Design of Extended Recursive Wiener Fixed-Point Smoother and Filter in Continuous-Time Stochastic Systems

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(Received October 18, 2006)

Abstract.

This paper designs the extended recursive Wiener fixed-point smoother and filter in continuous-time wide-sense stationary stochastic systems. It is assumed that the signal is observed with the nonlinear mechanism of the signal and with the additional white observation noise. The estimators use the information of the system matrix F for the state vector $x(k)$, the observation vector C for the state vector , the variance $K(k,k)=K(0)$ of the state vector, the nonlinear observation function and the variance of the white observation noise. F, C and $K(0)$ are usually calculated from the auto-covariance function of the signal. It is noteworthy, from a simulation example for the estimation of a stochastic signal in the phase demodulation problem, that the proposed extended recursive Wiener estimators are superior in estimation accuracy to the extended Kalman estimators based on the state-space model.

Keywords. Continuous-time stochastic systems, Extended recursive Wiener estimators, Covariance information, Fixed-point smoother, Non-linear modulation

1. Introduction

The estimation problem using the covariance information has been investigated extensively in the area of the signal detection and estimation problems [1]-[5] for communication systems. Also, the extended Kalman filter [6],[7],[8],[9] is useful in the wide area of engineering such as signal demodulation problems etc. In [7], the robust extended Kalman filter is proposed for the discrete-time nonlinear systems with norm-bounded parameter uncertainties in Krein space. It is a characteristic that both the Kalman filter and the extended Kalman filter use the full information of the state-space model.

The estimation technique using the covariance information might be classified into two kinds from the expressions for the auto-covariance function of the signal to be estimated. One kind is expressed in the form of the semi-degenerate kernel [10]-[12]. The semi-degenerate kernel expresses the auto-covariance function of the signal as a finite sum of products of deterministic functions. In section III of [10], by starting with the Wiener-Hopf integral equation having the semi-degenerate kernel for the process of having rational spectral densities, the linear least-squares filtering problem is introduced. In [11], by assuming that the signal process is expressed by the autoregressive (AR) model, in terms of the signal auto-covariance function in the form of the semi-degenerate kernel, the recursive least-squares (RLS) filter is derived from the Wiener-Hopf equation in linear discrete-time stochastic systems. In [12], by using the semi-degenerate kernel expression of the covariance information of the signal and observation noise, least-squares second-order polynomial estimators are derived in systems including uncertain observations. Furthermore, in [13]-[15], for the observation having the nonlinear mechanism of the signal, the extended recursive estimators using the covariance information are designed in stochastic systems. It might be advantageous that the estimation technique using the auto-covariance function of the signal in the form of the semi-degenerate kernel can be applied when the state-space model is not realizable from the covariance information, e.g. for the triangular signal, the rectangular signal [16], etc. As for the other kind of expression in discrete-time wide-sense stationary stochastic systems, the auto-covariance function of the state vector is represented in terms of the system matrix F, the observation vector C for the state vector $x(k)$and the variance $K(k,k)=K(0)$ of the state vector. The recursive Wiener estimators, using this type of information, are investigated in [3],[10],[17],[18].

In [13]-[15], the extended estimators are designed from the RLS Wiener estimators [17] like the derivation of the extended Kalman filter [6] from the Kalman filter in linear stochastic systems. The observation mechanism in the extended recursive Wiener estimators is same as that in the extended Kalman filter.

In [19], by using the information of F, C, $K(0)$ and R, the RLS Wiener fixed-point smoother and filter are designed in the case of $H(t)=1$, which indicates that the observation equation (1) does not include the linear modulation. This paper, in the estimation of the continuous-time wide-sense stationary stochastic signal, newly designs the extended recursive Wiener fixed-point smoother and filter. The estimators use the information of F, C, and $K(0)$, which are calculated from the auto-covariance function of the signal, together with the variance of white observation noise, R. In [Theorem 1], by using the the information of F, C, $K(0)$ and R, the RLS Wiener fixed-point smoothing and filtering algorithms are presented by applying the estimation technique in [19] to the case of the observation equation (1) with the linear modulation. In [Theorem 2], the extended recursive Wiener fixed-point smoother and filter are proposed in continuous-time wide-sense stationary stochastic systems. The signal is observed with the additional noise through the nonlinear mechanism of the linear stochastic signal. The estimators in [Theorem 2] are derived by extending the RLS Wiener estimators in [Theorem 1] with the linear modulation to the observation equation (17) with the nonlinear modulation of the signal like the derivation of the extended Kalman filter from the RLS Kalman filter. Also, in [Theorem 3], by referring to [6], the recursive Wiener fixed-point smoother and filter are proposed in terms of the second-order approximation of the nonlinear observation mechanism.

