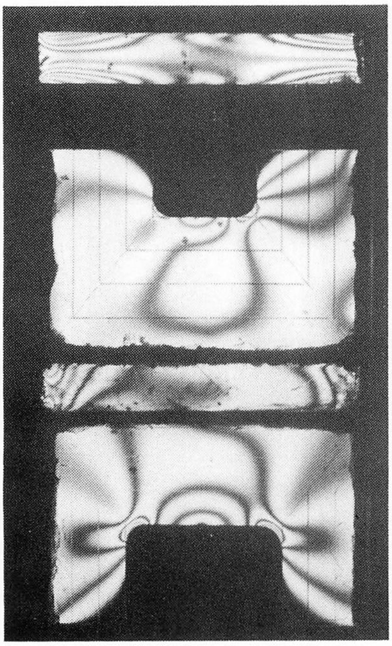


Photo.-8 Isochromatics ( dark field )

$\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=0.48$ $\frac{h_{OR}}{h}=0.32$ $\frac{\ell_{WL}}{D}=2.185$ $\frac{\ell_{WR}}{D}=2.85$

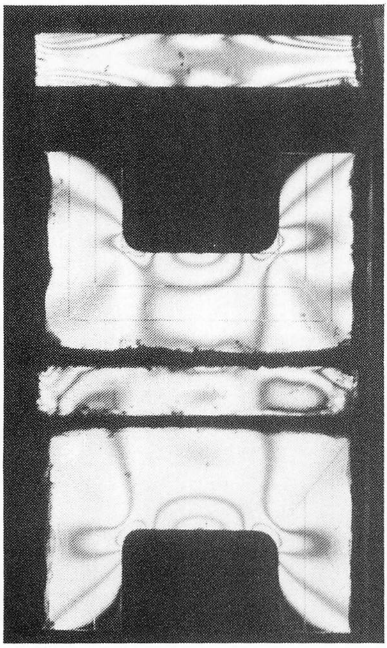


Photo.-9 Isochromatics ( dark field )

$\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.48$ $\frac{\ell_{WL}}{D}=\frac{\ell_{WR}}{D}=2.185$

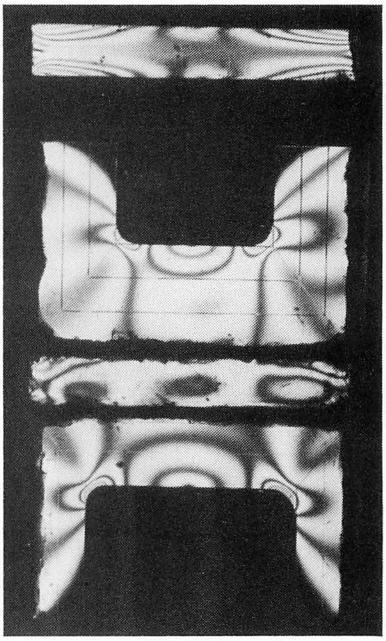


Photo.-10 Isochromatics ( dark field )

$\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=0.64$ $\frac{h_{OR}}{h}=0.48$ $\frac{\ell_{WL}}{D}=1.53$ $\frac{\ell_{WR}}{D}=2.185$

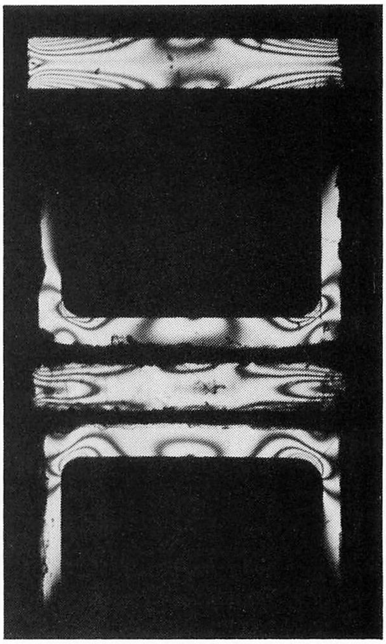


Photo.-11 Isochromatics ( dark field )

$\frac{b}{t}=3.0$ $\frac{h_{OL}}{h}=\frac{h_{OR}}{h}=0.80$ $\frac{\ell_{WL}}{D}=\frac{\ell_{WR}}{D}=0.875$

## § 4. Experimental Results

The shearing force ratio $Q_W/(Q_C+Q_W)$ of the wing walls to column with wall plus wing walls is presented in Fig. 7 and increases almost linearly to $\ell_W/D=4.0$

