Calcuclation of Light Distribution on a Focal Plane of a Contact Lens Carved Fresnel Zone Plate

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Abstract

A bifocal lens function has been difficult to achieve in a contact lens. As a solution to the problem, a diffraction bifocal contact lens was developed by Freeman in 1982.

This report shows the results of the calculation of light distribution condensed by a diffraction type contact lens. This lens gives near vision focal length by first order subsidiary light, and far vision focal length by 0th order subsidiary light of the refraction power of the lens.

The method followed in this investigation is described as follows.

First, diffraction light intensity was theoretically calculated. In the 2nd step, the shape of the grating was measured using a special micrometer. In the 3rd step, light distribution on the focal plane was calculated according to the dimensions measured and compared with the measurement of actual lens light distribution.

1. Introduction

A diffraction bifocal contact lens was developed by Freeman in 1982[1] and has been sold as Diffrax lens[2] (commodity name) in the United Kingdom. Recently, this type lens has been applied to an intraocular implant in United State[3].

The diffraction grating carved inner surface of the lens is called the Fresnel zone plate. This lens give near vision focal length by 1st order subsidiary light, and far vision focal length is given by 0th order subsidiary light of refraction power of the lens. This report shows the results of calculation, especially theoretical speculation on light distribution on the focal plane into a Diffrax lens. The shape of the grating was measured using a special micrometer named Tarysurf, manufactured by Rank Taylor Hobson Co., Ltd. in the United Kingdom.

According to the dimensions measured, and estimated parameters, light distribution on the focal plane was calculated and compared with actual light distribution of the same sample.

2. Theory

The formula concerning the calculation of light intensity value is deduced from the Fresnel-Kirchhof diffraction formula. A domain of integration is shown in Fig.1. In the figure, P_1 is a point on a diffraction plane: S_A. $\hat{n}$ is normal vector at P_1. r_{01} is a radius between P_1 and observing point: p_0, and r_{02} is a radius between light source: p_2 and P_1. u is a function and u_p is the value of the function. R is a radius between P_0 and an outer surface: S_R.

$k = 2\pi/\lambda$, λ: Light wave length.

u_p is presented as

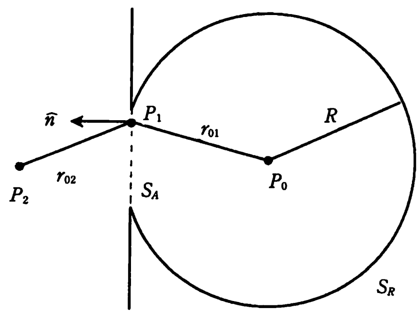


Fig.1 Integration domain.

$$u_p = \frac{1}{4\pi}\int_{s_A+s_R}\left[\frac{e^{ikr_{01}}}{r_{01}}\nabla u - u\nabla\left(\frac{e^{ikr_{02}}}{r_{01}}\right)\right]\vec{n}\,ds \tag{1}$$

If R is sufficiently large, the integration on S_R is neglected. Then, the Cartesian coordinates are taken in both the diffraction and the observation planes, where P_0 is denoted by (x_0, y_0) and p_1 by (x_i, y_i). g (x_i, y_i) is the Fresnel zone plate function. r_{01} is represented by Fresnel approximation as

$$r_{01} = z_i + \frac{(x_0 - x_i)^2 + (y_0 - y_i)^2}{2z_i} \tag{2}$$

Therefore, u_p is expressed by $u(x_0, y_0)$ as follows.

$$u(x_0, y_0) = \frac{1}{i\lambda z_i}e^{ikz_i}\int\int g(x_i, y_i)\exp\, ik\left(\frac{(x_0-x_i)^2+(y_0-y_i)^2}{2z_i}\right)dx_i\,dy_i \tag{3}$$

If the cylindrical coordinates are taken in these planes (Fig.2), (x_0, y_0) is changed to (ℓ, φ), and (x_i, y_i) to (r, θ). we have then