The phase demodulation problem to estimate the signal from the output that a phase was modulated is important in the analog and digital communication systems [20]. In a numerical simulation example concerned with the phase demodulation, the proposed extended recursive Wiener fixed-point smoother and filter in [Theorem 2] and [Theorem 3] are compared in estimation accuracy with the extended Kalman estimators, in terms of the first-order and second-order approximations of the nonlinear observation function, based on the state-space model.

2. Least-squares smoothing problem for linear modulation

Let an m-dimensional observation equation be given by

$$y(t) = H(t)z(t) + v(t),\ z(t) = Cx(t), \tag{1}$$

in linear continuous-time stochastic systems. Here, $z(t)$ is an $l\times 1$ signal vector, $H(t)$ is a linear modulation function of $z(t)$ and $x(t)$ is an $n\times 1$ state vector with the wide-sense stationary property. C is an $1\times n$ observation vector transforming $x(t)$ to $z(t)$. $v(t)$ is white observation noise. Also, let the state equation for the state vector $x(t)$ be expressed by

$$\frac{dx(t)}{dt} = Fx(t) + w(t), \tag{2}$$

where F is the state-transition matrix and $w(t)$ is white noise input. It is assumed that the signal and the observation noise are mutually independent and are zero mean. Let the auto-covariance function of $v(t)$ and $w(t)$ be expressed by

$$E[v(t)v^T(s)] = R\delta(t-s),\ R>0, \tag{3}$$

$$E[w(t)w^T(s)] = \Psi_w\delta(t-s),\ \Psi_w>0. \tag{4}$$

Here, $\delta(\cdot)$ denotes the Kronecker δ function.

Let $K(t,s)=K(t-s)$ represent the auto-covariance function of the state vector $x(t)$ and let $K(t,s)$ be expressed in the form of

$$K(t,s)=\begin{cases}A(t)B^T(s), & 0\le s\le t\\ B(t)A^T(s), & 0\le t\le s\end{cases} \tag{5}$$

in wide-sense stationary stochastic systems [19]. Here, $A(t)=e^{Ft}$, $B^T(s)=e^{-Fs}K(s,s)=e^{-Fs}K(0)$, where F represents the system matrix in the state equations of $x(t)$ and $K(0)$represents the variance of $x(t)$.

Let a fixed-point smoothing estimate $\hat{x}(t\mid T)$ of $x(t)$ be expressed by

$$\hat{x}(t\,|\,T)=\int_0^T h(t,s,T)y(s)ds\,,\ 0\le t\le T\,, \tag{6}$$

as a linear transformation of the observed value $y(s)$ during the time interval $0\le s\le T$. In (6), $h(t,s,T)$ is a time-varying impulse response function and t is the fixed point respectively.

Let us consider the estimation problem, which minimizes the mean-square value

$$J=E[\|x(t)-\hat{x}(t\,|\,T)\|^2] \tag{7}$$

of the fixed-point smoothing error. From an orthogonal projection lemma [6], [20],

$$x(t)-\int_0^T h(t,s,T)y(s)ds\perp y(s)\,,\ 0\le s,t\le T\,, \tag{8}$$

the optimal impulse response function satisfies the Wiener-Hopf integral equation

$$E[x(t)y^T(s)]=\int_0^T h(t,\tau,T)E[y(\tau)y^T(s)]d\tau\,. \tag{9}$$

Here, '⊥' denotes the notation of the orthogonality. Substituting (1) and (3) into (9), we obtain

$$h(t,s,T)R=K(t,s)C^TH^T(s)-\int_0^T h(t,\tau,T)H(\tau)CK(\tau,s)d\tau C^TH^T(s)\,. \tag{10}$$

3. RLS Wiener fixed-point smoothing and filtering algorithms

Under the linear least-squares estimation problem of the signal $z(t)$ in Section 2, [Theorem 1] shows the RLS Wiener fixed-point smoothing and filtering algorithms, which use the covariance information of the signal and observation noise.

