Chandrasekhar–type filter for white Gaussian plus colored observation noise

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An invariant imbedding method enables to derive Chandrasekhar-type filtering equations in the case of white Gaussian plus colored observation noise.

Key Words——Chandrasekhar-type equations ; filter ; estimation ; signal processing.

Abstract—The linear least-squares filter of Chandrasekhar-type is derived for white Gaussian plus colored observation noise. Here, it is assumed that system matrices for signal and colored noise are known in each state-space model, and that the variance of white Gaussian noise and the observed value are also given. The filtering estimate is calculated sequentially in linear continuous-time stationary systems.

1. Introduction

In stationary continuous-time systems, Chandrasekhar-type filter is designed (Kailath, 1973 ; Lindquist, 1974). Chandrasekhar-type equations consist of $2m \cdot n$ simultaneous differential equations for the Kalman gain, where n is the dimension of the system and m of the output. In the Kalman filter (1960), $n(n+1)/2$ Riccati-type nonlinear differential equations should be calculated to obtain the Kalman gain in the case of white Gaussian observation noise.

This paper, at first, clarifies that previous result by Lindquist (1974) can be obtained by another method based on the innovations approach. In detection and estimation theory (Trees, 1968), estimation problems are investigated for white Gaussian plus colored observation noise. However, the Chandrasekhar-type equations have not been derived for white Gaussian plus colored observation noise. Secondly, the Chandrasekhar-type filter for white Gaussian observation noise is extended to white Gaussian plus colored observation noise via an invariant imbedding method (Nakamori, 1990). The proposed filter calculates the estimate sequentially in linear stationary stochastic systems. The filter neces-

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sitates the information of system matrices for the signal and colored noise in each state-space model, the variance of white Gaussian noise and the observed value. The current filter might be compared with a previous filter (Nakamori, 1991) which also computes a filter gain directly by solving differential equations simultaneously. These equations contain Riccati-type nonlinear differential equations partly and can not be regarded as the Chandrasekhar-type equations strictly. Number of differential equations included in the calculation of the filtering estimate by the proposed technique is $2m^2+2m \cdot n+m+n$ which is less than $n^2+2m^2+3m \cdot n+m+n$ in the previous filter. It is considered that the current algorithm is more appropriate for fast calculation than the previous one.

2. Derivation of Chandrasekhar-type equations for white Gaussian noise

Kailath (1973) derives the Chandrasekhar-type equations from the Riccati-type nonlinear differential equations of the Kalman filter. In this section, the Chandrasekhar-type equations are obtained by a different method from the conventional derivation techniques (Kailath, 1973; Lindquist, 1974).

Let an m-dimensional observation equation be given by

$$y(t)=Hx(t)+v(t),\ z(t)=Hx(t), \tag{1}$$

where $y(t)$ is an observed value, H is an $m \times n$ observation matrix, $x(t)$ is a zero-mean signal process and $v(t)$ is white Gaussian observation noise with variance R.

$$E[v(t)v^T(s)]=R\delta(t-s) \tag{2}$$

It is assumed that $x(t)$ and $v(s)$ are uncorrelated.

$$E[x(t)v^T(s)]=0,\ 0 \leqq s,\ t<\infty \tag{3}$$

Let us assume that the filtering estimate $\hat{x}(t)$ *of* $x(t)$ is expressed by

$$\hat{x}(t)=\int_0^t g(t, s)\nu(s)ds, \tag{4}$$

where $g(t, s)$ is an impulse response function and $\nu(s)$ $(=y(s)-H\hat{x}(s))$ is called the innovations process (Kailath, 1968). The optimal impulse response function for the linear least-squares filtering estimate is given by

$$g(t, s)=E[x(t)\nu^T(s)]R^{-1} \tag{5}$$

(Kailath, 1968). If we denote the crosscovariance of $x(t)$ with $y(s)$ by $K_{xy}(t, s)$, we can rewrite (5) as

