On the Calculations of the Coefficients of the Cyclotomic Polynomials

Rep. Fac. Sci., Kagoshima Univ.,
No. 31, pp. 31~45 (1998)

On the Calculations of the Coefficients of the Cyclotomic Polynomials

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Received September 10, 1998

Abstract

We report some results of the calculations of the coefficints of the cyclotomic polynomials.We shall propose the calculations with the more good algorithm and more large size at an early date.

1 The cyclotomic polynomials

We first report on the results of the calculations of cyclotomic polynomial $\Phi_n(x)$,where $n = 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 = 4849845$.In this computer calculations we find the maximum value 669606,the minimum value -654589 among the coefficints of this 4849845-th cyclotomic polynomial.

Starting from the definitions of them,We consider the roots of polynomial $x^n - 1$ in the complex number field. Among the n roots,choosing primitive roots,let symbolize them $\zeta_1, \cdots, \zeta_m (m =$ the value of Euler function $\phi(n)$ at positive integer n). Usually well known,
the n-th cyclotomic polynomial is the following polynomial:

$$\Phi_n(x) = (x - \zeta_1) \cdots (x - \zeta_m)$$

The coefficients of this polynomial are rational integers,and the cyclotomic polynomial is irreducible over rational number field (see for instance,van der Waerden[1]).
it seems to plausible that from the concrete forms of $\Phi_n(x)$ for small n,the coefficient of polynomial $\Phi_n(x)$ has only the values -1, 0, 1. We understand this intuition to be invalid from the calculations in the case of $n = 105$.
We investigate the behavior how large cyclotomic polynomial coefficints by super computer.

$$\underline{Formula}\ \Phi_{mp}(z) = \begin{cases} \Phi_m(z^p) & if \quad p|m \\ \dfrac{\Phi_m(z^p)}{\Phi_m(z)} & otherwise \end{cases}$$

We shall remark in the case of n to be the product of distinct primes.
Above saying representing the equation
$\Phi_{4849845}(x) = x^{1658880} + \cdots - 654589x^{884974} + \cdots + 669606x^{848274} + \cdots + 1.$
By the way $\Phi_{105}(x) = x^{48} + \cdots - 2x^{41} + \cdots + 1.$
$\Phi_{3\times5\times7\times11}(x) = x^{480} + \cdots - 3x^{386} + 3x^{385} + \cdots + 1.$
In these polynomial there are at least 2 terms in this polynomial with coefficient maximum or minimum. Followings give same examples.
$\Phi_{3\times5\times7\times11\times13}(x) = x^{5670} + \cdots + 23x^{3466} + \cdots - 22x^{3440} + \cdots + 1.$

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$$\Phi_{3\times5\times7\times11\times13\times17}(x) = x^{92160} + \cdots - 532x^{53697} + \cdots + 500x^{53213} + \cdots + 1.$$

Looking the graphic of it,its frequency curve seems to be likes to the figure of Gauss normal distribution curves in any experimented case.
This calculation is not by algebraic manipulations soft on personal computers. In the methods of algebraic manipulation,it has the limit of $n = 3\times5\times7\times11\times13$ Its programme of integer numerical calculations is written by Fortran compiler language. Using super computer VPP700/56 in the Computer Center of Kyushu Univ. its job had the time of about 73 hours. The contents of programme are by only one PE(prosessor element)hitherto type(Neumann type).By paralell instruments,it can be of high speed .But this programme is now incompletion.We report the usage of super computers.
Formula

$$x^n - 1 = \prod_{d|n} \Phi_d(x)$$

This rigt side member give the factor decompositions of light side member.

$$\Phi_n(x) = \frac{x^n - 1}{\prod_{d|n, d\neq n} \Phi_d(x)}$$

We calculated with these formulas.
The files consists of above main programme and subroutines are supplied in the programme libraries in Computer center of Hokkaido Univ., Tohoku Univ., Osaka Univ., Kyushu Univ. respectively .
Please use our softwares.
contact e:mail address:koshiba@sci.kagoshima-u.ac.jp
The n-th cyclotomic polynomial Φ_n is defined explicitly by the formula

$$\Phi_n(z) = \sum_{k=0}^{\phi(n)} a_k(n) z^{\phi(n)-k}$$

where ϕ is *Euler's* function .

$$a_k(n) = -\frac{1}{k} \cdot \mu(n) \cdot \sum_{m=0}^{k-1} a_m(n)\mu((n, k-m))\phi((n, k-m)) \qquad for \quad k > 0$$

where μ denotes *Möbius* function and (a,b) the g.c.d of a and b.