[Theorem 1]

Let the observation equation, concerned with the linear modulation for the signal $z(t)$, be given by (1). Let the auto-covariance function of the state vector $x(t)$ be expressed by (5) and let the variance of white observation noise be R in wide-sense stationary stochastic systems. Then, the linear RLS algorithms, using the information of the system matrix F, the observation vector C and the variance $K(0)$ of the state vector, for the fixed-point smoothing and filtering estimates consist of (11)-

(16).

Fixed-point smoothing estimate of $z(t)$ at the fixed point t: $\hat{z}(t \mid T) = C\hat{x}(t \mid T)$

Fixed-point smoothing estimate of $x(t)$ at the fixed point t: $\hat{x}(t \mid T)$

$$\frac{\partial \hat{x}(t \mid T)}{\partial T} = h(t,T,T)(y(T) - H(T)\hat{z}(T \mid T)),\ \hat{z}(T \mid T) = C\hat{x}(T \mid T) \tag{11}$$

Smoother gain: $h(t,T,T)$

$$h(t,T,T) = (K_x(t,T)C^T H^T(T) - Q(t,T)C^T H^T(T))R^{-1} \tag{12}$$

$$\frac{\partial Q(t,T)}{\partial T} = h(t,T,T)(H(T)CK_x(T,T) - H(T)CG(T)) + Q(t,T)F^T,\ \ Q(t,t) = G(t) \tag{13}$$

Filtering estimate of $z(t)$: $\hat{z}(t \mid t) = H(t)C\hat{x}(t \mid t)$

Filtering estimate of state vector $x(t)$: $\hat{x}(t \mid t)$

$$\frac{d\hat{x}(t \mid t)}{dt} = F\hat{x}(t,t) + h(t,t,t)(y(t) - H(t)C\hat{x}(t \mid t)),\quad \hat{x}(0 \mid 0) = 0 \tag{14}$$

Auto-variance function of $\hat{x}(t \mid t)$: $G(t)$

$$\frac{dG(t)}{dt} = FG(t) + G(t)F^T + h(t,t,t)H(t)C(K_x(t,t) - G(t)),\ \ G(0) = 0 \tag{15}$$

Filter gain: $h(t,t,t)$

$$h(t,t,t) = (K_x(t,t)C^T H^T(T) - G(T)C^T H^T(t))R^{-1} \tag{16}$$

Proof. In [19], the RLS Wiener fixed-point smoother and filter, using the information of F, C, $K(0)$ and R, are designed in the case of $H(t) = 1$ which does not include the linear modulation in the observation equation (1). The RLS Wiener fixed-point smoothing and filtering algorithms in [Theorem 1] are derived by applying the estimation technique in [19] to the case of the observation equation (1) with the linear modulation. The algorithms use the information of F, C, $K(0)$ and R (Q.E.D.).

4. Extended recursive Wiener estimation algorithms in case of nonlinear modulation

Let an observation equation with the nonlinear mechanism be given by

$$y(t)=f(z(t),t)+v(t)\,,\ z(t)=Cx(t)\,, \tag{17}$$

where the signal $z(t)$ and the observation noise $v(t)$ have the same stochastic properties as those in Section 2.

Like the design of the extended Kalman filter, in the design of the extended recursive Wiener estimators using the covariance information, the modulating function is put as

$H(t)=\left.\dfrac{\partial f(z(t),t)}{\partial z(t)}\right|_{z(t)=\hat{z}(t|t)}$ in [Theorem 1] after expanding the nonlinear observation function in a first-order Taylor series about $\hat{z}(t\,|\,t)$ [6]. Here, $\hat{z}(t\,|\,t)=C\hat{x}(t\,|\,t)$ represents the filtering estimate of the signal $z(t)$. Also, $H(T)\hat{z}(T\,|\,T)$ and $H(t)C\hat{x}(t\,|\,t)$ in [Theorem 1] are replaced with $f(\hat{z}(T\,|\,T),T)$ and $f(\hat{z}(t,t),t)$ respectively.