$$\begin{aligned} g(t, s)&=E[x(t)(y(s)-H\hat{x}(s))^T]R^{-1} \\ &=\{K_{xy}(t, s)-E[x(t)\hat{x}^T(s)]H^T\}R^{-1} \end{aligned}$$

$$= \{K_{xy}(t, s) - E[\hat{x}(t)\,\hat{x}^T(s)]\,H^T\}\,R^{-1}$$

$$= \{K_{xy}(t, s) - E[(\int_0^t g(t, s')\,\nu(s')\,ds')(\int_0^s g(s, s'')\,\nu(s'')\,ds'')^T]\,H^T\}\,R^{-1}, \quad (6)$$

where we used the orthogonal projection lemma (Sage and Melsa, 1971) of $x(t) - \hat{x}(t)$ with $\hat{x}(s)$. If we put $s=t$, apply the property of convolution integral for stationary process to (6) and note that the variance of the innovations process equals that of white Gaussian observation noise, we have

$$g(t, t) = (K_{xy}(t, t) - \int_0^t g(s', 0)\,R g^T(s', 0)\,ds' H^T)\,R^{-1}. \quad (7)$$

If we differentiate (7) with respect to t, we obtain

$$dg(t, t)/dt = -g(t, 0)\,R g^T(t, 0)\,H^T R^{-1}. \quad (8)$$

The initial value on the differential equation (8) at $t=0$ is $g(0, 0) = K_{xy}(0, 0)\,R^{-1}$ *from* (7). $g(t, 0)$ is expressed by $g(t, 0) = E[x(t)\,\nu^T(0)]\,R^{-1}$ and this identity coincides with that obtained by Lindquist (1974). The differential equation for $g(t, 0)$ is written as

$$dg(t, 0)/dt = (k_x - g(t, t)\,H)\,g(t, 0),\ g(0, 0) = K_{xy}(0, 0)\,R^{-1}, \quad (9)$$

where k_x denotes the system matrix of the state-space model for the signal $x(t)$. The Chandrasekhar-type equations, which consist of 2m · n differential equations (8) and (9), calculate the Kalman gain $g(t, t)$ directly. In the Kalman filter, n (n+1)/2 Riccati-type nonlinear differential equations should be calculated in evaluating the Kalman gain.

3. Filtering problems for white Gaussian plus colored noise

In this section, linear least-squares filtering problems are introduced in the presence of white Gaussian plus colored observation noise.

Let an m-dimensional observation equation be give by

$$y(t) = Hx(t) + v_c(t) + v(t) \quad (10)$$

in the case of white Gaussian plus colored observation noise, where $x(t)$ and $v(t)$ have the same statistical properties with the case of white Gaussian observation noise in section 2. It is assumed that the signal $x(\cdot)$, white Gaussian noise $v(\cdot)$ and colored noise $v_c(\cdot)$ are uncorrelated mutually as written by

$$E[x(t)\,v_c^T(s)] = 0,\ E[v_c(t)\,v^T(s)] = 0,\ 0 \leqq s,\ t < \infty, \quad (11)$$

besides (3). Let us assume that the filtering estimate $\hat{x}(t)$ is given by

$$\hat{x}(t) = \int_0^t h(t, s)\,y(s)\,ds, \quad (12)$$

where $h(t, s)$ denotes n×m impulse response function. Minimizing the mean-square value of filtering error $x(t)-\hat{x}(t)$

$$J=E\left[(x(t)-\hat{x}(t))^T(x(t)-\hat{x}(t))\right], \tag{13}$$

we obtain the Wiener-Hopf integral equation (Sage and Melsa, 1971) :

$$E\left[x(t)y^T(s)\right]=\int_0^t h(t,s')E\left[y(s')y^T(s)\right]ds'. \tag{14}$$

Let $K_c(t,s)$ denote the autocovariance of $v_c(t)$. If we substitute (10) into (14), and use (2), (3) and (11), we obtain

$$h(t,s)R=K_{xy}(t,s)-\int_0^t h(t,s')\left(HK_{xy}(s',s)+K_c(s',s)\right)ds' \tag{15}$$

(Thees, 1968).

