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A TABLE OF THE EXPLICIT FORMULAS FOR THE SUMS OF POWERS $S_p(n) = \sum_{k=1}^{n} k^p$ FOR $p = 1\ (1)\ 61$, II

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Abstract

Let $S_p(n) = \sum_{k=1}^{n} k^p$. In the paper [1] we provide a list of the explicit formulas of $S_p(n)$ for $p = 1, 2, 3, \cdots, 61$. In the present paper we present a list of $S_p(n)$ in alternative forms.

Introduction

Let $S_p(n) = \sum_{k=1}^{n} k^p$. The following result is well known :

PROPOSITION A. I) *If p is an odd number which is greater than or equal to 3, then we have*

$$S_p(n) = n^2(n+1)^2\{\text{a polynomial in } n\}.$$

II) *If p is a positive even number, then we have*

$$S_p(n) = n(n+1)(2n+1)\{\text{a polynomial in } n\}.$$

In the paper [1] we provide a list of the explicit formulas of $S_p(n)$ for $p = 1, 2, 3, \cdots, 61$ in the form stated in the above Proposition A.

As a refinement of the Proposition A we have the following proposition, see [2] and [3].

PROPOSITION B. I) *If p is an odd number which is greater than or equal to 3, then we have*

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$$S_p(n) = n^2(n+1)^2\{\text{a polynomial in } n(n+1)\}.$$

II) *if p is a positive even number, then we have*

$$S_p(n) = n(n+1)(2n+1)\{\text{a polynomial in } n(n+1)\}.$$

All the known proofs of the Proposition B make use of the symmetry of the Bernoulli polynomials. Here we give an elementary proof of it.

PROOF OF THE PROPOSITION B. First we consider the case I). Noting p is an odd number, we have obviously

$$(k+1)^{p+1} - (k-1)^{p+1} = 2\left\{\binom{p+1}{1}k^p + \binom{p+1}{3}k^{p-2} + \cdots + \binom{p+1}{p-2}k^3 + \binom{p+1}{p}k\right\},$$

from which we obtain

$$(n+1)^{p+1} + n^{p+1} - 1 = \sum_{k=1}^{n}\{(k+1)^{p+1} - (k-1)^{p+1}\}$$
$$= 2\left\{\binom{p+1}{1}S_p(n) + \binom{p+1}{3}S_{p-2}(n) + \cdots + \binom{p+1}{p-2}S_3(n)\right\} + (p+1)n(n+1).$$

Thus we have

$$(1) \quad \binom{p+1}{1}S_p(n) + \binom{p+1}{3}S_{p-2}(n) + \cdots + \binom{p+1}{p-2}S_3(n) = \frac{1}{2}\{(n+1)^{p+1} + n^{p+1} - (p+1)n(n+1) - 1\},$$

which is due to Mr. Shigeru Shirasaka, Kagoshima Technical College. Denoting the both members of (1) by $f_p(n)$ we get

$$(2) \quad \binom{p+1}{1}S_p(n) + \binom{p+1}{3}S_{p-2}(n) + \cdots + \binom{p+1}{p-2}S_3(n) = f_p(n),$$

and

$$(3) \quad (n+1)^{p+1} + n^{p+1} = 2f_p(n) + (p+1)n(n+1) + 1.$$

From (3) it follows that

$$2f_{p+4}(n)+(p+5)n(n+1)+1=(n+1)^{p+5}+n^{p+5}$$
$$=\{(n+1)^{p+3}+n^{p+3}\}\{(n+1)^2+n^2\}-n^2(n+1)^2\{(n+1)^{p+1}+n^{p+1}\}$$
$$=\{2f_{p+2}(n)+(p+3)n(n+1)+1\}\{2n(n+1)+1\}$$
$$-n^2(n+1)^2\{2f_p(n)+(p+1)n(n+1)+1\}.$$

After some simplifications we have

$$(4)\quad f_{p+4}(n)=\{2n(n+1)+1\}f_{p+2}(n)-n^2(n+1)^2 f_p(n)$$
$$-\frac{1}{2}(p+1)n^3(n+1)^3+\frac{1}{2}(2p+5)n^2(n+1)^2.$$

Now if we set

$$(5)\qquad f_p(n)=n^2(n+1)^2 g_p(n)$$

then we can prove that

(6) $g_p(n)$ is a polynomial in $n(n+1)$.

The proof of (6) proceeds by induction in odd p.

i) If $p=3$, then noting (3) we get

$$f_p(n)=f_3(n)=n^2(n+1)^2.$$

Hence $g_3(n)=1$, which shows the validity of (6).

ii) If $p=5$, we obtain similarly

$$f_p(n)=f_5(n)=\frac{1}{2}n^2(n+1)^2(2n^2+2n+9),$$

which implies $g_p(n)=n(n+1)+\frac{9}{2}$. Hence in this case also (6) holds.

iii) Suppose (6) is true for p, and $p+2$, then we obtain from (4) and (5)

$$(7)\quad g_{p+4}(n)=\{2n(n+1)+1\}g_{p+2}(n)-n^2(n+1)^2 g_p(n)$$
$$-\frac{1}{2}(p+1)n(n+1)+\frac{1}{2}(2p+5).$$

By the hypotheses of induction the right-hand side of (7) is a polynomial in $n(n+1)$. Hence (6) is also true for $p+4$. This completes the proof of (6).

We are now in a position to prove the part I) of Proposition B. From (2) and

(5) we obtain

$$(8)\quad \binom{p+1}{1} S_p(n) = -\binom{p+1}{3} S_{p-2}(n) - \cdots - \binom{p+1}{p-2} S_3(n) + n^2(n+1)^2 g_p(n).$$

The proof of I) again proceeds by induction in odd p.

i) If $p=3$, we have obviously $S_p(n) = \frac{1}{4} n^2(n+1)^2$, so that the proposition holds in this case.

ii) If $p \geqq 5$, suppose that the proposition holds for 3, 5, 7, ⋯, $p-4$, $p-2$, then noting (6), (8), and the above hypotheses we see that the proposition holds for p. This completes the proof of the part I) of Proposition B.

Similarly, we can prove the part II). We omit the details.

On the basis of the Proposition B we obtain the table below. For simplicity we set $m = n(n+1)$ in the table.

REMARK. According to the part I) of Proposition B, we may put

$$S_p(n) = m^2 T_p(m) \qquad (p=3,\ 5,\ 7,\cdots)$$

where $m = n(n+1)$. Then, (7) and (8) imply that

$$(9)\quad \binom{p+1}{1} T_p(m) = -\binom{p+1}{3} T_{p-2}(m) - \cdots - \binom{p+1}{p-2} T_3(m) + h_p(m) \qquad (p=5,\ 7,\ 9,\cdots),$$

where $h_p(m)$ are defined recursively by means of

$$(10)\quad h_{p+4}(m) = (2m+1)h_{p+2}(m) - m^2 h_p(m) - \frac{1}{2}(p+1)m + \frac{1}{2}(2p+5) \qquad (p=3,\ 5,\ 7,\cdots),$$

$h_3(m)=1$, $h_5(m) = m + \frac{9}{2}$. If we eliminate $h_p(m)$ from (9) and (10), we obtain the recurrence formula for $T_p(m)$:

$$\begin{aligned}(p+5)T_{p+4}(m) = {} & \left\{\binom{p+3}{1}(2m+1) - \binom{p+5}{3}\right\} T_{p+2}(m) \\ & - \sum_{k=0}^{\frac{p-3}{2}} \left\{\binom{p+1}{2k+1} m^2 - \binom{p+3}{2k+3}(2m+1) + \binom{p+5}{2k+5}\right\} T_{p-2k}(m) \\ & - \frac{1}{2}(p+1)m + \frac{1}{2}(2p+5) \qquad (p=3,\ 5,\ 7,\cdots),\end{aligned}$$

$$T_3(m) = \frac{1}{4},\quad T_5(m) = \frac{1}{12}(2m-1).$$

Conjectures

Observing the results in [1] and the present article we make the following conjectures.