Consequently, the extended recursive Wiener fixed-point smoothing and filtering algorithms in case of the observation equation, which has the nonlinear mechanism of the signal $z(t)$, is summarized in [Theorem 2]. It is noted that the proposed extended recursive Wiener estimators are sub-optimal because of the Taylor series approximation of the modulating function.

[Theorem 2]

Let the observation equation, which has nonlinear mechanism of the signal, be given by the (17). Let the auto-covariance function of the state vector $x(t)$ be expressed by (5) and let the variance of white observation noise be R in wide-sense stationary stochastic systems. Then, the extended recursive Wiener fixed-point smoothing and filtering algorithms, using the covariance information of the signal and observation noise, consist of (18)-(26).

Fixed-point smoothing estimate of the signal $z(t)$ at the fixed point t: $\hat{z}(t\,|\,T)$

$$\hat{z}(t\,|\,T)=C\hat{x}(t\,|\,T) \tag{18}$$

Fixed-point smoothing estimate of the state vector $x(t)$ at the fixed point t: $\hat{x}(t\,|\,T)$

$$\frac{\partial \hat{x}(t\,|\,T)}{\partial T}=h(t,T,T)(y(T)-f(\hat{z}(T\,|\,T),T))\,,\ \hat{z}(T\,|\,T)=C\hat{x}(T\,|\,T) \tag{19}$$

Smoother gain: $h(t,T,T)$

$$h(t,T,T)=(K_x(t,T)C^TH^T(T)-Q(t,T)C^TH^T(T))R^{-1} \tag{20}$$

$$\frac{\partial Q(t,T)}{\partial T}=h(t,T,T)(H(T)CK_x(T,T)-H(T)CG(T))+Q(t,T)F^T,\ \ Q(t,t)=G(t) \tag{21}$$

Filtering estimate of the signal $z(t)$:

$$\hat{z}(t\,|\,t)=C\hat{x}(t\,|\,t) \tag{22}$$

Filtering estimate of the state vector $x(t)$: $\hat{x}(t\,|\,t)$

$$\frac{d\hat{x}(t\,|\,t)}{dt}=F\hat{x}(t,t)+h(t,t,t)(y(t)-f(\hat{z}(t\,|\,t),t)),\ \ \hat{x}(0\,|\,0)=0 \tag{23}$$

Auto-variance function of $\hat{x}(t\,|\,t)$: $G(t)$

$$\frac{dG(t)}{dt}=FG(t)+G(t)F^T+h(t,t,t)H(t)C(K_x(t,t)-G(t)),\ \ G(0)=0 \tag{24}$$

Filter gain: $h(t,t,t)$

$$h(t,t,t)=(K_x(t,t)C^TH^T(T)-G(T)C^TH^T(t))R^{-1} \tag{25}$$

Here, the function is given by

$$H(t)=\left.\frac{\partial f(z(t),t)}{\partial z(t)}\right|_{z(t)=\hat{z}(t|t)} \tag{26}$$

The difference of the extended recursive Wiener estimators from the extended Kalman estimators is based on the information used. The extended recursive Wiener estimators use the information of F, C, $K(0)$ and R. The extended Kalman estimators use the information of F, C and the variance of the white noise input, Ψ_w, in (4). Both estimators use the information of nonlinear modulation function. Since $G(t)$ is the auto-variance function of the filtering estimate $\hat{x}(t\,|\,t)$, the Kalman filtering algorithm for the filtering error variance function $P(t\,|\,t)$ is obtained by substituting $G(t)=K(t,t)-P(t\,|\,t)$ into (24) in the extended recursive Wiener estimation algorithms of [Theorem 2].

In [Theorem 3], in terms of the second-order approximation of the nonlinear observation function, the extended recursive Wiener fixed-point smoother and filter are presented.

[Theorem 3]

Let the nonlinear observation equation of the signal be given by the (17). Let the same covariance information of the signal and the observation noise be given as described in section 2 in wide-sense stationary stochastic systems. Then the extended recursive Wiener fixed-point smoothing and filtering algorithms, in terms of the second-order approximation of the nonlinear observation function, consist of (27)-(34).