2 The Fortran programme in this calculations

At once,we proposed here the programme list of our calculations.

```
      PROGRAM MOTOKO
C*********************************************************************
C*                                                                   *
C*     THE COEFFICIENTS OF CYCLOTOMIC POLYNOMIALS WITH HIGHER DEGREE *
C*                                                                   *
C*********************************************************************
      IMPLICIT INTEGER (A-S,U-Z)
```

```
      INTEGER TEST
C
      PARAMETER(KPARA=5000000)
C
C
      DIMENSION ENBUN(KPARA),PRODUC(0:KPARA),R(0:KPARA)
      DIMENSION CYC(0:KPARA)
C
      DIMENSION YAKUSU(1000),EULE(1000)
      ENBUN(1)=1
      READ(5,*)N
C
      N123=N
C     OUTPUT N123,N
      CALL NAME(N)
C
      CALL EULER(R,N,KPARA)
C
      ZAHL=0
      TEST=1
      DO 50   I=1,N
      M=MOD(N,I)
      IF(M.NE.0)            GOTO 50
      ZAHL=ZAHL+1
C
      IF(ZAHL.GT.300) THEN
C   THIS NUMBER 300  MAY BE A PROBLEM ]?
C
      TEST=-1
      ENDIF
      YAKUSU(ZAHL)=I
      EULE(ZAHL)=R(I)
 50   CONTINUE
      IF(TEST.EQ.-1) GOTO 222
C  YAKUSU(ZAHL): THIS MEMORY HAS DIVISOR OF NUMBER N
C  EULE(ZAHL): THIS MEMORY HAS THE VALUE OF EULER FUNCTION OF YAKUSU(ZAH
C
      CALL KREAD(WRALL,OUTALL,PART)
C
      JJJ=N123+ZAHL
*
      DO 10 I=2,ZAHL
*
      COL=1
C
      PRODUC(1)=1
      PRODUC(0)=-1
      DD=1
C
```

```
      DO 20 J=2,I-1
C
      M=MOD(YAKUSU(I),YAKUSU(J))
      IF(M.NE.0) GOTO 25
C
      DO 30 K=0,EULE(J)
      CYC(K)=ENBUN(COL+K)
 30   CONTINUE
C
      DR=DD+EULE(J)
C
      IF(DR.GT.KPARA)  GOTO 333
      DO 12 IK=0,DR
  12  R(IK)=0
C
      DO 38 IK=0,DD
      DO 35 JK=0,EULE(J)
      K=IK+JK
      R(K)=R(K)+PRODUC(IK)*CYC(JK)
 35   CONTINUE
 38   CONTINUE
C
      DD=DR
      DO 40 K=0,DD
      PRODUC(K)=R(K)
 40   CONTINUE
C
 25   COL=COL+EULE(J)+1
C     OUTPUT COL
 20   CONTINUE
C
      DO 60 K=0,YAKUSU(I)
      CYC(K)=0
 60   CONTINUE
      CYC(YAKUSU(I))=1
      CYC(0)=-1
C
      DS=YAKUSU(I)-DD
      NM=MAX(YAKUSU(I),DD)
      IF(NM.GT.KPARA)                    GOTO 444
C
      N4=DS/2-1
C      N5=N4+2
      IF(N4.EQ.0)THEN
      N4=1
C     N5=2
      ENDIF
      R(DS)=CYC(YAKUSU(I))/PRODUC(DD)
      DO 130 IW=DS,N4,-1
```

```
      DO 135 JW=DD,0,-1
      K=IW+JW
      CYC(K)=CYC(K)-R(IW)*PRODUC(JW)
 135  CONTINUE
      R(IW-1)=CYC(DD+IW-1)/PRODUC(DD)
 130  CONTINUE
      DO 17 IX=0,N4
      R(IX)=R(DS-IX)
 17    CONTINUE
C
      DO 70 K=0,EULE(I)
      ENBUN(COL+K)=R(K)
 70   CONTINUE
      KKK=COL+EULE(I)+1
C
      WRITE(16,100)I,YAKUSU(I),DS