CONJECTURE 1. *If either $p\equiv 1 \pmod 4$ and $p\geqq 13$ or $p\equiv 3 \pmod 4$ and $p\geqq 15$, denoting*

$$S_p(n)=n^2(n+1)^2\sum_{i=0}^{p-3} c(p,\ i)\, n^{p-3-i},$$

we have

$$\text{the sign of } c(p,\ i)=\begin{cases}(-1)^{[\frac{i}{2}]} & \text{if } i=0,\ 1,\ 2\\ (-1)^{[\frac{i-1}{2}]} & \text{if } i=3,\ 4,\ 5,\cdots,\ p-6\\ (-1)^{[\frac{i}{2}]} & \text{if } i=p-5,\ p-4,\ p-3.\end{cases}$$

Thus the pattern of arrangement of the signs of $c(p,\ i)$ is as follows:

either $+++--++--++ \cdots --++--++-++-$

or $+++--++--++ \cdots ++--++--+--+.$

CONJECTURE 2. *If either $p\equiv 0 \pmod 4$ and $p\geqq 12$ or $p\equiv 2 \pmod 4$ and $p\geqq 14$, denoting*

$$S_p(n)=n(n+1)(2n+1)\sum_{i=0}^{p-2} c(p,\ i)\, n^{p-2-i},$$

we have

$$\text{the sign of } c(p,\ i)=\begin{cases}(-1)^{[\frac{i}{2}]} & \text{if } i=0\\ (-1)^{[\frac{i-1}{2}]} & \text{if } i=1,\ 2,\ 3,\cdots,\ p-3\\ (-1)^{[\frac{i}{2}]} & \text{if } i=p-2.\end{cases}$$

Hence the pattern is as follows:

either $+++--++--++ \cdots ++--++--++--+-$

or $+++--++--++ \cdots --++--++--++-+.$

CONJECTURE 3. *When p is an odd number and $p\geqq 3$, putting*

$$S_p(n)=m^2\sum_{i=0}^{\frac{p-3}{2}} c(p,\ i)\, m^{\frac{p-3}{2}-i}$$

where $m=n(n+1)$, we have

$$\textit{the sign of } c(p,\ i) = (-1)^i \quad \textit{if} \quad i=0,\ 1,\ 2, \cdots, \frac{p-3}{2}.$$

In other words, $c(p,\ i)$ is alternating in sign.

CONJECTURE 4. *Similarly, when p is a positive even number, putting*

$$S_p(n) = (2n+1)\, m \sum_{i=0}^{\frac{p-2}{2}} c(p,\ i)\, m^{\frac{p-2}{2}-i},$$

we have again

$$\textit{the sign of } c(p,\ i) = (-1)^i \quad \textit{if} \quad i=0,\ 1,\ 2, \cdots, \frac{p-2}{2}.$$

TABLE

NOTATION $S_p(n) = \sum_{k=1}^{n} k^p$ *and* $m = n(n+1)$

$2S_1(n) = m$

$6S_2(n) = (2n+1)\, m$

$4S_3(n) = m^2$

$30S_4(n) = (2n+1)\, m(3m - 1)$

$12S_5(n) = m^2(2m - 1)$

$42S_6(n) = (2n+1)\, m(3m^2 - 3m + 1)$

$24S_7(n) = m^2(3m^2 - 4m + 2)$

$90S_8(n) = (2n+1)\, m(5m^3 - 10m^2 + 9m - 3)$

$20S_9(n) = m^2(2m^3 - 5m^2 + 6m - 3)$

$66S_{10}(n) = (2n+1)\, m(3m^4 - 10m^3 + 17m^2 - 15m + 5)$

$24S_{11}(n) = m^2(2m^4 - 8m^3 + 17m^2 - 20m + 10)$

$$2730S_{12}(n)=(2n+1)m(105m^5-525m^4+1435m^3-2360m^2+2073m-691)$$

$$420S_{13}(n)=m^2(30m^5-175m^4+574m^3-1180m^2+1382m-691)$$

$$90S_{14}(n)=(2n+1)m(3m^6-21m^5+84m^4-220m^3+359m^2-315m+105)$$

$$48S_{15}(n)=m^2(3m^6-24m^5+112m^4-352m^3+718m^2-840m+420)$$

$$510S_{16}(n)=(2n+1)m(15m^7-140m^6+770m^5-2930m^4+7595m^3-12370m^2+10851m-3617)$$

$$180S_{17}(n)=m^2(10m^7-105m^6+660m^5-2930m^4+9114m^3-18555m^2+21702m-10851)$$

$$3990S_{18}(n)=(2n+1)m(105m^8-1260m^7+9114m^6-47418m^5+1\,78227m^4-4\,60810m^3+7\,50167m^2-6\,58005m+2\,19335)$$

$$840S_{19}(n)=m^2(42m^8-560m^7+4557m^6-27096m^5+1\,18818m^4-3\,68648m^3+7\,50167m^2-8\,77340m+4\,38670)$$

$$6930S_{20}(n)=(2n+1)m(165m^9-2475m^8+22770m^7-1\,55100m^6+7\,95795m^5-29\,81895m^4+77\,04835m^3-125\,41460m^2+110\,00493m-36\,66831)$$

$$660S_{21}(n)=m^2(30m^9-495m^8+5060m^7-38775m^6+2\,27370m^5-9\,93965m^4+30\,81934m^3-62\,70730m^2+73\,33662m-36\,66831)$$

$$690S_{22}(n)=(2n+1)m(15m^{10}-275m^9+3135m^8-27060m^7+1\,81533m^6-9\,28151m^5+34\,75154m^4-89\,77920m^3+146\,13279m^2-128\,17695m+42\,72565)$$

$$16560S_{23}(n)=m^2(690m^{10}-13800m^9+1\,73052m^8-16\,59680m^7+125\,25777m^6-731\,91336m^5+3197\,14168m^4-9911\,62368m^3+20166\,32502m^2-23584\,55880m+11792\,27940)$$

$$13650S_{24}(n)=(2n+1)m(273m^{11}-6006m^{10}+83083m^9-8\,85885m^8+75\,22515m^7-502\,69856m^6+2567\,94972m^5-9612\,97596m^4+24833\,74425m^3-40421\,36250m^2+35454\,61365m-11818\,20455)$$

$$1092S_{25}(n)=m^2(42m^{11}-1001m^{10}+15106m^9-1\,77177m^8+16\,71670m^7-125\,67464m^6+733\,69992m^5-3204\,32532m^4+9933\,49770m^3-20210\,68125m^2+23636\,40910m-11818\,20455)$$

