Reduced-Order Estimators Using Covariance Data of Signal in Linear Discrete-Time Systems

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(Received 3 October, 1995)

Abstract — The recursive least-squares filter and fixed-point smoother are designed in linear discrete-time systems. Here, it is assumed that the system matrix, the observation vector of the signal generating model and the variances of the state and white Gaussian observation noise processes are known. It is shown that, for the signal process modeled by the AR (autoregressive) process of order n, the system matrix in the state-space model of the signal is calculated by the signal autocovariance data $K_z(i)$, $i = 0, 1, 2, \cdots, n$, by appropriate choices of observation vector, the system matrix and the state. Also, the components of the variance matrix of the state consist of $K_z(i)$, $i = 0, 1, 2, \cdots\cdot, n-1$. Hence, the proposed estimation technique requires the signal autocovariance data $K_z(i)$, $i = 0, 1, 2, \cdots, n$, the variance of observation noise and the observed value in calculating the filtering and fixed-point smoothing estimates.

Furthermore, it is examined to ascertain that the estimation accuracy based on the reduced-order AR model is almost equivalent to that based on the AR model of optimum order.

1. Introduction

The Kalman filter [1] assumes full knowledge of the state-space model, which generates the signal process, in signal estimation problems.

In continuous-time stochastic systems, the Wiener filter [2] is designed. The auto-covariance function of the observed value is formulated by use of the observation matrix, the system matrix of the state-space model, the crossvariance of the state with the observed value and the variance of white Gaussian observation noise. The Wiener filter uses these information and the observed value. Recently, in the estimation problems relevant to the communication systems for white Gaussian plus colored observation noise, recursive predictor [3] is developed by use of the covariance information of the signal and the noise in linear continuous-time systems.

This paper extends the continuous-time Wiener filter to the fixed-point smoother and the

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filter in linear discrete-time systems when the signal is observed with additive white Gaussian noise. The estimation algorithms calculate the estimates recursively and are suitable for on-line implementations. By appropriate choices of the observation vector, the system matrix and the state, for the stochastic signal modeled by the AR process of order n, the algorithms require $(n+1)$ data of the signal autocovariance, the variance of the observation noise and the observed value. The proposed algorithms are applied to a digital simulation example for the signal process fitted to the AR (autoregressive) model. As the model order of the AR model is reduced, the computation time of the estimates is shortened. In terms of a numerical simulation example, we confirm that the estimation accuracy by use of the reduced-order AR model is as same as that based on the AR model of optimum order. Here, the optimum order is selected based on an information-theoretic criterion (AIC) [4].

The current estimation technique using the covariance information is advantageous over the Kalman approach that requires the full information of the state-space model. In the proposed approach, by the appropriate choices of the observation vector etc., the technique enables us to estimate the signal by use of only the finite autocovariance data of the signal, the variance of the observation noise and the observed value.

2. Least-squares estimation problems in linear discrete-time systems

Let a scalar observation equation be given by

$$y(k) = Hx(k)+v(k),\ z(k) = Hx(k), \tag{1}$$

where $y(k)$ is an observed value, H is a $1\times n$ observation vector, $x(k)$ is a zero-mean signal process and $v(k)$ is white Gaussian observation noise with the variance R.

$$E[v(k)v(s)] = R\delta_D(k-s) \tag{2}$$

Here, $\delta_D(k-s)$ is the Kronecker Delta function, which satisfies $\delta_D(k-s) = 1$ for $k = s$ and $\delta_D(k-s) = 0$ for $k \neq s$. It is assumed that $x(k)$ and $v(s)$ are uncorrelated.

$$E[x(k)v(s)] = 0,\ 0 \leqq s,\ t < \infty \tag{3}$$

Let us assume that the fixed-point smoothing estimate $\hat{x}(k, L)$ of $x(k)$ at the fixed-point k is expressed by

$$\hat{x}(k, L) = \sum_{i=1}^{L} h(k, i, L)y(i), \tag{4}$$

where $h(k, i, L)$ is an impulse response function. Minimizing the mean-square value of the fixed-point smoothing error $x(k)-\hat{x}(k, L)$

$$J = E\{[x(k)-\hat{x}(k, L)]^T [x(k)-\hat{x}(k, L)]\}, \tag{5}$$

we obtain the Wiener-Hopf equation [5]:

$$E[x(k)y(s)] = \sum_{i=1}^{L} h(k, i, L)E[y(i)y(L)]. \tag{6}$$

Let $K_{xy}(k, s)$ denote the crosscovariance function of $x(k)$ with $y(s)$ and $K_z(k, s)$ the autocovariance function of $z(k)$. If we substitute (1) into (6), and use (2) and (3), we obtain

$$h(k, s, L)R = K_{xy}(k, s) - \sum_{i=1}^{L} h(k, i, L)K_z(i, s). \tag{7}$$

(7) is the basic equation which the optimal impulse response function $h(k, s, L)$ satisfies in linear least-squares smoothing problems. It is clear that $K_z(k, s)$ is expressed by

$$\begin{aligned} K_z(k, s) &= H\Phi^{k-s}K_{xy}(s, s)1(k-s) + K_{xy}{}^T(k, k)(\Phi^T)^{s-k}H^T1(s-k), \\ K_{xy}(s, s) &= K_x(s, s)H^T, \end{aligned} \tag{8}$$

where Φ is the stable system matrix in the state-space model for $x(k)$, $K_x(s, s)$ is the autovariance function of $x(s)$ and $1(k-s)$ represents the unit step function.

3. Recursive least-squares algorithms for the filtering and fixed-point smoothing estimates

In [Theorem 1], the recursive least-squares algorithms for the filtering and fixed-point smoothing estimates are shown in linear discrete-time systems.