(15) is the integral equation which the optimal impulse response function $h(t,s)$ satisfies in linear least-squares filtering problems for white Gaussian plus colored observation noise.

4. Derivation of Chandrasekhar-type filtering equations for white Gaussian plus colored observation noise

In [Theorem 1], the Chandrasekhar-type equations for linear least-squares filtering estimate are derived based on the invariant imbedding method (Nakamori, 1990).

[Theorem 1]

Let k_x and k_c be system matrices in the state-space models for the signal $x(t)$ and colored noise $v_c(t)$. Then the sequential algorithm for the filtering estimate $\hat{x}(t)$ consists of the following Chandrasekhar-type equations (16)－(21) for white Gaussian plus colored observation noise.

Filtering estimate of $x(t)$: $\hat{x}(t)$

$$d\hat{x}(t)/dt=k_x\hat{x}(t)+h(t,t)\left(y(t)-H\hat{x}(t)-e(t)\right),\ \hat{x}(0)=0 \tag{16}$$

Filtering estimate of $v_c(t)$: $e(t)$

$$de(t)/dt=k_c e(t)+\Phi(t,t)\left(y(t)-H\hat{x}(t)-e(t)\right),\ e(0)=0 \tag{17}$$

Filter gain for the filtering estimate of $x(t)$: $h(t,t)$

$$dh(t,t)/dt=-h(t,0)h^T(t,0)H^T-h(t,0)\Phi^T(t,0),\ h(0,0)=K_{xy}(0,0)R^{-1} \tag{18}$$

Filter gain for the filtering estimate of $v_c(t)$: $\Phi(t,t)$

$$d\Phi(t,t)/dt=-\Phi(t,0)h^T(t,0)H^T-\Phi(t,0)\Phi^T(t,0),\ \Phi(0,0)=K_c(0,0)R^{-1} \tag{19}$$

$$dh(t,0)/dt=k_x h(t,0)-h(t,t)\left(Hh(t,0)+\Phi(t,0)\right),\ h(0,0)=K_{xy}(0,0)R^{-1} \tag{20}$$

$$d\Phi(t,0)/dt=k_c\Phi(t,0)-\Phi(t,t)\left(Hh(t,0)+\Phi(t,0)\right),\ \Phi(0,0)=K_c(0,0)R^{-1} \tag{21}$$

Proof

If we differentiate (15) with respect to t, we obtain

$$\partial h(t, s)/\partial t R = k_x K_{xy}(t, s) - h(t, t)(HK_{xy}(t, s) + K_c(t, s)) - \int_0^t \partial h(t, s')/\partial t (HK_{xy}(s', s) + K_c(s', s))\, ds'. \quad (22)$$

Introducing a function $\Phi(t, s)$, which satisfies

$$\Phi(t, s) R = K_c(t, s) - \int_0^t \Phi(t, s')(HK_{xy}(s', s) + K_c(s', s))\, ds', \quad (23)$$

we obtain

$$\partial h(t, s)/\partial t = k_x h(t, s) - h(t, t)(Hh(t, s) + \Phi(t, s)) \quad (24)$$

form (15), (22) and (23).

Let us assume that the signal $x(t)$ and colored noise $v_c(t)$ are wide-sense stationary processes. From (24) and the differential equation, which is derived by differentiating (15) with respect to s and by using (15) and (23), we obtain a differential equation for $h(t, t)$.

$$dh(t, t)/dt = -h(t, 0) h^T(t, 0) H^T - h(t, 0)\Phi^T(t, 0) \quad (25)$$

Here, we took into consideration of wide-sense stationarity for the stochastic process $x(t)$. The initial condition on the differential equation (25) at $t=0$ is $h(0, 0) = K_{xy}(0, 0) R^{-1}$ from (15).