 100  FORMAT(1H ,'ORDER=',I4,'   DIVISOR=',I8,'   EULER NUMBER=',I7)
C
 10   CONTINUE
      IF(WRALL.EQ.1)CALL WRITE(R,DS)
      CALL MAXMIN(R,DS,AX,IN)
C
      IF(PART.EQ.-1)  GOTO 200
      IF(OUTALL.EQ.1)CALL  DISTRB(R,CYC,DS,AX,IN,N)
      IF(OUTALL.EQ.-1)CALL DIST01(R,CYC,DS,AX,IN,N)
C
 200  CONTINUE
      STOP
 444  STOP'ARRAY MEMORY HAS BEEN FOOL : DEG A(X) > DEG B(X)'
 333  STOP'ARRAY MEMORY HAS BEEN OVER '
 222  STOP'NUMBER OF DIVISORS ARE TOO LARGE'
C
      END
      SUBROUTINE EULER(EUL,N,KPARA)
C.............EULER FUNCTION FOR THE CALCULATIOMS OF CYCLOTOMIC POLYNOM
      IMPLICIT INTEGER (A-Z)
C
      DIMENSION EUL(0:KPARA)
C
      DO 30 I=1,N
 30   EUL(I)=0
      EUL(1)=1
C
      IF(N.GT.KPARA)    GOTO 555
C
      DO 10 I=1,N
      K=MOD(N,I)
      IF(K.NE.0)            GOTO 10
      S=0
```

```
      DO 20 J=1,I-1
      M=MOD(I,J)
      IF(M.NE.0)            GOTO 20
      S=S+EUL(J)
 20   CONTINUE
      EUL(I)=I-S
 10   CONTINUE
C
      RETURN
 555  STOP 'ARRAY MEMORY HAS BEEN TOO LARGE'
      END
      SUBROUTINE WRITE(K,N1)
      DIMENSION K(0:N1),N(100),NN(100)
      J=1
      DO 30 I=0,N1
      N(J)=K(I)
      NN(J)=I
      IF(J.NE.4) GOTO 20
      WRITE(16,123)(NN(L),N(L),L=1,4)
  123 FORMAT(1H ,4(I9,'.=',I7:,':'))

      J=0
  20  J=J+1
  30   CONTINUE
C
      N2=N1+1
      M=MOD(N2,4)
      IF(J.LE.4)  WRITE(16,123)(NN(L),N(L),L=1,M)
      END
      SUBROUTINE NAME(N)
      WRITE(16,100)
      WRITE(16,200)
      WRITE(16,300)N
      WRITE(16,200)
      WRITE(16,100)
      WRITE(16,400)
 100  FORMAT(1H ,79('*'))
 200  FORMAT(1H ,'*',77(' '),'*')
 300  FORMAT(1H ,'*',15X,'THE   COMPUTATIONS OF ',I8,'-TH CYCLOTOMIC',
     *2X,'POLYNOMIAL:     *')
 400  FORMAT(80X)
      RETURN
      END
      SUBROUTINE MAXMIN(K,I,AX,IN)
      IMPLICIT INTEGER (A-Z)
      DIMENSION K(0:I)
      AX=K(0)
      IN=K(0)
      DO 10 L=1,I
```

```
      IF(K(L).GT.AX) AX=K(L)
      IF(K(L).LT.IN) IN=K(L)
 10   CONTINUE
      RETURN
      END
      SUBROUTINE DISTRB(K,M,DS,AX,IN,TH)
      IMPLICIT INTEGER (A-Z)
      DIMENSION K(0:DS),J(-9999999:9999999),M(0:DS)
      DO 10 I=-9999999,9999999
 10   J(I)=0
C
      CALL LABEL(TH)
C
      DO 20 L=IN,AX
      DO 30 I=0,DS
      IF(K(I).NE.L) GOTO 30
      J(L)=J(L)+1
      M(J(L))=I
 30   CONTINUE
      IF(J(L).NE.0) CALL BUNPU(M,J(L),L)
 20   CONTINUE
C
C   TEST
      S=0
      DO 50 I=IN,AX
 50   S=S+J(I)
      MM=DS+1
      IF(MM.NE.S) WRITE(16,*)S,'THIS RESULTS WAS BAD]]'
C
      WRITE(18,*)'THE HISTOGRAMME OF THE VALUE OF THE COEFFICIENTS'
      DO 40 I=IN,AX
      WRITE(18,100)I,J(I)