$378S_{26}(n)=(2n+1)m(7m^{12}-182m^{11}+3003m^{10}-38753m^{9}+4\,06120m^{8}-34\,34184m^{7}+229\,26780m^{6}-1170\,91548m^{5}+4383\,04419m^{4}-11322\,85290m^{3}+18429\,93525m^{2}-16165\,36467m+5388\,45489)$

$56S_{27}(n)=m^{2}(2m^{12}-56m^{11}+1001m^{10}-14092m^{9}+1\,62448m^{8}-15\,26304m^{7}+114\,63390m^{6}-669\,09456m^{5}+2922\,02946m^{4}-9058\,28232m^{3}+18429\,93525m^{2}-21553\,81956m+10776\,90978)$

$870S_{28}(n)=(2n+1)m(15m^{13}-455m^{12}+8827m^{11}-1\,35564m^{10}+17\,18002m^{9}-179\,24270m^{8}+1514\,08620m^{7}-10105\,62744m^{6}+51608\,54643m^{5}-1\,93182\,00399m^{4}+4\,99051\,76965m^{3}-8\,12294\,18480m^{2}+7\,12483\,83087m-2\,37494\,61029)$

$60S_{29}(n)=m^{2}(2m^{13}-65m^{12}+1358m^{11}-22594m^{10}+3\,12364m^{9}-35\,84854m^{8}+336\,46360m^{7}-2526\,40686m^{6}+14745\,29898m^{5}-64394\,00133m^{4}+1\,99620\,70786m^{3}-4\,06147\,09240m^{2}+4\,74989\,22058m-2\,37494\,61029)$

$14322S_{30}(n)=(2n+1)m(231m^{14}-8085m^{13}+1\,82182m^{12}-32\,83280m^{11}+494\,83434m^{10}-6241\,77554m^{9}+65048\,25250m^{8}-5\,49324\,52344m^{7}+36\,66189\,40611m^{6}-187\,22644\,97565m^{5}+700\,82716\,31088m^{4}-1810\,46272\,79060m^{3}+2946\,84492\,83827m^{2}-2584\,75238\,28015m+861\,58412\,76005)$

$7392S_{31}(n)=m^{2}(231m^{14}-8624m^{13}+2\,08208m^{12}-40\,40960m^{11}+659\,77912m^{10}-9078\,94624m^{9}+1\,04077\,20400m^{8}-9\,76576\,93056m^{7}+73\,32378\,81222m^{6}-427\,94617\,08720m^{5}+1868\,87243\,49568m^{4}-5793\,48072\,92992m^{3}+11787\,37971\,35308m^{2}-13785\,34604\,16080m+6892\,67302\,08040)$

$1\,17810S_{32}(n)=(2n+1)m(1785m^{15}-71400m^{14}+18\,49260m^{13}-386\,44060m^{12}+6830\,22158m^{11}-1\,02439\,67916m^{10}+12\,90624\,05978m^{9}-134\,46328\,29430m^{8}+1135\,45004\,37885m^{7}-7577\,87158\,31676m^{6}+38698\,84780\,42362m^{5}-1\,44857\,64812\,58186m^{4}+3\,74213\,99414\,05445m^{3}-6\,09098\,74698\,44470m^{2}+5\,34255\,94815\,63381m-1\,78085\,31605\,21127)$

$7140S_{33}(n)=m^{2}(210m^{15}-8925m^{14}+2\,46568m^{13}-55\,20580m^{12}+1050\,80332m^{11}-17073\,27986m^{10}+2\,34658\,91996m^{9}-26\,89265\,65886m^{8}+252\,32223\,19530m^{7}-1894\,46789\,57919m^{6}+11056\,81365\,83532m^{5}-48285\,88270\,86062m^{4}+1\,49685\,59765\,62178m^{3}-3\,04549\,37349\,22235m^{2}+3\,56170\,63210\,42254m-1\,78085\,31605\,21127)$

$210S_{34}(n)=(2n+1)m(3m^{16}-136m^{15}+4012m^{14}-96220m^{13}+19\,70878m^{12}-346\,58988m^{11}+5191\,69562m^{10}-65389\,86454m^{9}+6\,81213\,73265m^{8}-57\,52285$

$68156m^7 + 383\ 89988\ 37626m^6 - 1960\ 50699\ 59594m^5 + 7338\ 57452\ 87261m^4$
$- 18957\ 90256\ 31030m^3 + 30857\ 30302\ 54741m^2 - 27065\ 72251\ 28535m$
$+ 9021\ 90750\ 42845)$

$72S_{35}(n) = m^2(2m^{16} - 96m^{15} + 3009m^{14} - 76976m^{13} + 16\ 89324m^{12} - 319\ 92912m^{11}$
$+ 5191\ 69562m^{10} - 71334\ 39768m^9 + 8\ 17456\ 47918m^8 - 76\ 69714\ 24208m^7$
$+ 575\ 84982\ 56439m^6 - 3360\ 86913\ 59304m^5 + 14677\ 14905\ 74522m^4$
$- 45498\ 96615\ 14472m^3 + 92571\ 90907\ 64223m^2 - 1\ 08262\ 89005\ 14140m$
$+ 54131\ 44502\ 57070)$

$19\ 19190S_{36}(n) = (2n+1)m(25935m^{17} - 13\ 22685m^{16} + 440\ 89500m^{15} - 12027\ 61560m^{14}$
$+ 2\ 82604\ 87710m^{13} - 57\ 58497\ 26270m^{12} + 1011\ 35712\ 45778m^{11} - 15144$
$73041\ 11376m^{10} + 1\ 90734\ 84609\ 30913m^9 - 19\ 86987\ 96953\ 20655m^8 + 167$
$78397\ 21072\ 19650m^7 - 1119\ 76679\ 87263\ 02636m^6 + 5718\ 44446\ 49593\ 29117m^5$
$- 21405\ 29413\ 77607\ 15161m^4 + 55296\ 77119\ 94832\ 30305m^3 - 90005\ 16889$
$90570\ 78180m^2 + 78945\ 81465\ 91604\ 32119m - 26315\ 27155\ 30534\ 77373)$

$1\ 03740S_{37}(n) = m^2(2730m^{17} - 1\ 46965m^{16} + 51\ 87000m^{15} - 1503\ 45195m^{14} + 37680$
$65028m^{13} - 8\ 22642\ 46610m^{12} + 155\ 59340\ 37812m^{11} - 2524\ 12173\ 51896m^{10}$
$+ 34679\ 06292\ 60166m^9 - 3\ 97397\ 59390\ 64131m^8 + 37\ 28532\ 71349\ 37700m^7$
$- 279\ 94169\ 96815\ 75659m^6 + 1633\ 84127\ 57026\ 65462m^5 - 7135\ 09804$
$59202\ 38387m^4 + 22118\ 70847\ 97932\ 92122m^3 - 45002\ 58444\ 95285\ 39090m^2$
$+ 52630\ 54310\ 61069\ 54746m - 26315\ 27155\ 30534\ 77373)$