The function $h(t, 0)$ in (25) satisfies

$$h(t, 0) R = K_{xy}(t, 0) - \int_0^t h(t, s')(HK_{xy}(s', 0) + K_c(s', 0))\, ds' \quad (26)$$

from (15). If we differentiate (26) with respect to t, we obtain

$$\partial h(t, 0)/\partial t R = k_x K_{xy}(t, 0) - h(t, t)(HK_{xy}(t, 0) + K_c(t, 0)) - \int_0^t \partial h(t, s')/\partial t (HK_{xy}(s', 0) + K_c(s', 0))\, ds'. \quad (27)$$

A differential equation for $h(t, 0)$ becomes

$$dh(t, 0)/dt = k_x h(t, 0) - h(t, t)(Hh(t, 0) + \Phi(t, 0)) \quad (28)$$

from (15), (23) and (27).

The function $\Phi(t, 0)$ in (25) satisfies

$$\Phi(t, 0) R = K_c(t, 0) - \int_0^t \Phi(t, s')(HK_{xy}(s', 0) + K_c(s', 0))\, ds' \quad (29)$$

from (23). If we differentiate (29) with respect to t, and use (26) and (29), we obtain a differential equation for $\Phi(t, 0)$ *as*

$$d\Phi(t, 0)/dt = k_c \Phi(t, 0) - \Phi(t, t)(Hh(t, 0) + \Phi(t, 0)). \quad (30)$$

The initial condition on the differential equation $\Phi(t, 0)$ at $t=0$ is $\Phi(0, 0)=K_c(0, 0)R^{-1}$ from (29).

Now, the function $\Phi(t, t)$ in (30) is unknown. If we differentiate (23) with respect to t, and use (15) and (23), we obtain a differential equation for $\Phi(t, s)$ *as*

$$\partial\Phi(t, s)/\partial t=k_c\Phi(t, s)-\Phi(t, t)(Hh(t, s)+\Phi(t, s)). \qquad (31)$$

From (31) and the differential equation, which is derived by differentiating (23) with respect to s and by using (15), (23), (26) and (29), we obtain a differential equation for $\Phi(t, t)$.

$$d\Phi(t, t)/dt=-\Phi(t, 0)h^T(t, 0)H^T-\Phi(t, 0)\Phi^T(t, 0) \qquad (32)$$

Here, we used wide-sense stationarity for the colored noise process. The initial condition on the differential equation (32) at $t=0$ is $\Phi(0, 0)=K_c(0, 0)R^{-1}$ from (29).

If we differentiate (12) for the filtering estimate $\hat{x}(t)$ with respect to t, we obtain

$$\begin{aligned} d\hat{x}(t)/dt&=h(t, t)y(t)+\int_0^t \partial h(t, s)/\partial t y(s)\,ds \\ &=h(t, t)y(t)+\int_0^t (k_x h(t, s)-h(t, t)(Hh(t, s)+\Phi(t, s)))y(s)\,ds \end{aligned} \qquad (33)$$

from (24). If we introduce a function $e(t)$ given by

$$e(t)=\int_0^t \Phi(t, s)y(s)\,ds, \qquad (34)$$

and use (12), we obtain a differential equation for the filtering estimate.

$$d\hat{x}(t)/dt=k_x\hat{x}(t)+h(t, t)(y(t)-H\hat{x}(t)-e(t)) \qquad (35)$$

The initial condition on the differential equation (35) at $t=0$ is $\hat{x}(0)=0$ from (12).

If we differentiate (34) with respect to t, we obtain

$$de(t)/dt=\Phi(t, t)y(t)+\int_0^t \partial\Phi(t, s)/\partial t y(s)\,ds. \qquad (36)$$

If we substiute (31) into (36), and use (12) and (34), we obtain

$$de(t)/dt=k_c e(t)+\Phi(t, t)(y(t)-H\hat{x}(t)-e(t)). \qquad (37)$$

The initial condition on the differential equation (37) at $t=0$ is $e(0)=0$ from (34).