  40  CONTINUE
      RETURN
 100  FORMAT(1H ,'NUMBER   OF TERMS WITH VALUE ',I7,'=',I8,' : ')
      END
      SUBROUTINE LABEL(TH)
      INTEGER TH
      WRITE(17,100)
      WRITE(17,200)
      WRITE(17,300)TH
      WRITE(17,200)
      WRITE(17,100)
      WRITE(17,400)
 100  FORMAT(1H ,79('*'))
 200  FORMAT(1H ,'*',77(' '),'*')
 300  FORMAT(1H ,'*',7X,' THE DISTRIBUTIONS OF THE VALUE OF',I10,'
     *COEFFICIENTS ',7X,'*')
 400  FORMAT(80X)
```

```
      RETURN
      END
      SUBROUTINE BUNPU(M,K,L)
      DIMENSION M(0:K)
*     WRITE(17,*)L
      WRITE(17,150)L
 150  FORMAT(1H ,'COEFFICIENT=',I7,':')

      WRITE(17,*)(M(I),I=1,K)
      WRITE(17,*)K
      DO 10 I=1,2
 10   WRITE(17,*)
      RETURN
      END
      SUBROUTINE DIST01(K,M,DS,AX,IN,TH)
      IMPLICIT INTEGER (A-Z)
      DIMENSION K(0:DS),J(-9999999:9999999),M(0:DS)
      DO 10 I=-9999999,9999999
 10   J(I)=0
C
      CALL LABEL(TH)
C
      AX0=AX-50
      IN0=IN+50
C
      DO 20 L=IN,AX
      DO 30 I=0,DS
      IF(K(I).NE.L) GOTO 30
      J(L)=J(L)+1
      M(J(L))=I
 30   CONTINUE
      IF((L.LE.AX0).AND.(L.GE.IN0)) GOTO 20
      IF(J(L).NE.0) CALL BUNPU(M,J(L),L)
 20   CONTINUE
C
C   TEST
      S=0
      DO 50 I=IN,AX
 50   S=S+J(I)
      MM=DS+1
      IF(MM.NE.S) WRITE(16,*)S,'THIS RESULTS WAS BAD]]'
C
      WRITE(18,*)'THE HISTOGRAMME OF THE VALUE OF THE COEFFICIENTS'
      DO 40 I=IN,AX
      WRITE(18,100)I,J(I)
 40   CONTINUE
      RETURN
 100  FORMAT(1H ,'NUMBER   OF TERMS WITH VALUE ',I7,'=',I8,' : ')
      END
```

```
      SUBROUTINE KREAD(WRALL,OUTALL,PART)
C     CHARACTER CARD*10
      CHARACTER*10 CARD
      INTEGER WRALL,OUTALL,PART
      WRITE(16,*)'  DO YOU USE THE SUBROUTINE WRITE  ALL OR PARTLY ??'
C
      READ(5,100)CARD
 100  FORMAT(A10)
      WRITE(16,*)CARD
C
      IF(CARD.EQ.'YES') THEN
      WRALL=1
      GOTO 10
      ENDIF
C
      IF(CARD.EQ.'NO') THEN
      WRALL=-1
      GOTO 10
      ENDIF
C
      STOP' NONUSE  ROUTINE    WRITE  '
 10   CONTINUE
      WRITE(16,*)'  DO YOU USE THE SUBROUTINE DISTRB OR DIST01 ?'
C
      READ(5,200)CARD
 200  FORMAT(A10)
      WRITE(16,*)CARD
C
      IF(CARD.EQ.'YES') THEN
      OUTALL=1
      GOTO 20
      ENDIF
C
      IF(CARD.EQ.'NO') THEN
      OUTALL=-1
      GOTO 20
      ENDIF
C
      STOP' NONUSE  ROUTINE    DISTRB OR DIST01 '
 20   CONTINUE
      READ(5,300)CARD
 300  FORMAT(A10)
C
      IF(CARD.EQ.'YES') THEN
      PART=1
      GOTO 30
      ENDIF
C
      IF(CARD.EQ.'NO') THEN
```

```
      PART=-1
      GOTO 30
      ENDIF
C
      STOP'NONUSE ROUTINE ******PART*******'
 30   CONTINUE
      RETURN
      END
```