$8190S_{38}(n) = (2n+1)m(105m^{18} - 5985m^{17} + 2\ 23839m^{16} - 68\ 91528m^{15} + 1841$
$04186m^{14} - 43026\ 60390m^{13} + 8\ 75562\ 84424m^{12} - 153\ 72376\ 75064m^{11}$
$+ 2301\ 77986\ 93071m^{10} - 28988\ 38953\ 05147m^9 + 3\ 01986\ 36936\ 28015m^8 - 25$
$50011\ 38908\ 05268m^7 + 170\ 18415\ 79411\ 47951m^6 - 869\ 09935\ 55134\ 13493m^5$
$+ 3253\ 21461\ 69300\ 16242m^4 - 8404\ 10146\ 85130\ 94360m^3 + 13679\ 14537\ 08838$
$02977m^2 - 11998\ 32507\ 71811\ 84105m + 3999\ 44169\ 23937\ 28035)$

$1680S_{39}(n) = m^2(42m^{18} - 2520m^{17} + 99484m^{16} - 32\ 43072m^{15} + 920\ 52093m^{14}$
$- 22947\ 52208m^{13} + 5\ 00321\ 62528m^{12} - 94\ 59924\ 15424m^{11} + 1534\ 51991\ 28714m^{10}$
$- 21082\ 46511\ 31016m^9 + 2\ 41589\ 09549\ 02412m^8 - 22\ 66676\ 79029\ 38016m^7$
$+ 170\ 18415\ 79411\ 47951m^6 - 993\ 25640\ 63010\ 43992m^5 + 4337\ 61948\ 92400$
$21656m^4 - 13446\ 56234\ 96209\ 50976m^3 + 27358\ 29074\ 17676\ 05954m^2$
$- 31995\ 53353\ 91498\ 24280m + 15997\ 76676\ 95749\ 12140)$

$94710S_{40}(n) = (2n+1)m(1155m^{19} - 73150m^{18} + 30\ 50355m^{17} - 1052\ 57625m^{16}$
$+ 31722\ 24990m^{15} - 8\ 42809\ 97240m^{14} + 196\ 70127\ 83780m^{13} - 4001\ 39088$
$10260m^{12} + 70247\ 20969\ 95669m^{11} - 10\ 51824\ 46405\ 30198m^{10} + 132\ 46511$

$41512\ 08839m^9 - 1379\ 95296\ 74378\ 24065m^8 + 11652\ 49567\ 24773\ 24885m^7 - 77767\ 10538\ 62890\ 93368m^6 + 3\ 97142\ 37253\ 61643\ 26456m^5 - 14\ 86584\ 19698\ 23741\ 09528m^4 + 38\ 40325\ 92264\ 43234\ 42995m^3 - 62\ 50802\ 27295\ 42962\ 51670m^2 + 54\ 82737\ 08842\ 54315\ 63071m - 18\ 27579\ 02947\ 51438\ 54357)$

$5\ 68260S_{41}(n) = m^2(13530m^{19} - 8\ 99745m^{18} + 394\ 94070m^{17} - 14385\ 20875m^{16} + 4\ 59039\ 61620m^{15} - 129\ 58203\ 32565m^{14} + 3225\ 90096\ 53992m^{13} - 70310\ 15405\ 23140m^{12} + 13\ 29293\ 35277\ 64198m^{11} - 215\ 62401\ 51308\ 69059m^{10} + 2962\ 40164\ 37452\ 15854m^9 - 33946\ 84299\ 89704\ 71999m^8 + 3\ 18501\ 54838\ 10468\ 80190m^7 - 23\ 91338\ 49062\ 83896\ 21066m^6 + 139\ 56717\ 66341\ 37749\ 01168m^5 - 609\ 49952\ 07627\ 73384\ 90648m^4 + 1889\ 44035\ 39410\ 07133\ 95354m^3 - 3844\ 24339\ 78668\ 92194\ 77705m^2 + 4495\ 84441\ 25088\ 53881\ 71822m - 2247\ 92220\ 62544\ 26940\ 85911)$

$99330S_{42}(n) = (2n+1)m(1155m^{20} - 80850m^{19} + 37\ 37965m^{18} - 1436\ 73915m^{17} + 48512\ 62647m^{16} - 14\ 53892\ 27704m^{15} + 385\ 73317\ 22748m^{14} - 8999\ 40381\ 44220m^{13} + 1\ 83054\ 63278\ 01177m^{12} - 32\ 13585\ 84132\ 24202m^{11} + 481\ 17377\ 36514\ 40233m^{10} - 6059\ 81946\ 26056\ 39731m^9 + 63128\ 04367\ 28905\ 15550m^8 - 5\ 33061\ 07158\ 85522\ 78464m^7 + 35\ 57574\ 02290\ 12938\ 15112m^6 - 181\ 67879\ 28661\ 55255\ 25912m^5 + 680\ 06045\ 40812\ 83082\ 16603m^4 - 1756\ 81525\ 18219\ 47943\ 72890m^3 + 2859\ 52416\ 28292\ 06413\ 18593m^2 - 2508\ 16111\ 24648\ 16824\ 44015m + 836\ 05370\ 41549\ 38941\ 48005)$

$9240S_{43}(n) = m^2(210m^{20} - 15400m^{19} + 7\ 47593m^{18} - 302\ 47140m^{17} + 10780\ 58366m^{16} - 3\ 42092\ 30048m^{15} + 96\ 43329\ 30687m^{14} - 2399\ 84101\ 71792m^{13} + 52301\ 32365\ 14622m^{12} - 9\ 88795\ 64348\ 38216m^{11} + 160\ 39125\ 78838\ 13411m^{10} - 2203\ 57071\ 36747\ 78084m^9 + 25251\ 21746\ 91562\ 06220m^8 - 2\ 36916\ 03181\ 71343\ 45984m^7 + 17\ 78787\ 01145\ 06469\ 07556m^6 - 103\ 81645\ 30663\ 74431\ 57664m^5 + 453\ 37363\ 60541\ 88721\ 44402m^4 - 1405\ 45220\ 14575\ 58354\ 98312m^3 + 2859\ 52416\ 28292\ 06413\ 18593m^2 - 3344\ 21481\ 66197\ 55765\ 92020m + 1672\ 10740\ 83098\ 77882\ 96010)$