Fixed-point smoothing estimate of the signal $z(t)$ at the fixed point t : $\hat{z}(t\,|\,T)$

$$\hat{z}(t\,|\,T)=C\hat{x}(t\,|\,T) \tag{27}$$

Fixed-point smoothing estimate of the state vector $x(t)$ at the fixed point t : $\hat{x}(t\,|\,T)$

$$\frac{\partial\hat{x}(t\,|\,T)}{\partial T}=h(t,T,T)(y(T)-f(\hat{z}(T\,|\,T),T)-\frac{1}{2}\frac{\partial^2 h(\hat{z}(T\,|\,T),T)}{[\partial\hat{z}(T\,|\,T)]^2}:C(K_x(0)-G(T))C^T),$$

$$\hat{z}(T\,|\,T)=C\hat{x}(T\,|\,T) \tag{28}$$

Smoother gain: $h(t,T,T)$

$$h(t,T,T)=(K_x(t,T)C^TH^T(T)-Q(t,T)C^TH^T(T))R^{-1} \tag{29}$$

$$\frac{\partial Q(t,T)}{\partial T}=h(t,T,T)(H(T)CK_x(T,T)-H(T)CG(T))+Q(t,T)F^T,\quad Q(t,t)=G(t) \tag{30}$$

Filtering estimate of the signal $z(t)$:

$$\hat{z}(t\,|\,t)=C\hat{x}(t\,|\,t) \tag{31}$$

Filtering estimate of the state vector : $x(t)$: $\hat{x}(t\,|\,t)$

$$\frac{d\hat{x}(t\,|\,t)}{dt}=F\hat{x}(t,t)+h(t,t,t)(y(t)-f(\hat{z}(t\,|\,t),t)-\frac{1}{2}\frac{\partial^2 h(\hat{z}(t\,|\,t),t)}{[\partial\hat{z}(t\,|\,t)]^2}:C(K_x(0)-G(t))C^T),$$

$$\hat{x}(0\,|\,0)=0 \tag{32}$$

Auto-variance function of $\hat{x}(t\,|\,t)$: $G(t)$

$$\frac{dG(t)}{dt}=FG(t)+G(t)F^T+h(t,t,t)H(t)C(K_x(t,t)-G(t))-\Xi(t),\quad G(0)=0 \tag{33}$$

$$\Xi(t)_{kl}=\left\{\frac{1}{2}\sum_{i,j=1}^{N}(K_x(0)-G(t))_{ik}(K_x(0)-G(t))_{lj}+(K_x(0)-G(t))_{kj}(K_x(0)-G(t))_{li}\frac{\partial^2 h(\hat{z}(t\,|\,t),t)}{\partial\hat{x}_i(t\,|\,t)\partial\hat{x}_j(t\,|\,t)}\right\}^T$$

$$\times R^{-1}\left\{y(t)-h(\hat{z}(t\,|\,t),t)-\frac{\partial^2 h(\hat{z}(t\,|\,t),t)}{[\partial\hat{z}(t\,|\,t)]^2}:C(K_x(0)-G(t))C^T\right\}$$

Filter gain: $h(t,t,t)$

$$h(t,t,t)=(K_x(t,t)C^TH^T(T)-G(T)C^TH^T(t))R^{-1} \tag{34}$$

Here, the function $H(t)$ is given by

$$H(t) = \left. \frac{\partial f(z(t),t)}{\partial z(t)} \right|_{z(t)=\hat{z}(t|t)}$$

Proof. The extended filtering algorithm in [Theorem 3] is obtained by substituting the filtering error variance function $P_{\tilde{x}}(t)$ expressed by $P_{\tilde{x}}(t) = K(0) - G(t)$ into the extended Kalman filter based on the second-order approximation [6]. The extended fixed-point smoothing algorithm is obtained by applying the second-order approximation to the nonlinear observation function in [Theorem 2]. (Q.E.D.)