3 The List of Cyclotomic Polynomials $\Phi_n(x)$ for $1 \leqq n \leqq 105$

We listed below all calculations of the our polynomials for n less than 105.

$$
\begin{aligned}
\Phi_1(x) &= x-1,\\
\Phi_2(x) &= x+1,\\
\Phi_3(x) &= x^2+x+1,\\
\Phi_4(x) &= x^2+1,\\
\Phi_5(x) &= x^4+x^3+x^2+x+1,\\
\Phi_6(x) &= x^2-x+1,\\
\Phi_7(x) &= x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_8(x) &= x^4+1,\\
\Phi_9(x) &= x^6+x^3+1,\\
\Phi_{10}(x) &= x^4-x^3+x^2-x+1,\\
\Phi_{11}(x) &= x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{12}(x) &= x^4-x^2+1,\\
\Phi_{13}(x) &= x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{14}(x) &= x^6-x^5+x^4-x^3+x^2-x+1,\\
\Phi_{15}(x) &= x^8-x^7+x^5-x^4+x^3-x+1,\\
\Phi_{16}(x) &= x^8+1,\\
\Phi_{17}(x) &= x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+\\
&\quad x^5+x^4+x^3+x^2+x+1,\\
\Phi_{18}(x) &= x^6-x^3+1,\\
\Phi_{19}(x) &= x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+\\
&\quad x^7+x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{20}(x) &= x^8-x^6+x^4-x^2+1,\\
\Phi_{21}(x) &= x^{12}-x^{11}+x^9-x^8+x^6-x^4+x^3-x+1,\\
\Phi_{22}(x) &= x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,\\
\Phi_{23}(x) &= x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+\\
&\quad x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{24}(x) &= x^8-x^4+1,\\
\Phi_{25}(x) &= x^{20}+x^{15}+x^{10}+x^5+1,\\
\Phi_{26}(x) &= x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,\\
\Phi_{27}(x) &= x^{18}+x^9+1,\\
\Phi_{28}(x) &= x^{12}-x^{10}+x^8-x^6+x^4-x^2+1,\\
\Phi_{29}(x) &= x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+\\
&\quad x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+\\
&\quad x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{30}(x) &= x^8+x^7-x^5-x^4-x^3+x+1,\\
\Phi_{31}(x) &= x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+\\
&\quad x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+\\
&\quad x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,\\
\Phi_{32}(x) &= x^{16}+1,\\
\Phi_{33}(x) &= x^{20}-x^{19}+x^{17}-x^{16}+x^{14}-x^{13}+x^{11}-x^{10}+x^9-x^7+x^6-\\
&\quad x^4+x^3-x+1,
\end{aligned}
$$

$$\Phi_{34}(x) = x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{35}(x) = x^{24}-x^{23}+x^{19}-x^{18}+x^{17}-x^{16}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^8+x^7-x^6+x^5-x+1,$$

$$\Phi_{36}(x) = x^{12}-x^6+1,$$

$$\Phi_{37}(x) = x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{38}(x) = x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{39}(x) = x^{24}-x^{23}+x^{21}-x^{20}+x^{18}-x^{17}+x^{15}-x^{14}+x^{12}-x^{10}+x^9-x^7+x^6-x^4+x^3-x+1,$$

$$\Phi_{40}(x) = x^{16}-x^{12}+x^8-x^4+1,$$

$$\Phi_{41}(x) = x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{42}(x) = x^{12}+x^{11}-x^9-x^8+x^6-x^4-x^3+x+1,$$

$$\Phi_{43}(x) = x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{44}(x) = x^{20}-x^{18}+x^{16}-x^{14}+x^{12}-x^{10}+x^8-x^6+x^4-x^2+1,$$

$$\Phi_{45}(x) = x^{24}-x^{21}+x^{15}-x^{12}+x^9-x^3+1,$$

$$\Phi_{46}(x) = x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{47}(x) = x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{48}(x) = x^{16}-x^8+1,$$

$$\Phi_{49}(x) = x^{42}+x^{35}+x^{28}+x^{21}+x^{14}+x^7+1,$$

$$\Phi_{50}(x) = x^{20}-x^{15}+x^{10}-x^5+1,$$

$$\Phi_{51}(x) = x^{32}-x^{31}+x^{29}-x^{28}+x^{26}-x^{25}+x^{23}-x^{22}+x^{20}-x^{19}+x^{17}-x^{16}+x^{15}-x^{13}+x^{12}-x^{10}+x^9-x^7+x^6-x^4+x^3-x+1\,,$$