$2\ 17350S_{44}(n) = (2n+1)m(2415m^{21} - 1\ 85955m^{20} + 94\ 83705m^{19} - 4037\ 88000m^{18} + 1\ 51819\ 24065m^{17} - 50\ 97118\ 24755m^{16} + 1525\ 37830\ 95180m^{15} - 40455\ 50815\ 69800m^{14} + 9\ 43770\ 73451\ 06565m^{13} - 191\ 96597\ 80164\ 63805m^{12} + 3370\ 01019\ 24792\ 18197m^{11} - 50459\ 47415\ 13727\ 37724m^{10} + 6\ 35477\ 72133\ 90898\ 46492m^9 - 66\ 20075\ 53010\ 85512\ 25220m^8 + 559\ 00742\ 25359\ 98973\ 63160m^7 - 3730\ 73627\ 67995\ 17615\ 69584m^6 + 19052\ 18720\ 20646\ 90384\ 78583m^5 - 71316\ 18871\ 45891\ 38168\ 92019m^4 + 1\ 84232\ 69178\ 33985\ 73805\ 11325m^3 - 2\ 99870\ 93588\ 10743\ 93781\ 99000m^2 + 2\ 63024\ 39752\ 43946\ 79020\ 96735m - 87674\ 79917\ 47982\ 26340\ 32245)$

$$9660S_{45}(n)=m^2(210m^{21}-16905m^{20}+9\ 03210m^{19}-403\ 78800m^{18}+15980\ 97270m^{17}-5\ 66346\ 47195m^{16}+179\ 45627\ 17080m^{15}-5056\ 93851\ 96225m^{14}+1\ 25836\ 09793\ 47542m^{13}-27\ 42371\ 11452\ 09115m^{12}+518\ 46310\ 65352\ 64338m^{11}-8409\ 91235\ 85621\ 22954m^{10}+1\ 15541\ 40387\ 98345\ 17544m^{9}-13\ 24015\ 10602\ 17102\ 45044m^{8}+124\ 22387\ 16746\ 66438\ 58480m^{7}-932\ 68406\ 91998\ 79403\ 92396m^{6}+5443\ 48205\ 77327\ 68681\ 36738m^{5}-23772\ 06290\ 48630\ 46056\ 30673m^{4}+73693\ 07671\ 33594\ 29522\ 04530m^{3}-1\ 49935\ 46794\ 05371\ 96890\ 99500m^{2}+1\ 75349\ 59834\ 95964\ 52680\ 64490m-87674\ 79917\ 47982\ 26340\ 32245)$$

$$9870S_{46}(n)=(2n+1)m(105m^{22}-8855m^{21}+4\ 95880m^{20}-232\ 70940m^{19}+9689\ 22955m^{18}-3\ 62194\ 31815m^{17}+121\ 42342\ 49649m^{16}-3632\ 42891\ 88888m^{15}+96329\ 28170\ 38011m^{14}-22\ 47178\ 99324\ 20665m^{13}+457\ 08094\ 56705\ 79874m^{12}-8024\ 15866\ 29227\ 46344m^{11}+1\ 20146\ 43231\ 10281\ 58276m^{10}-15\ 13102\ 87472\ 48021\ 72812m^{9}+157\ 62716\ 46363\ 63232\ 38860m^{8}-1331\ 02340\ 39447\ 52972\ 82128m^{7}+8883\ 06146\ 48332\ 71726\ 89457m^{6}-45364\ 16872\ 23998\ 48720\ 56455m^{5}+1\ 69807\ 25535\ 73769\ 21811\ 79620m^{4}-4\ 38666\ 84833\ 28962\ 70163\ 09700m^{3}+7\ 14007\ 03684\ 01870\ 25474\ 47845m^{2}-6\ 26273\ 66717\ 36077\ 71441\ 85905m+2\ 08757\ 88905\ 78692\ 57147\ 28635)$$

$$10080S_{47}(n)=m^2(210m^{22}-18480m^{21}+10\ 81920m^{20}-531\ 90720m^{19}+23254\ 15092m^{18}-9\ 15017\ 22480m^{17}+323\ 79579\ 99064m^{16}-10256\ 26988\ 86272m^{15}+2\ 88987\ 84511\ 14033m^{14}-71\ 90972\ 77837\ 46128m^{13}+1567\ 13467\ 08705\ 59568m^{12}-29627\ 66275\ 54070\ 63424m^{11}+4\ 80585\ 72924\ 41126\ 33104m^{10}-66\ 02630\ 72607\ 18640\ 26816m^{9}+756\ 61039\ 02545\ 43515\ 46528m^{8}-7098\ 79148\ 77053\ 49188\ 38016m^{7}+53298\ 36878\ 89996\ 30361\ 36742m^{6}-3\ 11068\ 58552\ 50275\ 34083\ 87120m^{5}+13\ 58458\ 04285\ 90153\ 74494\ 36960m^{4}-42\ 11201\ 74399\ 58041\ 93565\ 73120m^{3}+85\ 68084\ 44208\ 22443\ 05693\ 74140m^{2}-100\ 20378\ 67477\ 77243\ 43069\ 74480m+50\ 10189\ 33738\ 88621\ 71534\ 87240)$$

$$3\ 24870S_{48}(n)=(2n+1)m(3315m^{23}-3\ 04980m^{22}+186\ 74942m^{21}-9615\ 91774m^{20}+4\ 41172\ 02129m^{19}-182\ 60962\ 65630m^{18}+6815\ 96872\ 10797m^{17}-2\ 28416\ 24220\ 97959m^{16}+68\ 32534\ 43189\ 25369m^{15}-1811\ 89603\ 85573\ 88160m^{14}+42267\ 85821\ 40537\ 32312m^{13}-8\ 59736\ 00551\ 13236\ 92024m^{12}+150\ 92853\ 82424\ 57890\ 28164m^{11}-2259\ 86608\ 24182\ 87634\ 30920m^{10}+28460\ 35239\ 59614\ 28152\ 43308m^{9}-2\ 96485\ 10409\ 05207\ 75682\ 85364m^{8}+25\ 03557\ 13098\ 22680\ 92652\ 38475m^{7}-167\ 08385\ 29516\ 91913\ 32298\ 92748m^{6}+853\ 26664\ 97573\ 38893\ 71880\ 08706m^{5}-3193\ 94958\ 53708\ 69283\ 48975\ 75858m^{4}+8251\ 00079\ 14370\ 36990\ 36283\ 15215m^{3}-13429\ 94723\ 31828\ 24539\ 19573\ 41530m^{2}+11779\ 74707\ 48954\ 17141\ 12316\ 78487m-3926\ 58235\ 82984\ 72380\ 37438\ 92829)$$

$66300S_{49}(n)=m^2(1326m^{23} - 1\,27075m^{22} + 81\,19540m^{21} - 4370\,87170m^{20} + 2\,10081\,91490m^{19} - 91\,30481\,32815m^{18} + 3587\,35195\,84630m^{17} - 1\,26897\,91233\,87755m^{16} + 40\,19137\,90111\,32570m^{15} - 1132\,43502\,40983\,67600m^{14} + 28178\,57214\,27024\,88208m^{13} - 6\,14097\,14679\,38026\,37160m^{12} + 116\,09887\,55711\,21454\,06280m^{11} - 1883\,22173\,53485\,73028\,59100m^{10} + 25873\,04763\,26922\,07411\,30280m^9 - 2\,96485\,10409\,05207\,75682\,85364m^8 + 27\,81730\,14553\,58534\,36280\,42750m^7 - 208\,85481\,61896\,14891\,65373\,65935m^6 + 1218\,95235\,67961\,98419\,59828\,69580m^5 - 5323\,24930\,89514\,48805\,81626\,26430m^4 + 16502\,00158\,28740\,73980\,72566\,30430m^3 - 33574\,86808\,29570\,61347\,98933\,53825m^2 + 39265\,82358\,29847\,23803\,74389\,28290m - 19632\,91179\,14923\,61901\,87194\,64145)$