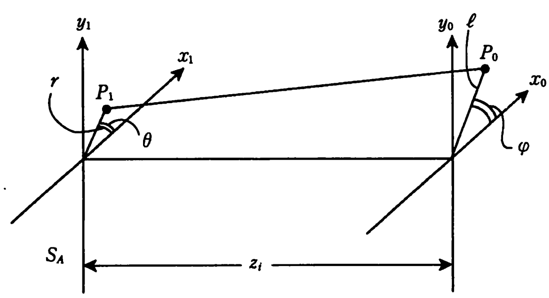


Fig.2 Cylindrical coordinates.

$$dx_i \times dy_i = r \times d\theta \times dr \tag{4}$$

f is the focal length of Fresnel zone plate.

$$(k \times \ell/z_i) = 2\pi\rho \text{ and } g(r, \theta) = g_n(r) \tag{5}$$

because of the circular symmetry of the zone plate. Equation (3) becomes

$$u(\rho) = \frac{2\pi}{i\lambda z}\int g(r)e^{izr^2}J_0(Rr)rdr. \tag{6}$$

$$\text{Where, } z = \pi/\lambda z_i \text{ and } R = (2\pi/\lambda z_i)\rho \tag{7}$$

When a convex lens is added in the optical system, $u(\rho)$ is changed to

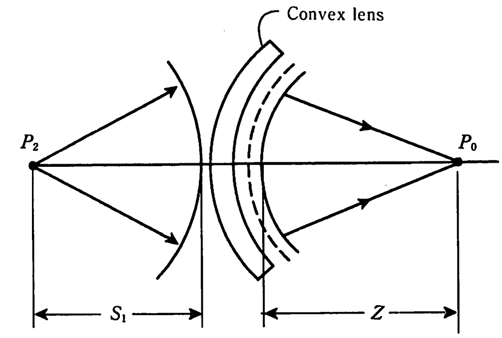


Fig.3 Addition of convex lens.

$$u(\rho) = \frac{2\pi}{i\lambda z}\int e^{-i\frac{kr^2}{2s_\ell}} g(r) e^{-\frac{kr^2}{2f_\ell}} e^{izr^2} J_0(Rr) r dr. \tag{8}$$

Where s_ℓ and z is presented as shown in Fig.3
and f_ℓ is focal length of the convex lens.
If $(r^2/2\lambda f) = \xi$, $u(\rho)$ becomes

$$u(\rho) = \frac{2\pi}{i\lambda z}\int g(\xi)\cdot e^{i2\pi\left(\frac{f}{z}-\frac{f}{f_l}-\frac{f}{s_l}\right)\xi} J_0\left(4\pi\frac{\rho\sqrt{\xi}}{z\sqrt{2\lambda f}}\right)\cdot d\xi \tag{9}$$

If the Fresnel zone plate is formed as shown in Fig.4, $g(\xi)$ is presented as follows.

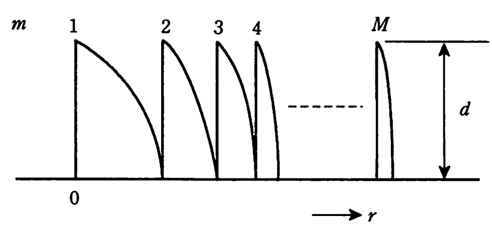


Fig.4 Fresnel zone plate applied to Diffrax lens.

$$g(\xi) = \sum_{M=0}^{M-1} G(\xi - m) \qquad \text{when } 0 \leq \xi - m < 1$$

$$x = \xi - m \quad \text{and} \quad G(x) = e^{i2\pi p(1-x)} \tag{10}$$

Where m is the grating number with the maximum M,

$$n = \left[\frac{f}{z} - \frac{f}{f_l} - \frac{f}{s_l}\right], \quad b_n = \frac{f}{z}, \quad \text{and } P = \Delta nd/\lambda \tag{11}$$

Δn is the refractive index difference between contact lens material and tear solution.
Then, $u(\rho)$ is expressed as