$$\Phi_{52}(x) = x^{24}-x^{22}+x^{20}-x^{18}+x^{16}-x^{14}+x^{12}-x^{10}+x^8-x^6+x^4-x^2+1,$$

$$\Phi_{53}(x) = x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{54}(x) = x^{18}-x^9+1,$$

$$\Phi_{55}(x) = x^{40}-x^{39}+x^{35}-x^{34}+x^{30}-x^{28}+x^{25}-x^{23}+x^{20}-x^{17}+x^{15}-x^{12}+x^{10}-x^6+x^5-x+1,$$

$$\Phi_{56}(x) = x^{24}-x^{20}+x^{16}-x^{12}+x^8-x^4+1,$$

$$\Phi_{57}(x) = x^{36}-x^{35}+x^{33}-x^{32}+x^{30}-x^{29}+x^{27}-x^{26}+x^{24}-x^{23}+x^{21}-x^{20}+x^{18}-x^{16}+x^{15}-x^{13}+x^{12}-x^{10}+x^9-x^7+x^6-x^4+x^3-x+1,$$

$$\Phi_{58}(x) = x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{59}(x) = x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{60}(x) = x^{16}+x^{14}-x^{10}-x^8-x^6+x^2+1,$$

$$\Phi_{61}(x) = x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{62}(x) = x^{30}-x^{29}+x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{63}(x) = x^{36}-x^{33}+x^{27}-x^{24}+x^{18}-x^{12}+x^9-x^3+1,$$

$$\Phi_{64}(x) = x^{32}+1,$$

$$\Phi_{65}(x) = x^{48}-x^{47}+x^{43}-x^{42}+x^{38}-x^{37}+x^{35}-x^{34}+x^{33}-x^{32}+x^{30}-x^{29}+x^{28}-x^{27}+x^{25}-x^{24}+x^{23}-x^{21}+x^{20}-x^{19}+x^{18}-x^{16}+x^{15}-x^{14}+x^{13}-x^{11}+x^{10}-x^6+x^5-x+1,$$

$$\Phi_{66}(x) = x^{20}+x^{19}-x^{17}-x^{16}+x^{14}+x^{13}-x^{11}-x^{10}-x^9+x^7+x^6-x^4-x^3+x+1,$$

$$\Phi_{67}(x) = x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{68}(x) = x^{32}-x^{30}+x^{28}-x^{26}+x^{24}-x^{22}+x^{20}-x^{18}+x^{16}-x^{14}+x^{12}-x^{10}+x^8-x^6+x^4-x^2+1,$$

$$\Phi_{69}(x) = x^{44}-x^{43}+x^{41}-x^{40}+x^{38}-x^{37}+x^{35}-x^{34}+x^{32}-x^{31}+x^{29}-x^{28}+x^{26}-x^{25}+x^{23}-x^{22}+x^{21}-x^{19}+x^{18}-x^{16}+x^{15}-x^{13}+x^{12}-x^{10}+x^9-x^7+x^6-x^4+x^3-x+1,$$

$$\Phi_{70}(x) = x^{24}+x^{23}-x^{19}-x^{18}-x^{17}-x^{16}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}-x^8-x^7-x^6-x^5+x+1,$$

$$\Phi_{71}(x) = x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{72}(x) = x^{24}-x^{12}+1,$$

$$\Phi_{73}(x) = x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1,$$

$$\Phi_{74}(x) = x^{36}-x^{35}+x^{34}-x^{33}+x^{32}-x^{31}+x^{30}-x^{29}+x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1,$$

$$\Phi_{75}(x) = x^{40}-x^{35}+x^{25}-x^{20}+x^{15}-x^5+1,$$

$$\Phi_{76}(x) = x^{36}-x^{34}+x^{32}-x^{30}+x^{28}-x^{26}+x^{24}-x^{22}+x^{20}-x^{18}+x^{16}-x^{14}+x^{12}-x^{10}+x^8-x^6+x^4-x^2+1,$$

$$\Phi_{77}(x) = x^{60}-x^{59}+x^{53}-x^{52}+x^{49}-x^{48}+x^{46}-x^{45}+x^{42}-x^{41}+x^{39}-x^{37}+x^{35}-x^{34}+x^{32}-x^{30}+x^{28}-x^{26}+x^{25}-x^{23}+x^{21}-x^{19}+x^{18}-x^{15}+x^{14}-x^{12}+x^{11}-x^8+x^7-x+1,$$