$43758S_{50}(n)=(2n+1)m(429m^{24} - 42900m^{23} + 28\,61430m^{22} - 1609\,96550m^{21} + 81024\,67945m^{20} - 36\,95212\,22070m^{19} + 1527\,20073\,17105m^{18} - 56981\,72866\,32715m^{17} + 19\,09388\,88360\,96624m^{16} - 571\,13563\,49312\,08688m^{15} + 15145\,66211\,07755\,07256m^{14} - 3\,53317\,08234\,23641\,31640m^{13} + 71\,86532\,00995\,35906\,71300m^{12} - 1261\,61132\,85559\,37534\,61544m^{11} + 18890\,21538\,74242\,19456\,26908m^{10} - 2\,37900\,01914\,87821\,90751\,26500m^9 + 24\,78318\,28932\,02890\,86967\,54625m^8 - 209\,27228\,17734\,15218\,89412\,90220m^7 + 1396\,65353\,42820\,62319\,92988\,33930m^6 - 7132\,45392\,05912\,40262\,74791\,73450m^5 + 26698\,21707\,99269\,03770\,01974\,47495m^4 - 68970\,09622\,98454\,19074\,09704\,39650m^3 + 1\,12260\,89736\,84198\,25844\,28118\,85155m^2 - 98466\,87812\,24507\,42029\,46177\,97225m + 32822\,29270\,74835\,80676\,48725\,99075)$

$3432S_{51}(n)=m^2(66m^{24} - 6864m^{23} + 4\,76905m^{22} - 279\,99400m^{21} + 14731\,75990m^{20} - 7\,03849\,94680m^{19} + 305\,44014\,63421m^{18} - 11996\,15340\,27940m^{17} + 4\,24308\,64080\,21472m^{16} - 134\,38485\,52779\,31456m^{15} + 3786\,41552\,76938\,76814m^{14} - 94217\,88862\,46304\,35104m^{13} + 20\,53294\,85998\,67401\,91800m^{12} - 388\,18810\,10941\,34626\,03552m^{11} + 6296\,73846\,24747\,39818\,75636m^{10} - 86509\,09787\,22844\,33000\,46000m^9 + 9\,91327\,31572\,81156\,34787\,01850m^8 - 93\,00990\,30104\,06763\,95294\,62320m^7 + 698\,32676\,71410\,31159\,96494\,16965m^6 - 4075\,68795\,46235\,65864\,42738\,13400m^5 + 17798\,81138\,66179\,35846\,67982\,98330m^4 - 55176\,07698\,38763\,35259\,27763\,51720m^3 + 1\,12260\,89736\,84198\,25844\,28118\,85155m^2 - 1\,31289\,17082\,99343\,22705\,94903\,96300m + 65644\,58541\,49671\,61352\,97451\,98150)$

$17490S_{52}(n)=(2n+1)m(165m^{25} - 17875m^{24} + 12\,94150m^{23} - 792\,64900m^{22} + 43579\,41445m^{21} - 21\,79988\,17465m^{20} + 992\,67605\,24065m^{19} - 41010\,69452\,29300m^{18} + 15\,30013\,41903\,38470m^{17} - 512\,67698\,66296\,84690m^{16} + 15335\,08520\,62360\,90840m^{15} - 4\,06662\,93304\,55942\,38000m^{14} + 94\,86604\,86020\,98286\,96500m^{13} - 1929\,59206\,42662\,04665\,12820m^{12} + 33874\,40778\,78253\,34252\,34204m^{11} - 5\,07204\,43107\,84625\,81655\,43488m^{10} + 63\,87642\,55737\,65174\,19446\,89729m^9$

$- 665\ 43127\ 77235\ 64320\ 77566\ 95055m^8 + 5618\ 98455\ 27025\ 09890\ 52031\ 10330m^7 - 37500\ 30614\ 65621\ 67773\ 24728\ 47788m^6 + 1\ 91507\ 19847\ 99880\ 06683\ 95714\ 99691m^5 - 7\ 16850\ 16325\ 48707\ 67446\ 01437\ 36543m^4 + 18\ 51854\ 92327\ 25337\ 60331\ 36593\ 47155m^3 - 30\ 14217\ 85450\ 16236\ 11314\ 42566\ 00580m^2 + 26\ 43846\ 86984\ 71168\ 59248\ 15247\ 31149m - 8\ 81282\ 28994\ 90389\ 53082\ 71749\ 10383)$

$5940S_{53}(n) = m^2(110m^{25} - 12375m^{24} + 9\ 31788m^{23} - 594\ 48675m^{22} + 34105\ 62870m^{21} - 17\ 83626\ 68835m^{20} + 850\ 86518\ 77770m^{19} - 36909\ 62507\ 06370m^{18} + 14\ 49486\ 39697\ 94340m^{17} - 512\ 67698\ 66296\ 84690m^{16} + 16237\ 14904\ 18970\ 37360m^{15} - 4\ 57495\ 79967\ 62935\ 17750m^{14} + 113\ 83925\ 83225\ 17944\ 35800m^{13} - 2480\ 90408\ 26279\ 77426\ 59340m^{12} + 46903\ 02616\ 77581\ 55118\ 62744m^{11} - 7\ 60806\ 64661\ 76938\ 72483\ 15232m^{10} + 104\ 52506\ 00297\ 97557\ 77276\ 74102m^9 - 1197\ 77629\ 99024\ 15777\ 39620\ 51099m^8 + 11237\ 96910\ 54050\ 19781\ 04062\ 20660m^7 - 84375\ 68882\ 97648\ 77489\ 80639\ 07523m^6 + 4\ 92447\ 08180\ 56834\ 45758\ 74695\ 70634m^5 - 21\ 50550\ 48976\ 46123\ 02338\ 04312\ 09629m^4 + 66\ 66677\ 72378\ 11215\ 37192\ 91736\ 49758m^3 - 135\ 63980\ 34525\ 73062\ 50914\ 91547\ 02610m^2 + 158\ 63081\ 21908\ 27011\ 55488\ 91483\ 86894m - 79\ 31540\ 60954\ 13505\ 77744\ 45741\ 93447)$

$43890S_{54}(n) = (2n+1)m(399m^{26} - 46683m^{25} + 36\ 56835m^{24} - 2429\ 73900m^{23} + 1\ 45387\ 86795m^{22} - 79\ 44566\ 32905m^{21} + 3967\ 94202\ 92490m^{20} - 1\ 80613\ 01446\ 09560m^{19} + 74\ 60991\ 44466\ 68370m^{18} - 2783\ 45548\ 70233\ 90410m^{17} + 93267\ 49523\ 54762\ 70486m^{16} - 27\ 89793\ 50742\ 18819\ 72432m^{15} + 739\ 81017\ 79334\ 21950\ 23276m^{14} - 17258\ 23933\ 85786\ 95214\ 90100m^{13} + 3\ 51035\ 60725\ 32519\ 56024\ 83264m^{12} - 61\ 62506\ 29789\ 50426\ 10982\ 01200m^{11} + 922\ 71738\ 49486\ 22405\ 75330\ 51763m^{10} - 11620\ 53892\ 60833\ 03372\ 48505\ 22807m^9 + 1\ 21056\ 71216\ 62483\ 13851\ 58049\ 28195m^8 - 10\ 22217\ 94860\ 50712\ 04208\ 73453\ 95748m^7 + 68\ 22137\ 64097\ 69421\ 49713\ 66373\ 94965m^6 - 348\ 39408\ 03472\ 76501\ 41572\ 10967\ 07951m^5 + 1304\ 10948\ 18168\ 33360\ 99518\ 48575\ 68234m^4 - 3368\ 93494\ 37033\ 78819\ 22924\ 65007\ 87920m^3 + 5483\ 53098\ 84966\ 78063\ 37983\ 97292\ 51599m^2 - 4809\ 74399\ 97560\ 02299\ 53399\ 04290\ 94015m + 1603\ 24799\ 99186\ 67433\ 17799\ 68096\ 98005)$