5. A numerical simulation example

Let a scalar observation equation with the nonlinear mechanism be given by

$$y(t) = f(z(t),t) + v(t), \; z(t) = Cx(t),$$

$$f(z(t),t) = \cos(2\pi f_c t + m_A z(t)), \; f_c = 10{,}000(Hz), \; m_A = 1.2 . \tag{35}$$

The nonlinear function in (35) appears on the phase modulation in analogue communication systems [1]. Here, f_c and m_A represent the carrier frequency and the phase sensitivity respectively. The observation function is given by

$$H(t) = \left. \frac{\partial f(z(t),t)}{\partial z(t)} \right|_{z(t)=\hat{z}(t,t)} = -m_A \sin(2\pi f_c t + m_A \hat{z}(t\,|\,t)) \tag{36}$$

Let $v(t)$ be white Gaussian observation noise having the mean zero and the variance R, which is expressed by $N(0,R)$.

Let the signal $z(t)$ be represented by the state vector $x(t)$, which consists of the state variables $x_1(t) = z(t), \quad x_2(t) = \dfrac{dz(t)}{dt}$, as

$$z(t) = Cx(t), \; x(t) = [x_1(t) \quad x_2(t)]^T, \; z(t) = x_1(t), \; C = [1 \quad 0]. \tag{37}$$

Let the state differential equations for the state vector $x(t)$ be given by

$$\frac{dx(t)}{dt} = Fx(t) + w(t), \; F = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, \; w(t) = \begin{bmatrix} 1 \\ \omega_n^2 \end{bmatrix} u(t), \tag{38}$$

$$E[u(t)u(s)] = \delta(t-s)$$

The transfer function $N(s)$ from $u(t)$ to $z(t)$ is given by $N(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, where ζ is the damping ratio and ω_n is the resonant frequency. Here, for example, these values are put as $\zeta = 0.7$ and $\omega_n = 16$.

The auto-covariance function $K_x(t,s)$ of the state vector $x(t)$ is expressed by

$$K_x(t,s) = e^{F(t-s)} K_x(s,s), \quad K_x(s,s) = \begin{bmatrix} \dfrac{1+\omega_n^2+4\zeta^2+4\zeta\omega_n}{4\zeta\omega_n} & -0.5 \\ -0.5 & \dfrac{\omega_n+\omega_n^3}{4\zeta} \end{bmatrix}. \tag{39}$$

The auto-covariance function $K_z(t,s)$ of the signal $z(t)$ is given by .

$$K_z(t,s) = CK_x(t,s)C^T$$

By substituting F, C and $K_x(0) = K_x(s,s)$ into the extended recursive Wiener estimation algorithms of [Theorem 2], the fixed-point smoothing estimate $\hat{z}(t|T)$ at the fixed point t and the filtering estimate $\hat{z}(t|t)$ of the signal are calculated recursively.

As an numerical integration method of the differential equations, the fourth-order Runge-Kutta method with the step-size $\Delta = 0.001$ is used here.

Fig.1 illustrates the signal $z(t)$ and the fixed-point smoothing estimate $\hat{z}(t|t+0.005)$ vs. t, $0 \le t \le 1.3$, for the observation noise $N(0,0.5^2)$.

Fig.2 illustrates the mean-square values (MSVs) of the filtering error $z(t)-\hat{z}(t|t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t|t+L)$ by the extended recursive Wiener fixed-point smoother and filter in [Theorem 2] vs. L, $\Delta \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$. Here, the MSVs of the filtering errors and the fixed-point smoothing errors are calculated by $\dfrac{\sum_{i=1}^{1300}(z(i)-\hat{z}(i,i))^2}{1300}$ and $\dfrac{\sum_{i=1}^{1300}\sum_{j=1}^{L}(z(i)-\hat{z}(i,i+L))^2}{1300 \cdot L}$ respectively.

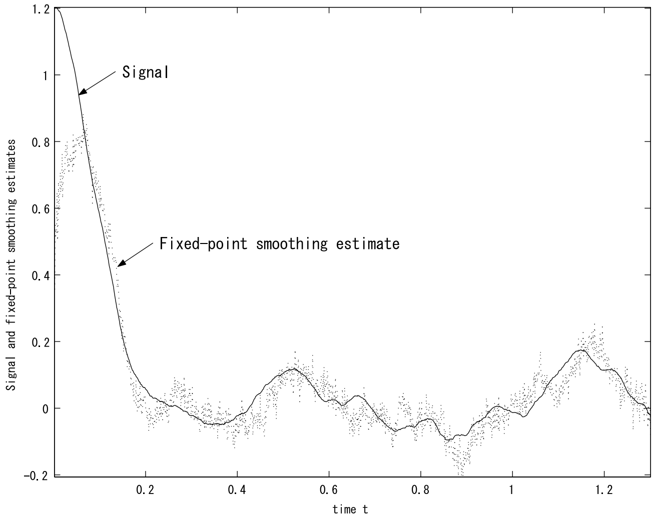

Fig.1 Signal $z(t)$ and the fixed-point smoothing estimate $\hat{z}(t \mid t+0.005)$ vs. t, $0 \le t \le 1.3$, for the observation noise $N(0,0.5^2)$.