$$\Phi_{78}(x) = x^{24}+x^{23}-x^{21}-x^{20}+x^{18}+x^{17}-x^{15}-x^{14}+x^{12}-x^{10}-x^9+x^7+x^6-x^4-x^3+x+1,$$

$$\Phi_{79}(x) = x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1,$$

$$\Phi_{80}(x) = x^{32}-x^{24}+x^{16}-x^{8}+1,$$

$$\Phi_{81}(x) = x^{54}+x^{27}+1,$$

$$\Phi_{82}(x) = x^{40}-x^{39}+x^{38}-x^{37}+x^{36}-x^{35}+x^{34}-x^{33}+x^{32}-x^{31}+x^{30}-x^{29}+x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^{9}+x^{8}-x^{7}+x^{6}-x^{5}+x^{4}-x^{3}+x^{2}-x+1,$$

$$\Phi_{83}(x) = x^{82}+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1,$$

$$\Phi_{84}(x) = x^{24}+x^{22}-x^{18}-x^{16}+x^{12}-x^{8}-x^{6}+x^{2}+1,$$

$$\Phi_{85}(x) = x^{64}-x^{63}+x^{59}-x^{58}+x^{54}-x^{53}+x^{49}-x^{48}+x^{47}-x^{46}+x^{44}-x^{43}+x^{42}-x^{41}+x^{39}-x^{38}+x^{37}-x^{36}+x^{34}-x^{33}+x^{32}-x^{31}+x^{30}-x^{28}+x^{27}-x^{26}+x^{25}-x^{23}+x^{22}-x^{21}+x^{20}-x^{18}+x^{17}-x^{16}+x^{15}-x^{11}+x^{10}-x^{6}+x^{5}-x+1,$$

$$\Phi_{86}(x) = x^{42}-x^{41}+x^{40}-x^{39}+x^{38}-x^{37}+x^{36}-x^{35}+x^{34}-x^{33}+x^{32}-x^{31}+x^{30}-x^{29}+x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^{9}+x^{8}-x^{7}+x^{6}-x^{5}+x^{4}-x^{3}+x^{2}-x+1,$$

$$\Phi_{87}(x) = x^{56}-x^{55}+x^{53}-x^{52}+x^{50}-x^{49}+x^{47}-x^{46}+x^{44}-x^{43}+x^{41}-x^{40}+x^{38}-x^{37}+x^{35}-x^{34}+x^{32}-x^{31}+x^{29}-x^{28}+x^{27}-x^{25}+x^{24}-x^{22}+x^{21}-x^{19}+x^{18}-x^{16}+x^{15}-x^{13}+x^{12}-x^{10}+x^{9}-x^{7}+x^{6}-x^{4}+x^{3}-x+1,$$

$$\Phi_{88}(x) = x^{40}-x^{36}+x^{32}-x^{28}+x^{24}-x^{20}+x^{16}-x^{12}+x^{8}-x^{4}+1,$$

$$\Phi_{89}(x) = x^{88}+x^{87}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1,$$

$$\Phi_{90}(x) = x^{24}+x^{21}-x^{15}-x^{12}-x^{9}+x^{3}+1,$$

$$\Phi_{91}(x) = x^{72}-x^{71}+x^{65}-x^{64}+x^{59}-x^{57}+x^{52}-x^{50}+x^{46}-x^{43}+x^{39}-x^{36}+x^{33}-x^{29}+x^{26}-x^{22}+x^{20}-x^{15}+x^{13}-x^{8}+x^{7}-x+1,$$

$$\Phi_{92}(x) = x^{44}-x^{42}+x^{40}-x^{38}+x^{36}-x^{34}+x^{32}-x^{30}+x^{28}-x^{26}+x^{24}-x^{22}+x^{20}-x^{18}+x^{16}-x^{14}+x^{12}-x^{10}+x^{8}-x^{6}+x^{4}-x^{2}+1,$$

$$\Phi_{93}(x) = x^{60}-x^{59}+x^{57}-x^{56}+x^{54}-x^{53}+x^{51}-x^{50}+x^{48}-x^{47}+x^{45}-x^{44}+x^{42}-x^{41}+x^{39}-x^{38}+x^{36}-x^{35}+x^{33}-x^{32}+x^{30}-x^{28}+x^{27}-x^{25}+x^{24}-x^{22}+x^{21}-x^{19}+x^{18}-x^{16}+x^{15}-x^{13}+x^{12}-x^{10}+x^{9}-x^{7}+x^{6}-x^{4}+x^{3}-x+1,$$