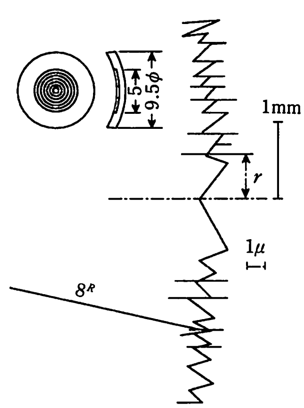
$$u(\rho) = \frac{2\pi}{i\lambda z}\lambda f e^{2\pi ip}\sum_{m=0}^{M-1} e^{2\pi inm}\int_0^1 e^{2\pi i(n-p)\xi} J_0(4\pi b_n \rho\sqrt{m+\xi})d\xi \tag{12}$$

If $u^*(\rho)$ is the conjugate of $u(\rho)$, the light energy $I(\rho)$ is presented as follows.

$$I(\rho) = u(\rho)^2 = u \times u^* \tag{13}$$

3. Calculation results and discussion

In this chapter, light distribution on a focal plane is speculated. First, light distribution was calculated according to an idealized Fresnel zone plate as shown in Fig.4. Fig.5 is a cross-section of a Diffrax lens sample. In the figure, horizontal dimension is expanded 50 times the vertical dimension. Every radius of the grating is shown in a column in the figure.



Radius, Focal length

m	r(mm)	f(mm)
1	0.72	439
2	0.85	323
3	1.20	430
4	1.48	490
5	1.64	479
6	1.80	483
7	1.95	486
8	2.11	498
9	2.22	490
10	2.32	482
Avg.		480

Notice:
Wave length: 558nm

Fig.5 Cross-section of a Diffrax lens.

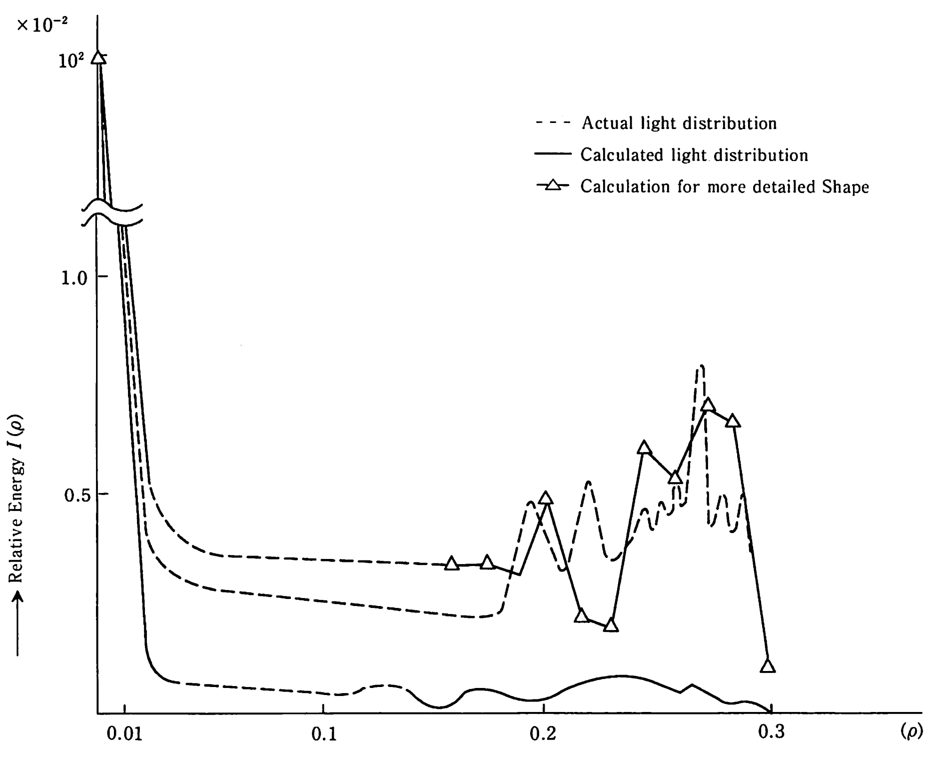


Fig.6 Light distribution on the 1st order diffraction light focus.