□

We readily notice that $e(t)$ represents a filtering estimate of colored noise $v_c(t)$ and $h(t, t)$ is the filter gain for $\hat{x}(t)$. Here, we should note that (16) and (17) are the innovations state-space models for the signal $x(t)$ and colored noise $v_c(t)$.

It should be pointed out that the filtering algorithm of Chandrasekhar-type in [Theorem 1] calculates the filter gain directly and does not include any Riccati-type nonlinear differential equations similarly with that in the case of white Gaussian observation noise.

5. Comparison of the present filter with previous one

Now, let us compare the present filter with that by Nakamori (1991). The filtering algorithm is summarized in [Theorem 2].

[Theorem 2]

Let k_x and k_c be the system matrices in the state-space models for the signal and colored noise. Then the filtering algorithm, which calculates the filtering estimate of $x(t)$ sequentially, consists of (38) – (47) for white Gaussian plus colored observation noise.

Filtering estimate of signal $x(t)$: $\hat{x}(t)$

$$d\hat{x}(t)/dt = k_x\hat{x}(t) + w(t, t)(y(t) - H\hat{x}(t) - f(t)),\ \hat{x}(0) = 0 \qquad (38)$$

Filtering estimate of colored noise $v_c(t)$: $f(t)$

$$df(t)/dt = k_c f(t) + J(t, t)(y(t) - H\hat{x}(t) - f(t)),\ f(0) = 0 \qquad (39)$$

Filter gain for the filtering estimate of $x(t)$: $w(t,t)$

$$w(t, t) = (K_{xy}(t, t) - Q(t))R^{-1} \qquad (40)$$

Filter gain for the filtering estimate of $v_c(t)$: $J(t, t)$

$$J(t, t) = (K_c(t, t) - S(t))R^{-1} \qquad (41)$$

$$dT(t)/dt = k_x T(t) + T(t)k_x^T + w(t, t)(K_{xy}^T(t, t) - HT(t) - U(t)),\ T(0) = 0 \qquad (42)$$

$$dU(t)/dt = k_c U(t) + U(t)k_x^T + J(t, t)(K_{xy}^T(t, t) - HT(t) - U(t)),\ U(0) = 0 \qquad (43)$$

$$dV(t)/dt = k_x V(t) + V(t)k_c^{\ T} + w(t, t)(K_c(t, t) - HV(t) - W(t)),\ V(0) = 0 \qquad (44)$$

$$dQ(t)/dt = k_x Q(t) + T(t)k_x^T H^T + V(t)k_c^T + w(t, t)(HK_{xy}(t, t) + K_c(t, t) - HQ(t) - S(t)),$$

$$Q(0) = 0 \qquad (45)$$

$$dS(t)/dt = k_c S(t) + U(t)k_x^T H^T + W(t)k_c^T + J(t, t)(HK_{xy}(t, t) + K_c(t, t) - HQ(t) - S(t)),$$

$$S(0) = 0 \qquad (46)$$

$$dW(t)/dt = k_c W(t) + W(t)k_c^T + J(t, t)(K_c(t, t) - HV(t) - W(t)),\ W(0) = 0 \qquad (47)$$

We notice that the filter in [Theorem 2] does not correspond to the Chandrasekhar-type algorithm, since the differential equations (45) and (46) are Riccati-type nonlinear differential equations.

Number of differential equations contained in the present filtering algorithm is $2(m^2 + m\cdot n) + m + n$ which might be compared with $n^2 + 2m^2 + 3m\cdot n + m + n$ in [Theorem 2]. This suggests that the current estimation technique is more appropriate for fast calculation of the filtering estimate than the previous one in linear time invariant continuous systems.