The empirical equations are

for $b/t=3.0$ ,

$$\frac{Q_W}{Q_W+Q_C}=0.13\left(\frac{\ell_W}{D}\right)+0.13 \qquad (5a)$$

for $b/t=6.0$

$$\frac{Q_W}{Q_W+Q_C}=0.13\left(\frac{\ell_W}{D}\right)+0.06 \qquad (5b)$$

where, $0.5\leqq\frac{\ell_W}{D}<4.5$

The shearing force ratio $Q_C/Q_0$ of the column with to without walls is presented in Fig. 8 and increases almost linearly to $\ell_W/D=2.5$

The emprical equations are

for $b/t=3.0$

$$\frac{Q_C}{Q_0}=1.90\left(\frac{\ell_W}{D}\right)+0.70 \qquad (6a)$$

for $b/t=6.0$

$$\frac{Q_C}{Q_0}=1.65\left(\frac{\ell_W}{D}\right)+0.78 \qquad (6b)$$

where, $0.5\leqq\frac{\ell_W}{D}\leqq2.5$

Although the plates over $\ell_W/D>2.5$ are somewhat scattered, it is assumed that $Q_C/Q_0$ is constant.

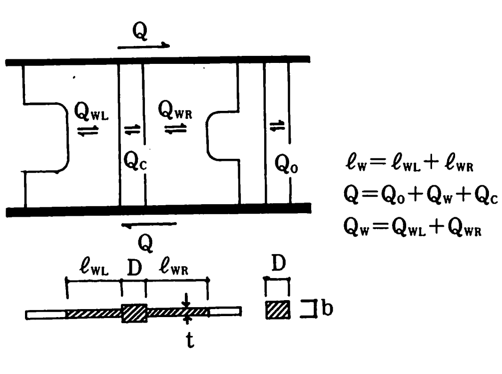



Fig. 6. Shearing Forces of Column without Wall, with Wall, and Wall Sign of Shape

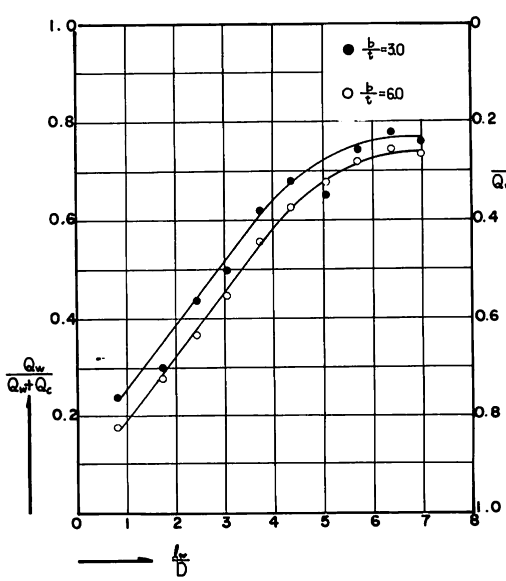



Fig. 7. Shearing Force Ratios of Wing Walls to Column with Walls plus Wing Walls along both Sides.

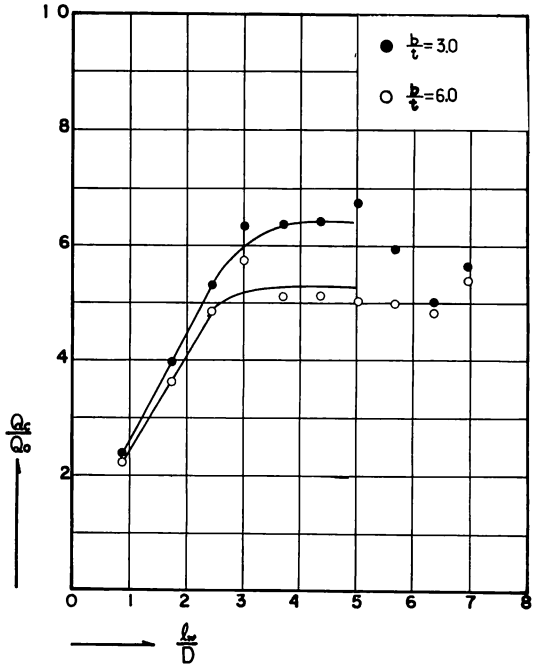



Fig. 8. Shearing Force Ratios of Column with Walls to without Wall.