From the radii, focal length is estimated as 480 millimeters. The convex lens focal length is estimated as 40 millimeters. Numbers of grating is 10. The material of the lens is PMMA (plastic name abreviation), whose refractive index is estimated at 1.48. The inner surface of the lens is filled with tear solution. The refractive index of the solution is estimated as 1.33. Then, Δn becomes 0.15. Average of the thickness of the grating is 1.6 micro-meters. When light wave-length is 558 nanometers, the light distribution of $I(\rho)$ was calculated. The results of the calculation for the idealized crosssectional figure on the lst order diffraction light is shown as a thin solid line in Fig.6. The thin line shows stray light existed outside of the focal point was small amount. On the other hand, actual light distribution on the lst order diffraction light focal length, is shown as a dashed line in the same figure. There is a larger amount of stray light than the calculated results.

Therefore, more accurate simulation was tried. Improved calculation was carried out for each triangle cross-sectional shape shown in Fig.5, according to the following formuli,

$$u_m(\rho) = u_{m1}(\rho) + u_{m2}(\rho) \tag{14}$$

$$u_{m1} = \frac{2\pi}{i\lambda z}\lambda f e^{2\pi i(p_1 \cdot m2/(m2-m1))} \int_{m1}^{m2} e^{2\pi i\left(n-\frac{p_1}{(m2-m1)}\right)\xi} d\xi \tag{15}$$

$$u_{m2} = \frac{2\pi}{i\lambda z}\lambda f e^{-2\pi i(p_2 \cdot m3/(m3-m2))} \int_{m2}^{m3} e^{2\pi i\left(n+\frac{p_2}{(m3-m2)}\right)\xi} d\xi \tag{16}$$

$$u(\rho) = \sum_{m=0}^{M-1} u_m(\rho) \tag{17}$$

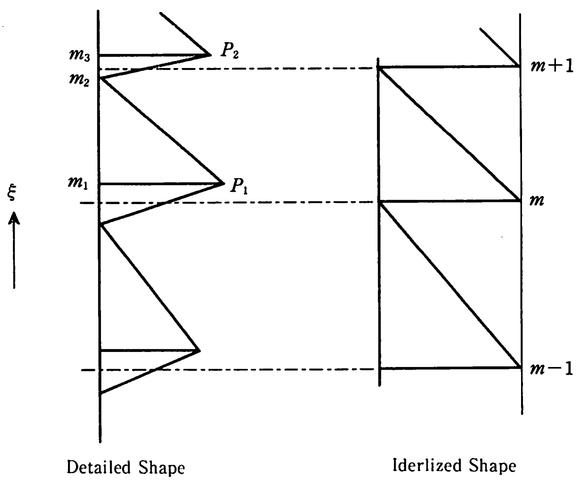


Fig.7 Cross-section of more actually detailed gratings.

In the formula, m indicates the grating numbers as shown in Fig.4 and Fig.7. m_1 is the distance from the center and p_1 is the thickness at the maximum thickness position in the m-th number grating. Further, m_2 is the distance at the minimum thickness position. Moreover, m_3 is the distance at the maximum thickness position whose thickness is p_2 in the next grating as shown in Fig.7. Function $u_{m1}(\rho)$ is for the part of the grating whose thickness is decreasing due to the distance. Further, $u_{m2}(\rho)$ is for the part of the grating whose thickness is increasing due to the distance. For m-th number grating, function $u_m(\rho)$ is obtained by formula (14) from u_{m1} and u_{m2}. For whole gratings, function $u(\rho)$ is obtained by summation of all gratings and Last, light intensity

$I(\rho)$ is obtained according to the formula (13). Subsequently, the results of the calculation is shown as a thick solid line connecting with open triangles in Fig.6. This curve is nearer to the actually measured results. It shows that if the Fresnel zone plate should be made more correctly to the idealized cross-sectional shape, the characteristics of the bifocal contact lens must be improved.

4. References

1) M.H. Freeman: UK patent No.2129157B, 5 Feb. 1986.
2) J.Stone : Optician, march 4, pp.21–34, 1988.
3) P.Percival: Indication for the multizone bifocal implant, J. Cataract Refract Surg., Vol.16, No.3, pp.193–197, 1990.