6. Digital simulation example

The signal processes $x_1(t)$ and $x_2(t)$ are generated by

$$\begin{bmatrix} dx_1(t)/dt \\ dx_2(t)/dt \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} u(t),$$

$$E[u(t)\,u(s)] = \delta(t-s)/3. \tag{48}$$

The observation equation is given by

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + v_c(t) + v(t). \tag{49}$$

We find that the crosscovariance of $x(t)$ with $y(t)$ is given by $K_{xy}(t, t) = K_{xy}(0, 0) = [1/8 \quad 0]^T$. Also, we assume that the colored noise process is generated by

$$dv_c(t)/dt = -0.7v_c(t) + r(t),\ E[r(t)\,r(s)] = \sqrt{2 \cdot 0.7 \cdot P},\ P=0.01. \tag{50}$$

The autocovariance $K_c(t, s)$ of $v_c(t)$ for $s=t$ is $K_c(t, t) = K_c(0, 0) = 0.01$ from (50). Fig. 1 shows colored noise processes for $v_c(0) = -0.1$ (graph (a)), $v_c(0) = -0.3$ (graph (b)) and $v_c(0) = -0.5$ (graph (c)). Fig. 2 shows the filtering estimate $\hat{x}_1(t)$ of $x_1(t)$ calculated by the present filtering algorithm of [Theorem 1]. Graph (a) illustrates the signal process $x_1(t)$. Graphs (b) and (c) illustrate the filtering estimates for white Gaussian observation noises $N(0, 0.1^2)$ and $N(0, 0.3^2)$ when initial condition of the colored noise process is $v_c(0) = -0.1$. Graph (d) illustrates the filtering estimate $\hat{x}_1(t)$ for white Gaussian observation

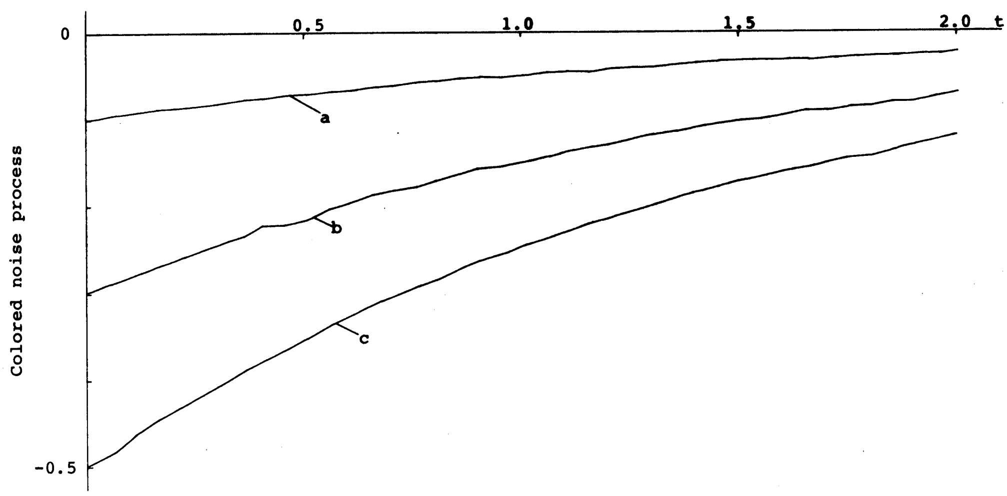

Fig. 1 Colored noise process $v_c(t)$ vs. t.
(a)……Colored noise process for the initial value of $v_c(t)$ at $t=0$ $v_c(0) = -0.1$.
(b)……Colored noise process for the initial value of $v_c(t)$ at $t=0$ $v_c(0) = -0.3$.
(c)……Colored noise process for the initial value of $v_c(t)$ at $t=0$ $v_c(0) = -0.5$.