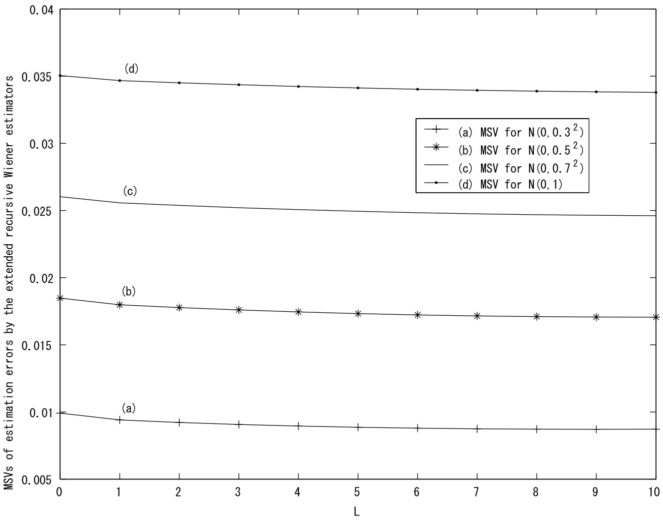

Fig.2 Mean-square values of the filtering error $z(t)-\hat{z}(t \mid t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t \mid t+L)$ by the extended recursive Wiener fixed-point smoother and filter in [Theorem 2] vs. L, $1 \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

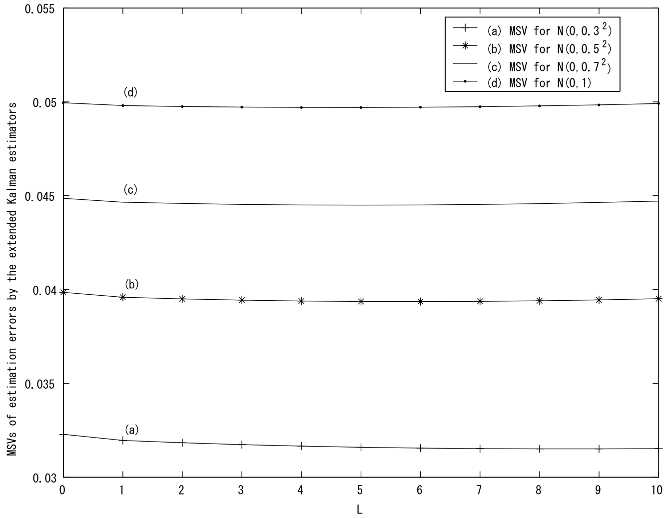

Fig.3 Mean-square values of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Kalman fixed-point smoother and filter [6] vs. L, $1 \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

Fig.3 illustrates the MSVs of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Kalman fixed-point smoother and filter [6] vs. L, $\Delta \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

Fig.4 illustrates the MSVs of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Wiener fixed-point smoother and filter in [Theorem 3] based on the second-order approximation vs. L, $1 \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

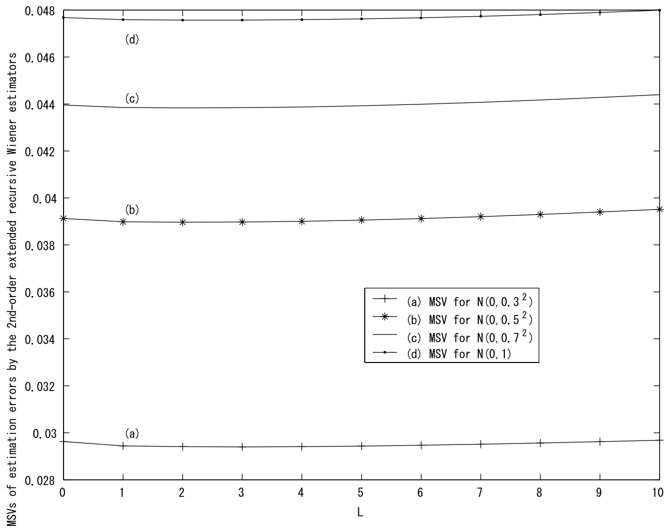

Fig.4 Mean-square values of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Wiener fixed-point smoother and filter in [Theorem 3] based on the second-order approximation vs. L, $1 \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