$6384S_{55}(n) = m^2(114m^{26} - 13832m^{25} + 11\ 25180m^{24} - 777\ 51648m^{23} + 48462\ 62265m^{22} - 27\ 63327\ 41880m^{21} + 1442\ 88801\ 06360m^{20} - 68804\ 95788\ 98880m^{19} + 29\ 84396\ 57786\ 67348m^{18} - 1171\ 98125\ 76940\ 59120m^{17} + 41452\ 22010\ 46561\ 20216m^{16} - 13\ 12844\ 00349\ 26503\ 39968m^{15} + 369\ 90508\ 89667\ 10975\ 11638m^{14} - 9204\ 39431\ 39086\ 37447\ 94720m^{13} + 2\ 00591\ 77557\ 32868\ 32014\ 19008m^{12} - 37\ 92311\ 56793\ 54108\ 37527\ 39200m^{11} + 615\ 14492\ 32990\ 81603\ 83553\ 67842m^{10} - 8451\ 30103\ 71514\ 93361\ 80731\ 07496m^9 + 96845\ 36973\ 29986\ 51081$

26439 42556 m^8 − 9 08638 17653 78410 70407 76403 51776 m^7 + 68 22137 64097 69421 49713 66373 94965 m^6 − 398 16466 32540 30287 33225 26819 51944 m^5 + 1738 81264 24224 44481 32691 31434 24312 m^4 − 5390 29590 99254 06110 76679 44012 60672 m^3 + 10967 06197 69933 56126 75967 94585 03198 m^2 − 12825 98399 93493 39465 42397 44775 84040 m + 6412 99199 96746 69732 71198 72387 92020)

49590 $S_{56}(n)$ = $(2n+1)m$(435 m^{27} − 54810 m^{26} + 46 31445 m^{25} − 3327 68475 m^{24} + 2 15963 60175 m^{23} − 128 42878 23000 m^{22} + 7006 77940 68840 m^{21} − 3 49817 50964 01480 m^{20} + 159 21447 30578 62530 m^{19} − 6576 87354 56163 20100 m^{18} + 2 45360 49112 01638 15890 m^{17} − 82 21480 43370 66549 51030 m^{16} + 2459 18735 41552 94022 98940 m^{15} − 65213 84716 47861 61197 17520 m^{14} + 15 21303 96692 84752 47312 59160 m^{13} − 309 43588 65905 37475 51607 25560 m^{12} + 5432 21416 32155 49697 00334 88871 m^{11} − 81337 01132 86844 82798 91783 39722 m^{10} + 10 24343 87996 35700 60265 19303 98781 m^9 − 106 71080 14719 99096 37632 34962 37115 m^8 + 901 07929 26911 08535 84239 26097 92465 m^7 − 6013 67542 85677 34775 87563 52509 14512 m^6 + 30710 73951 74514 42031 08987 16979 22204 m^5 − 1 14956 50717 82845 59562 27703 61243 60252 m^4 + 2 96969 69421 57518 17386 21365 25039 27355 m^3 − 4 83370 12975 57121 38357 44882 19831 97630 m^2 + 4 23976 19091 25617 74880 20609 14824 12159 m − 1 41325 39697 08539 24960 06869 71608 04053)

1740 $S_{57}(n)$ = m^2(30 m^{27} − 3915 m^{26} + 3 43070 m^{25} − 255 97575 m^{24} + 17277 08814 m^{23} − 10 70239 85250 m^{22} + 609 28516 58160 m^{21} − 31801 59178 54680 m^{20} + 15 16328 31483 67860 m^{19} − 657 68735 45616 32010 m^{18} + 25827 42011 79119 80620 m^{17} − 9 13497 82596 74061 05670 m^{16} + 289 31615 93123 87532 11640 m^{15} − 8151 73089 55982 70149 64690 m^{14} + 2 02840 52892 37966 99641 67888 m^{13} − 44 20512 66557 91067 93086 75080 m^{12} + 835 72525 58793 15338 00051 52134 m^{11} − 13556 16855 47807 47133 15297 23287 m^{10} + 1 86244 34181 15581 92775 48964 36142 m^9 − 21 34216 02943 99819 27526 46992 47423 m^8 + 200 23984 28202 46341 29830 94688 42770 m^7 − 1503 41885 71419 33693 96890 88127 28628 m^6 + 8774 49700 49861 26294 59710 61994 06344 m^5 − 38318 83572 60948 53187 42567 87081 20084 m^4 + 1 18787 87768 63007 26954 48546 10015 70942 m^3 − 2 41685 06487 78560 69178 72441 09915 98815 m^2 + 2 82650 79394 17078 49920 13739 43216 08106 m − 1 41325 39697 08539 24960 06869 71608 04053)

1770 $S_{58}(n)$ = $(2n+1)m$(15 m^{28} − 2030 m^{27} + 1 84527 m^{26} − 142 94709 m^{25} + 10030 04730 m^{24} − 6 46886 76000 m^{23} + 384 07529 36760 m^{22} − 20945 86274 55240 m^{21} + 10 45629 99483 45570 m^{20} − 475 89190 50981 89580 m^{19} + 19658 14903 92150 08310 m^{18} − 7 33376 71032 19195 32330 m^{17} + 245 73801 72469 82998 72410 m^{16} − 7350 44953 23257 06815 26480 m^{15} + 1 94922 55482 29228 48191 98856 m^{14}

$- 45\ 47139\ 40675\ 01921\ 29568\ 93320m^{13} + 924\ 89610\ 39597\ 15404\ 44759\ 34859m^{12} - 16236\ 75188\ 16708\ 07622\ 27688\ 16542m^{11} + 2\ 43114\ 28672\ 74344\ 14461\ 33068\ 59367m^{10} - 30\ 61738\ 16407\ 04931\ 49598\ 48483\ 58357m^{9} + 318\ 95590\ 90210\ 94301\ 35684\ 49248\ 76500m^{8} - 2693\ 30340\ 44871\ 74607\ 54050\ 65963\ 19928m^{7} + 17974\ 72501\ 76692\ 94941\ 38511\ 06178\ 51124m^{6} - 91793\ 63011\ 39723\ 61640\ 30041\ 60809\ 24724m^{5} + 3\ 43602\ 11655\ 34353\ 20916\ 20203\ 16171\ 92451m^{4} - 8\ 87634\ 96725\ 33713\ 71451\ 43057\ 81439\ 05130m^{3} + 14\ 44781\ 19368\ 38749\ 02220\ 98775\ 55393\ 21341m^{2} - 12\ 67254\ 20023\ 32006\ 27930\ 70163\ 99105\ 40315m + 4\ 22418\ 06674\ 44002\ 09310\ 23387\ 99701\ 80105)$

