The Velocity Distribution Functions and the Eddy Kinematic Viscosity of a Free Turbulent Jet Flow

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The velocity profile in the streamwise direction proposed by Hatta and Nozaki is compared with the other ones by using the mean width of the jet analogous to the displacement thickness of the boundary layer. In order to refer the experimental results and the different solutions of a free jet to each other, the constant contained in the eddy kinematic viscosity is also compared with the spread parameter contained in the solutions proposed by Görtler.

1. Introdudtion

On the study of the two-dimensional jet flow issuing from a finite width nozzle, the velocity distribution function in the streamwise direction was proposed by Hatta and Nozaki [1] in the diffusion regions both for the zone of flow establishment and for the zone of established flow. The velocity profiles of the jet were proposed by many researchers so far. Comparisons of the profiles with each other have not been done yet systematically. In this paper, by introducing the mean width of the jet, the velocity profiles are compared with each other. By using the mean width of the jet, the constant contained in the eddy kinematic viscosity has a universal value which is independent of the approximate velocity profile assumed as was stated in the previous paper [1]. In order to compare the experimental results and the different solutions of a free jet with each other, the constant contained in the eddy kinematic viscosity is also compared with the spread parameter in the solutions proposed by Görtler [2].

2. Eddy kinematic viscosity

In the diffusion region of the jet, the eddy kinematic viscosity ε has been usually assumed as

$$\varepsilon=\kappa' bU, \tag{1}$$

where κ' denotes the empirical constant, b the half width of the jet and U the center line velocity of the jet. In the approximate calculation of the jet, however, the value of b depends on the expression of the velocity profile assumed, hence the value of κ' also depends on this profile. But the value of ε must be independent of the expression of the velocity profile, so that Eq. (1) may be unsuitable for the expression for ε. Hatta and Nozaki [1] proposed in the previous paper that the eddy kinematic viscosity is assumed to be proportional to both the center line velocity and the half mean width of the jet b_m, namely,

$$\varepsilon=\kappa b_m U \tag{2}$$

where κ denotes the empirical constant. This assumption led to the fact that the constant κ is to have a universal value which is independent of the approximate velocity profile assumed.

3. Comparison of velocity profiles

By using a parameter

$$\eta=y/b,$$

the velocity distribution function in the streamwise direction is proposed by Hatta and Nozaki [1] as a polynomial of η:

$$f(\eta)=1-6\eta^2+8\eta^3-3\eta^4 \tag{3}$$

considering the boundary conditions at $\eta=0$ and $\eta=1$.

The velocity profile proposed by Görtler is given by

$$f(\eta)=1-\tanh^2\eta$$

where

$$\eta=\sigma y/x.$$

The other profiles are also given as follows:

Simson [3] : $f(\eta)=(1-\eta^{7/4})^2$,

Cosine function : $f(\eta)=(1+\cos\pi\eta)/2$

and

Kimura and Mitsuoka [4] : $f(\eta)=1-3\eta^2+2\eta^3$,

where

$$\eta=y/b.$$

In order to compare the velocity profile expressed as Eq. (3) with other ones, it is not appropriate to compare those about the flow issuing from a finite width nozzle, since the effects of the length of the zone of flow establishment is introduced. Then those profiles are compared with each other by applying the method of calculation of the flow issuing from a finite width nozzle to the flow issuing from an infinitesimal width nozzle.

A two-dimensional steady jet is issuing into a semi-finite space from an infinitesimal width nozzle, as shown in Fig. 1. Let the origin O be taken at the center of the nozzle exit and the axes of x and y in the direction along and perpendicular to a jet center line respectively.

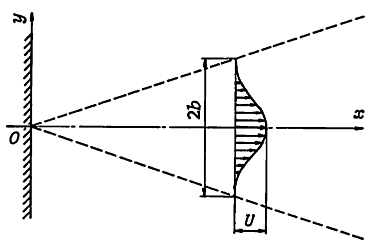

Fig. 1 Schema of a free jet flow issuing from an infinitesimal width nozzle

As far as inside of the jet, the boundary layer approximations may be applied right across the section for the equations of motion. For a turbulent jet, if u and v are the components of mean velocity, the equation of motion in the x-direction is given by

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\varepsilon\frac{\partial^2 u}{\partial y^2}. \tag{4}$$

In Eq. (4), ε is assumed to be constant with respect to y. Integration of Eq. (4) with respect to y gives the momentum integral equation

$$\frac{d}{dx}\int_0^\infty u^2 dy=0. \tag{5}$$

By integrating Eq. (5) with respect to x, we obtain

$$\int_0^\infty u^2 dy=\text{const.} \tag{6}$$

Let J denotes the momentum of the fluid issuing from the infinitesimal nozzle exit per unit time and ρ the density of fluid, the momentum integral equation becomes

$$\int_0^\infty u^2 dy=\frac{J}{2\rho}. \tag{7}$$

By multiplying Eq. (4) by u and then integrating it from $y=0$ to $y=\infty$, the energy integral equation is obtained as follows:

$$\frac{1}{2}\frac{d}{dx}\int_0^\infty u^3 dy+\varepsilon\int_0^\infty\left(\frac{\partial u}{\partial y}\right)^2 dy=0. \tag{8}$$

For example, by using Eq. (3) as a velocity profile, Eq. (7) becomes

$$U^2 b=\frac{7}{4}\frac{J}{\rho} \tag{9}$$

and using Eq. (2) for ε, Eq. (8) becomes

$$\frac{83}{715}\frac{d}{dx}(U^3 b)+\frac{96}{175}\kappa U^3=0. \tag{10}$$

Substituting Eq. (9) into Eq. (10) and integrating it with respect to x, we obtain

$$-\frac{1}{2U^2}+\frac{54912}{20335}\frac{\kappa\rho}{J}x=0 \tag{11}$$

by putting $U=\infty$ at $x=0$. From Eq. (11), the center line velocity of the jet issuing from an infinitesimal width nozzle is given by

$$U=\left(\frac{20335}{109824}\right)^{1/2}\left(\frac{J}{\kappa\rho x}\right)^{1/2}. \tag{12}$$