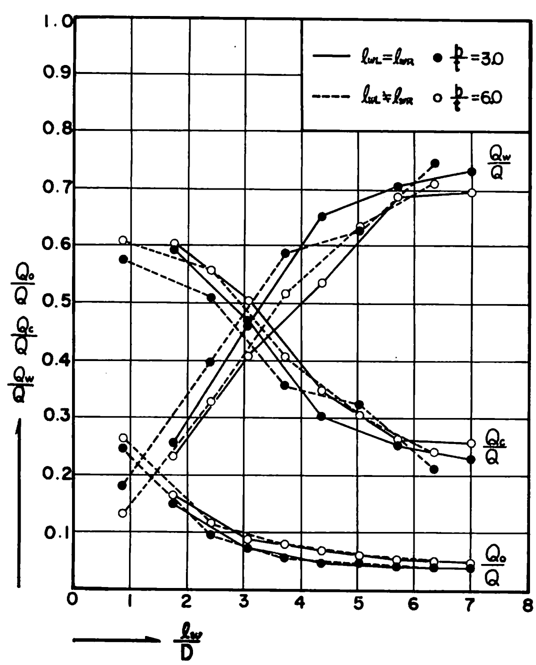



Fig. 9. Each Shearing Force Ratio, a Column with or without Walls to Total Shearing Force.

## §5. Comparison of the Experimental Results through the Epoxy Resin Model with those through the Reinforced Concrete Model

According to the data presented in §4, the experimental results through the epoxy resin model were compared with those through the reinforced concrete model.

The dimensions of the specimen compared are shown in Fig. 10

The two reinforced concrete specimens of the same dimensions were tested by Drs. Y. Higashi and M. Ohkubo[3]. One specimen was monotonically loaded, and the other was cyclically loaded. The shearing forces were obtained through the measure by strain gauges.

The shearing force ratio of the column with to without wing walls was obtained from Fig. 8 and that of the column with only spandrel wall was obtained by using next equation in refference (2).

$$\frac{Q}{Q_0}=e^{-1.67\sqrt{b/t}}\left(\frac{h-h_0}{D}\right)^{\frac{5}{2}}+1.0 \qquad (7)$$

The comparisons of the reinforced concrete experiments with the epoxy model experiments are shown in Table. − 2.

## §6. Considerations and Conclusions

In comparisons in §5, these degree of errors are practically permissible considering followings,

i ) As both specimens are quite small, the measured values are not always accurate extremely

ii ) The walls of the compared reinforced concrete specimen is eccentric.

The data, which are obtained from this experiment, may be applied to the actual structure calcurations.

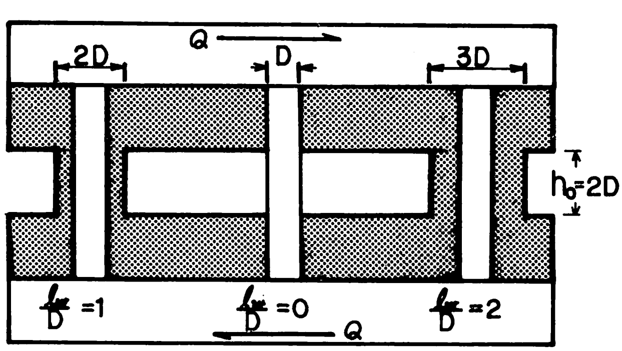



Fig.10. the Specimen (b / t =4.0) of Experiment by Higashi and Ohkubo[3]

Table-2 Comparison of Experimental Values by Authors with those by Higashi and Ohkubo[3)]

| Opening Ratio $h_0/(D+\ell_w)$ | 2 | 1 | 2/3 |
|---|---|---|---|
| Wall Length Ratio $\ell_w/D$ | 1 | 0 | 2 |
| | Shearing Force Ratio (Q) | | |
| Reinforced Concrete (1) | 0.28 | 0.17 | 0.55 |
| Experiment (2) | 0.31 | 0.14 | 0.55 |
| Epoxy Model Experiment | 0.26 | 0.17 | 0.57 |

(1) monotonic increasing load

(2) cyclic increasing load

## Reference


1) Ikuo Tokuhiro and Akio Sasaki "Studies on Elastic Rigidity of Columns with Spandrel Walls (I)" Transactions of Architectural Institute of Japan, No. 304 Jun. 1981

2) Ikuo Tokuhiro and Takuma Kyutoku " Studies on Elastic Rigidity of Columns with Spandrel Walls (II) " Transactions of the Architectural Institute of Japan, No. 318, Aug. 1982

3) Masamichi Ohkubo, Hiroaki Eto and Youichi Higashi " Cracks and Statical Hysterisis-Loops of Reinforced Concrete Frames with Spandrel Walls Cast Simultaneously " Transactions of the Architectural Institute of Japan No. 169, Feb. 1971

4) Masamichi Ohkubo " studies on Rigidities, Strength and Hysteretic Characteristics of Reinforced Concrete Frames with Spandrel or Wing Walls " Transaction of the Architectural Institute of Japan, No. 186, Aug. 1972