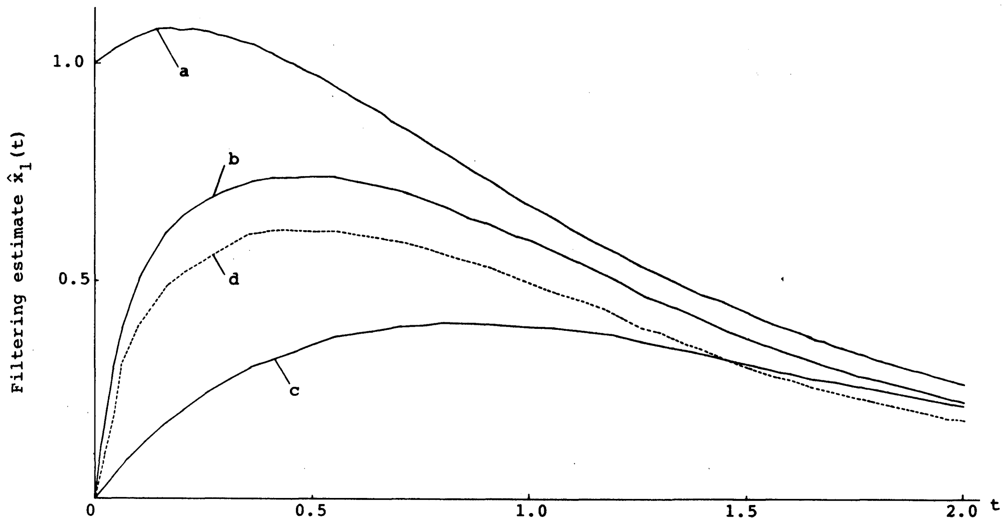

Fig. 2 Filtering estimate $\hat{x}_1(t)$ of x_1 (t) calculated by the present filtering equations of [Theorem 1].

(a)……Signal process $x_1(t)$.

(b)……Filtering estimate $\hat{x}_1(t)$ for white Gaussian observation noise $N(0,0.1^2)$ when initial value of the colored noise process is $v_c(0)=-0.1$.

(c)……Filtering estimate $\hat{x}_1(t)$ for white Gaussian observation noise $N(0,0.3^2)$ when initial value of the colored noise process is $v_c(0)=-0.1$.

(d)……Filtering estimate $\hat{x}_1(t)$ for white Gaussian observation noise $N(0,0.1^2)$ when $v_c(0)=-0.3$.

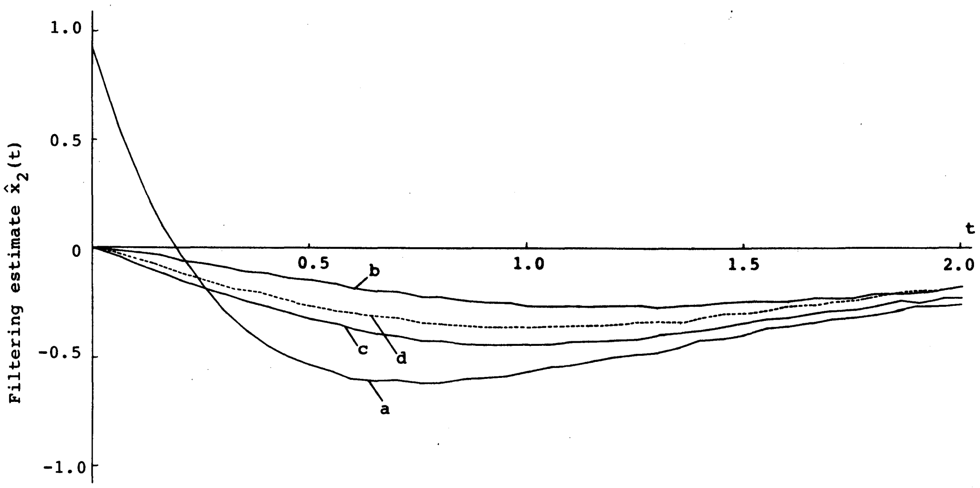

Fig. 3 Filtering estimate $\hat{x}_2(t)$ of $x_2(t)$ calculated by the present filtering equations of [Theorem 1].