By substituting Eq. (12) into Eq. (9), the half width of the jet is given by

$$b=\frac{27456}{2905}\kappa x. \tag{13}$$

From the results of the above calculation, U and b are expressed in the form

$$U=C_1\left(\frac{J}{\kappa\rho x}\right)^{1/2} \tag{14}$$

and

$$b=C_2\kappa x, \tag{15}$$

respectively, for any velocity profile assumed, where C_1 and C_2 are constants determined only by the velocity profile. Substituting Eqs. (14) and (15) into Eq. (1), the eddy kinematic viscosity becomes

$$\varepsilon = C_3 \kappa^{3/2} \left(\frac{J}{\rho} \right)^{1/2} x^{1/2}, \tag{16}$$

where

$$C_3 = C_1 C_2.$$

Figure 2 shows the results of calculation for each velocity profile. The value of the mean width of the jet is also shown in the same figure. As the result, there are not so much differences among them except that the Görtler's profile differs from the other ones at the jet boundary.

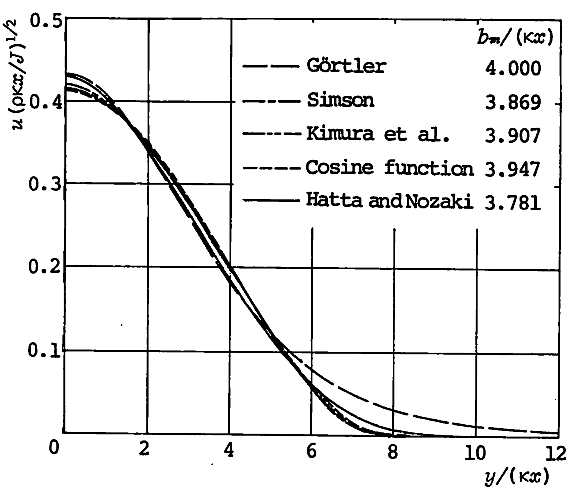

Fig. 2 Comparison of the velocity profiles

4. Relations between the empirical constant and spread parameter

So far, the solutions of a two-dimensional free jet flow issuing from an infinitesimal width nozzle by Görtler[2] have been used for the approximate calculations of the flow where the jet flow takes place. As has been stated in the last section, the spread parameter σ appears. Therefore, it is available to obtain the relations between the empirical constant contained in the eddy kinematic viscosity κ and the spread parameter σ, for comparing the experimental results and the different solutions of a free jet with each other.

The relations between κ and σ can be determined by referring to the expressions of the center line velocity of the jet, the mean width of the jet or the eddy kinematic viscosity expressed by using those parameters, but the results differ a little each other according to the approximate expression of the velocity profile in the author's calculations.

The center line velocity of the jet by Görtler is given by

$$U = \frac{\sqrt{3}}{2} \left(\frac{J\sigma}{\rho x} \right)^{1/2}. \tag{15}$$

On the other hand, that by the author is given by Eq. (10). Comparing Eq. (10) with Eq. (15), we obtain

$$\sigma=0.2469/\kappa. \tag{16}$$

In comparison of the expression for the mean width of the jet, using Eq. (2) and after some calculations, the mean width of the jet of Görtler's solution is given by

$$b_m=\frac{x}{\sigma} \tag{17}$$

and that by the author is given by

$$b_m=\frac{54912}{14525}\kappa x. \tag{18}$$

Equating Eq. (17) to Eq. (18), we obtain

$$\sigma=0.2645/\kappa. \tag{19}$$

Furthermore, the expression of the eddy kinematic viscosity

$$\varepsilon=1.135\frac{Ub_{1/2}}{4\sigma} \tag{20}$$

is used by Görtler, where $b_{1/2}$ denotes half the width at half depth. Comparing Eq. 20) with Eq. (2), we obtain

$$\sigma=0.25/\kappa. \tag{21}^*$$

The relations between σ and κ show somewhat different values according to the procedure of comparison. For instance, $\sigma=7.67$ proposed by Reichardt [5] from his experiment corresponds to $\kappa=0.0322$ when we use Eq. (16).

5. Conclusions

The velocity profile in the streamwise direction by Hatta and Nozaki is compared with the other ones by using the mean width of the jet. As the result, there are not so much differences among them except that the Görtler's profile differes from the other ones at the jet boundary.

In order to refer the experimental results and the different solutions of a free jet to each other, the constant contained in the eddy kinematic viscosity κ is also compared with the spread parameter σ contained in the solution proposed by Görtler. The relations between κ and σ show somewhat different values according to the procedure of comparison. For instance, $\sigma=7.67$ proposed by Reichardt corresponds to $\kappa=0.0322$ when we use the expressions for the center line velocity of the jet.

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* The factor 1.125 which appears in reference 2) is incorrect and it should be 1.135 from the author's reexamination.

References

1) Hatta, K. and Nozaki, T., Bull. JSME, Vol. 18, No. 118 (1975-4), p. 349.
2) Görtler, H., ZAMM, Bd. 22, Nr. 5(1942-10), S. 244.
3) Simson, A.K., Trans. ASME, Ser. I, Vol. 85, No. 2 (1966-6), p. 295.
4) Kimura, M. and Mitsuoka, T., Proc. of the 3rd Fluidics Symposium (in Japanese), (1968-8), p. 49, SICE.
5) Reichardt, H., VDI-Forschungsheft 414 (1942).