$360S_{59}(n) = m^2(6m^{28} - 840m^{27} + 79083m^{26} - 63\ 53204m^{25} + 4629\ 25260m^{24} - 3\ 10505\ 64480m^{23} + 192\ 03764\ 68380m^{22} - 10928\ 27621\ 50560m^{21} + 5\ 70343\ 63354\ 61220m^{20} - 271\ 93823\ 14846\ 79760m^{19} + 11794\ 88942\ 35290\ 04986m^{18} - 4\ 63185\ 29072\ 96333\ 88840m^{17} + 163\ 82534\ 48313\ 21999\ 14940m^{16} - 5188\ 55261\ 10534\ 40104\ 89280m^{15} + 1\ 46191\ 91611\ 71921\ 36143\ 99142m^{14} - 36\ 37711\ 52540\ 01537\ 03655\ 14656m^{13} + 792\ 76808\ 91083\ 27489\ 52650\ 87022m^{12} - 14987\ 77096\ 76961\ 30112\ 87096\ 76808m^{11} + 2\ 43114\ 28672\ 74344\ 14461\ 33068\ 59367m^{10} - 33\ 40077\ 99716\ 78107\ 08652\ 89254\ 81844m^{9} + 382\ 74709\ 08253\ 13161\ 62821\ 39098\ 51800m^{8} - 3591\ 07120\ 59828\ 99476\ 72067\ 54617\ 59904m^{7} + 26962\ 08752\ 65039\ 42412\ 07766\ 59267\ 76686m^{6} - 1\ 57360\ 50876\ 68097\ 62811\ 94357\ 04244\ 42384m^{5} + 6\ 87204\ 23310\ 68706\ 41832\ 40406\ 32343\ 84902m^{4} - 21\ 30323\ 92140\ 80912\ 91483\ 43338\ 75453\ 72312m^{3} + 43\ 34343\ 58105\ 16247\ 06662\ 96326\ 66179\ 64023m^{2} - 50\ 69016\ 80093\ 28025\ 11722\ 80655\ 96421\ 61260m + 25\ 34508\ 40046\ 64012\ 55861\ 40327\ 98210\ 80630)$

$567\ 86730S_{60}(n) = (2n+1)m(4\ 65465m^{29} - 674\ 92425m^{28} + 65827\ 61185m^{27} - 54\ 83084\ 60700m^{26} + 4147\ 30152\ 83700m^{25} - 2\ 89171\ 53788\ 52300m^{24} + 186\ 20009\ 74746\ 19800m^{23} - 11050\ 75092\ 54156\ 30000m^{22} + 6\ 02600\ 93126\ 24686\ 71390m^{21} - 300\ 81446\ 22272\ 21987\ 05830m^{20} + 13690\ 72079\ 87369\ 96958\ 24330m^{19} - 5\ 65535\ 61312\ 43597\ 90908\ 99600m^{18} + 210\ 98145\ 68466\ 07092\ 90362\ 23650m^{17} - 7069\ 51276\ 06816\ 68462\ 16852\ 93830m^{16} + 2\ 11461\ 36108\ 30452\ 32617\ 32830\ 52920m^{15} - 56\ 07628\ 26013\ 93522\ 73147\ 61216\ 51600m^{14} + 1308\ 14350\ 49489\ 38954\ 13514\ 26037\ 16845m^{13} - 26607\ 86755\ 20361\ 09567\ 25951\ 74197\ 38805m^{12} + 4\ 67106\ 89088\ 13094\ 44711\ 61017\ 79625\ 38089m^{11} - 69\ 94031\ 77603\ 09659\ 20482\ 03138\ 16699\ 19828m^{10} + 880\ 81594\ 45317\ 69024\ 82458\ 56250\ 86854\ 02074m^{9} - 9175\ 88099\ 35138\ 55760\ 99235\ 95006\ 62549\ 00870m^{8} + 77482\ 28146\ 90648\ 06019\ 58636\ 74175\ 31627\ 58940m^{7} - 5\ 17105\ 75972\ 52761\ 12836\ 66082\ 62518\ 30381\ 84408m^{6} + 26\ 40764\ 45071\ 43002\ 53626\ 75706\ 79520\ 27548\ 05741m^{5} - 98\ 84915\ 25455\ 40894\ 61960\ 40581\ 03145\ 24197\ 27473m^{4} + 255\ 35920\ 78037\ 72219\ 13377\ 78828\ 26199\ 91547\ 74435m^{3} - 415\ 64178\ 37058\ 81115\ 43884\ 70747\ 89186\ 05690\ 79360m^{2} + 364\ 56994\ 21451\ 26671\ 61209\ 14982$

23946 07381 24473m − 121 52331 40483 75557 20403 04994 07982 02460 41491)

18 61860$S_{61}(n) = m^2$(30030m^{29} − 44 99495m^{28} + 4539 83530m^{27} − 3 91648 90050m^{26} + 307 20752 06200m^{25} − 22243 96445 27100m^{24} + 14 89600 77979 69584m^{23} − 920 89591 04513 02500m^{22} + 52400 08097 93451 01860m^{21} − 27 34676 92933 83817 00530m^{20} + 1303 87817 13082 85424 59460m^{19} − 56553 56131 24359 79090 89960m^{18} + 22 20857 44049 06009 77932 86700m^{17} − 785 50141 78535 18718 01872 54870m^{16} + 24877 80718 62406 15602 03862 41520m^{15} − 7 00953 53251 74190 34143 45152 06450m^{14} + 174 41913 39931 91860 55135 23471 62246m^{13} − 3801 12393 60051 58509 60850 24885 34115m^{12} + 71862 59859 71245 29955 63233 50711 59706m^{11} − 11 65671 96267 18276 53413 67189 69449 86638m^{10} + 160 14835 35512 30731 78628 82954 70337 09468m^9 − 1835 17619 87027 71152 19847 19001 32509 80174m^8 + 17218 28477 09032 90226 57474 83150 07028 35320m^7 − 1 29276 43993 13190 28209 16520 65629 57595 46102m^6 + 7 54504 12877 55143 58179 07344 79862 93585 15926m^5 − 32 94971 75151 80298 20653 46860 34381 74732 42491m^4 + 102 14368 31215 08887 65351 11531 30479 96619 09774m^3 − 207 82089 18529 40557 71942 35373 94593 02845 39680m^2 + 243 04662 80967 51114 40806 09988 15964 04920 82982m − 121 52331 40483 75557 20403 04994 07982 02460 41491)

References

[1] T. Origuchi, H. Kiriyama, and Y. Matsuoka, *A table of the explicit formulas for the sums of powers* $S_p(n) = \sum_{k=1}^{n} k^p$ *for* p=1(1) 61, Rep. Fac. Sci., Kagoshima Univ., (Math., Phys. & Chem.) No.20 (1987), 11−31.

[2] Problem No.738 proposed by S.Rienstra, Nieuw Arch. Wisk. 3 (1985), 313.

[3] Solution to the above problem by A. A. Jagers, *ibid.* 5 (1987), 103−104.