$$\Phi_{94}(x) = x^{46}-x^{45}+x^{44}-x^{43}+x^{42}-x^{41}+x^{40}-x^{39}+x^{38}-x^{37}+x^{36}-x^{35}+x^{34}-x^{33}+x^{32}-x^{31}+x^{30}-x^{29}+x^{28}-x^{27}+x^{26}-x^{25}+x^{24}-x^{23}+x^{22}-x^{21}+x^{20}-x^{19}+x^{18}-x^{17}+x^{16}-x^{15}+x^{14}-x^{13}+x^{12}-x^{11}+x^{10}-x^{9}+x^{8}-x^{7}+x^{6}-x^{5}+x^{4}-x^{3}+x^{2}-x+1,$$

$$
\begin{aligned}
\Phi_{95}(x) = {} & x^{72}-x^{71}+x^{67}-x^{66}+x^{62}-x^{61}+x^{57}-x^{56}+x^{53}-x^{51}+x^{48}- \\
& x^{46}+x^{43}-x^{41}+x^{38}-x^{36}+x^{34}-x^{31}+x^{29}-x^{26}+x^{24}-x^{21}+ \\
& x^{19}-x^{16}+x^{15}-x^{11}+x^{10}-x^{6}+x^{5}-x+1, \\
\Phi_{96}(x) = {} & x^{32}-x^{16}+1, \\
\Phi_{97}(x) = {} & x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+ \\
& x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+ \\
& x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+ \\
& x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+ \\
& x^{49}+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+ \\
& x^{37}+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+ \\
& x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+ \\
& x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1, \\
\Phi_{98}(x) = {} & x^{42}-x^{35}+x^{28}-x^{21}+x^{14}-x^{7}+1, \\
\Phi_{99}(x) = {} & x^{60}-x^{57}+x^{51}-x^{48}+x^{42}-x^{39}+x^{33}-x^{30}+x^{27}-x^{21}+x^{18}- \\
& x^{12}+x^{9}-x^{3}+1, \\
\Phi_{100}(x) = {} & x^{40}-x^{30}+x^{20}-x^{10}+1, \\
\Phi_{101}(x) = {} & x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+ \\
& x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{79}+ \\
& x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+ \\
& x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+ \\
& x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+x^{46}+ \\
& x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+ \\
& x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+x^{24}+ \\
& x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+ \\
& x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1, \\
\Phi_{102}(x) = {} & x^{32}+x^{31}-x^{29}-x^{28}+x^{26}+x^{25}-x^{23}-x^{22}+x^{20}+x^{19}-x^{17}- \\
& x^{16}-x^{15}+x^{13}+x^{12}-x^{10}-x^{9}+x^{7}+x^{6}-x^{4}-x^{3}+x+1, \\
\Phi_{103}(x) = {} & x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{92}+ \\
& x^{91}+x^{90}+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}+ \\
& x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+ \\
& x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+ \\
& x^{57}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}+x^{48}+x^{47}+ \\
& x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+ \\
& x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{25}+ \\
& x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+ \\
& x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x+1, \\
\Phi_{104}(x) = {} & x^{48}-x^{44}+x^{40}-x^{36}+x^{32}-x^{28}+x^{24}-x^{20}+x^{16}-x^{12}+x^{8}-x^{4}+1, \\
\Phi_{105}(x) = {} & x^{48}+x^{47}+x^{46}-x^{43}-x^{42}-2x^{41}-x^{40}-x^{39}+x^{36}+x^{35}+x^{34}+ \\
& x^{33}+x^{32}+x^{31}-x^{28}-x^{26}-x^{24}-x^{22}-x^{20}+x^{17}+x^{16}+x^{15}+ \\
& x^{14}+x^{13}+x^{12}-x^{9}-x^{8}-2x^{7}-x^{6}-x^{5}+x^{2}+x+1
\end{aligned}
$$

References

[1] van der Waerden, Algebra, Erster Teil, 129, Springer, 1966.

[2] Y.Koshiba, On the Coefficients of the Cyclotomic Polynomials, in Japanese, Kôhô, Computer Center of Kyushu University (1997), 141–145.