(a)……Signal process $x_2(t)$.

(b)……Filtering estimate $\hat{x}_2(t)$ for white Gaussian observation noise $N(0,0.1^2)$ when initial value of the colored noise process is $v_c(0)=-0.1$.

(c)……Filtering estimate $\hat{x}_2(t)$ for white Gaussian observation noise $N(0,0.3^2)$ when initial value of the colored noise process is $v_c(0)=-0.1$.

(d)……Filtering estimate $\hat{x}_2(t)$ for white Gaussian observation noise $N(0,0.1^2)$ when $v_c(0)=-0.3$.

noise $N(0,0.1^2)$ when $v_c(0)=-0.3$. Fig. 3 shows the filtering estimate $\hat{x}_2(t)$ of $x_2(t)$ calculated by the algorithm of [Theorem 1]. Graph (a) illustrates the signal process $x_2(t)$. Graphs (b) and (c) illustrate the filtering estimates for white Gaussian observation noises $N(0,0.1^2)$ and $N(0,0.3^2)$ when initial condition of the colored noise process is $v_c(0)=-0.1$. Graph (d) illustrates the filtering estimate $\hat{x}_2(t)$ for white Gaussian observation noise $N(0,0.1^2)$ when $v_c(0)=-0.3$. Table 1 summarizes the mean-square values of filtering errors $x_1(t)-\hat{x}_1(t)$ and $x_2(t)-\hat{x}_2(t)$. The mean-square values are calculated by

Table 1. Mean-square values of filtering errors $x_1(t)-\hat{x}_1(t)$ and $x_2(t)-\hat{x}_2(t)$: $\sum_{k=1}^{2000}(x_i(k\Delta)-\hat{x}_i(k\Delta))^2/2000$, i=1, 2. Here, sampling interval for numerical integration scheme by the fourth-order Runge-Kutta method is $\Delta=0.001$.

	Mean-square value of filtering error $x_1(t)-\hat{x}_1(t)$			Mean-square value of filtering error $x_2(t)-\hat{x}_2(t)$		
	$v_c(0)=-0.1$	$v_c(0)=-0.3$	$v_c(0)=-0.5$	$v_c(0)=-0.1$	$v_c(0)=-0.3$	$v_c(0)=-0.5$
$N(0,0.1^2)$	0.57469×10^{-1}	0.10019	0.15983	0.36399×10^{-1}	0.51633×10^{-1}	0.73802×10^{-1}
$N(0,0.3^2)$	0.22825	0.26791	0.31323	0.83834×10^{-1}	0.10063	0.11987
$N(0,0.5^2)$	0.34995	0.37786	0.40763	0.13182	0.14433	0.15766
$N(0,0.7^2)$	0.41926	0.43814	0.45773	0.16201	0.17065	0.17960

$\sum_{k=1}^{2000}(x_i(k\Delta)-\hat{x}_i(k\Delta))^2/2000$, i=1, 2. Here, we used the fourth-order Runge-Kutta method and its sampling interval for numerical integration scheme is $\Delta=0.001$. The mean-square value decreases as the variance of white Gaussian noise and colored noise in Fig. 1 become small.

The filtering estimates $\hat{x}_1(t)$ and $\hat{x}_2(t)$ are also calculated by using the filtering algorithm of [Theorem 2]. As a result, the filtering algorithm of [Theorem 2] has the same estimation accuracy for $\hat{x}_1(t)$ and $\hat{x}_2(t)$ with that of [Theorem 1].

7. Conclusions

In this paper, the Chandrasekhar-type filter was devised for white Gaussian plus colored observation noise in linear continuous stationary stochastic systems. The number of differential equations included in the present filtering algorithm is less that that in [Theorem 2]. Furthermore, the present filter has the same estimation accuracy with that in [Theorem 2]. Therefore, the proposed filter is more suitable for fast calculation than the previous filter (Nakamori, 1991).

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