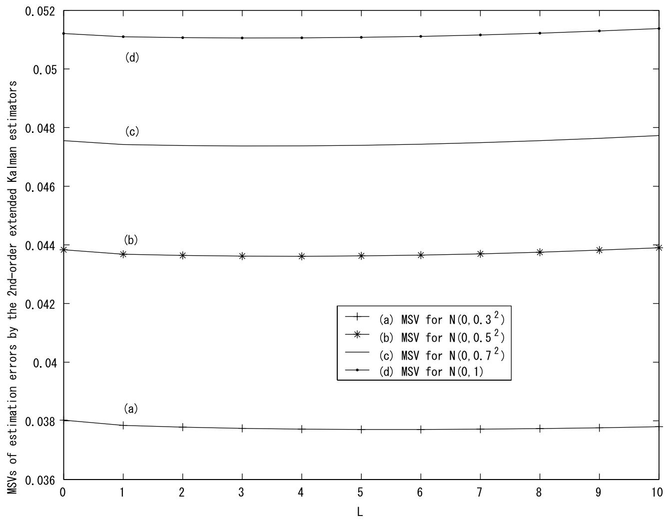

Fig.5 Mean-square values of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Kalman fixed-point smoother and filter based on the second-order approximation [6] vs. L, $1 \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

Fig.5 illustrates the MSVs of the filtering error $z(t)-\hat{z}(t\,|\,t)$ and the fixed-point smoothing error $z(t)-\hat{z}(t\,|\,t+L)$ by the extended recursive Kalman fixed-point smoother and filter based on the second-order approximation [6] vs. L , $\Delta \le L \le 10\Delta$, for the observation noises $N(0,0.3^2)$, $N(0,0.5^2)$, $N(0,0.7^2)$ and $N(0,1)$.

The MSV of the filtering error $z(t)-\hat{z}(t\,|\,t)$ corresponds to the case of $L=0$ in Fig.2-Fig.5. In Fig.2-Fig.5, as the value of the variance of the observation noise becomes large, there is a tendency that the MSVs of the fixed-point smoothing errors and the filtering errors become large. As the value of L becomes large, the MSV of the smoothing errors becomes small gradually. The MSV of the fixed-point smoothing errors is less than that of the filtering errors in these figures. From Fig.2 to Fig.5, it is seen that the estimation accuracy is feasible in the order of the extended recursive Wiener estimators, the extended recursive Wiener estimqtors based on the second-order approximation, the extended kalman estimators and the extended Kalman estimators based on the second-order approximation.

The estimation accuracy of the extended Kalman estimators is inferior to the extended recursive Wiener estimators. This reason might be explained as follows. The extended Kalman estimators use the additional information of the input noise variance Ψ_w in the state-space model for the signal. The precise and less information used in the estimators might result in the improved estimation accuracy.

6. Conclusions

In this paper, for the signal modulated nonlinearly, the extended recursive Wiener fixed-point smoother and filter have been designed in continuous-time wide-sense stationary stochastic systems. The extended recursive Wiener estimators themselves might be regarded as the alternative ones of the extended Kalman estimators although the kind of the information used is different.

The numerical simulation example has shown that the extended recursive Wiener fixed-point smoothing and filtering algorithms in [Theorem 2] and [Theorem 3] are feasible. It is advantageous that its estimation accuracy is preferable to the extended Kalman estimators. Also, the recursive Wiener estimators are feasible in estimation accuracy to the recursive Wiener estimators and the extended Kalman estimators based on the second-order approximation. The recursive extended Wiener estimators proposed in this paper is applicable to the estimation problem with the nonlinear observation mechanism genarally in continuous-time stochastic systems.

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