[Theorem 1]

Let the autocovariance function $K_z(k, s)$ of $z(k)$ be expressed by (8), let $K_x(k, s)$ be the autocovariance function of $x(k)$ and let the variance of white Gaussian obsevation noise be R . Then, the recursive least-squares algorithms for the filtering and fixed-point smoothing estimates consist of (9)−(14) in linear discrete-time systems.

Fixed-point smoothing estimate of $x(k) : \hat{x}(k, L)$

$$\hat{x}(k, L) = \hat{x}(k, L-1) + h(k, L, L)[y(L) - H\Phi\hat{x}(L-1, L-1)] \tag{9}$$

$$\begin{aligned} h(k, L, L) = {} & [K_x(k, k)(\Phi^T)^{L-k}H^T/R - q(k, L-1)\Phi^T H^T/R]/[1 + HK_x(L, L)H^T/R \\ & - H\Phi S(L-1)\Phi^T H^T/R] \end{aligned} \tag{10}$$

$$q(k, L) = q(k, L-1)\Phi^T + h(k, L, L)H[K_x(L, L) - \Phi S(L-1)\Phi^T],\ q(L, L) = S(L) \tag{11}$$

Filtering estimate of $x(L) : \hat{x}(L, L)$

$$\hat{x}(L, L) = \Phi\hat{x}(L-1, L-1) + G(L)[y(L) - H\Phi\hat{x}(L-1, L-1)],\ \hat{x}(0, 0) = 0 \tag{12}$$

$$S(L) = \Phi S(L-1)\Phi^T + G(L)H[K_x(L, L) - \Phi S(L-1)\Phi^T],\ S(0) = 0 \tag{13}$$

$$G(L) = [K_x(L, L)H^T - \Phi S(L-1)\Phi^T H^T]/[R + HK_x(L, L)H^T - H\Phi S(L-1)\Phi^T H^T] \tag{14}$$

Proof

If we subtract the equation obtained by putting $L \to L-1$ in (7) form (7), we have

$$[h(k, s, L) - h(k, s, L-1)]R = -h(k, L, L)HK_x(L, s)H^T - \sum_{i=1}^{L} [h(k, i, L) - h(k, i, L-1)]HK_x(i, s)H^T. \tag{15}$$

Let us introduce a new function $\Lambda(s, L-1)$ which satisfies

$$\Lambda(s, L-1)R = K_x(L, s)H^T - \sum_{i=1}^{L-1} \Lambda(i, L-1)HK_x(i, s)H^T. \tag{16}$$

From (15) and (16), we obtain

$$h(k, s, L) - h(k, s, L-1) = -h(k, L, L)H\Lambda(s, L-1). \tag{17}$$

Let us introduce a function $J(s, L-1)$ which satisfies

$$J(s, L-1)R = \Phi^{-s}K_x(s, s)H^T - \sum_{i=1}^{L-1} J(i, L-1)HK_x(i, s)H^T. \tag{18}$$

From (16) and (18), $\Lambda(s, L-1)$ is expressed by

$$\Lambda(s, L-1) = \Phi^L J(s, L-1). \tag{19}$$

If we subtract $J(s, L-2)$ from $J(s, L-1)$, we have

$$(J(s, L-1) - J(s, L-2))R = -J(L-1, L-1)HK_x(L-1, s)H^T - \sum_{i=1}^{L-2} [J(i, L-1) - J(i, L-2)]HK_x(i, s)H^T. \tag{20}$$

From (16), (19) and (20), we obtain an equation for $J(s, L)$.

$$\begin{aligned} J(s, L) &= J(s, L-1) - J(L, L)H\Lambda(s, L-1) \\ &= J(s, L-1) - J(L, L)H\Phi^L J(s, L-1) \end{aligned} \tag{21}$$

By introducing a function

$$r(L-1) = \sum_{i=1}^{L-1} J(i, L-1)HK_x(i, i)(\Phi^T)^{-i}, \tag{22}$$

we have an expression for $J(L-1, L-1)R$ as

$$\begin{aligned} J(L-1, L-1)R &= \Phi^{-(L-1)}K_x(L-1, L-1)H^T - \sum_{i=1}^{L-1} J(i, L-1)HK_x(i, L-1)H^T \\ &= \Phi^{-(L-1)}K_x(L-1, L-1)H^T - \sum_{i=1}^{L-1} J(i, L-1)HK_x(i, i)(\Phi^T)^{-i}(\Phi^T)^{L-1}H^T \\ &= \Phi^{-(L-1)}K_x(L-1, L-1)H^T - r(L-1)(\Phi^T)^{L-1}H^T \end{aligned} \tag{23}$$

from (18).

If we subtract $r(L-2)$ from $r(L-1)$ and use (21) and (22), we obtain

$$\begin{aligned} r(L-1)-r(L-2) &= J(L-1, L-1)HK_x(L-1, L-1)(\Phi^T)^{-(L-1)}+ \\ &\sum_{i=1}^{L-2}[J(i, L-1)-J(i, L-2)]HK_x(i, i)(\Phi^T)^{-i} \\ &= J(L-1, L-1)H[K_x(L-1, L-1)(\Phi^T)^{-(L-1)}-\Phi^{L-1}r(L-2)]. \end{aligned} \quad (24)$$

The transition matrix for the system of $x(k)$ is stable. However, the stability for the matrix $(\Phi^T)^{-(L-1)}$ is not guaranteed. Then, we introduce a new function $S(L) = \Phi^L r(L)(\Phi^T)^L$. From (24), $S(L)$ is developed as follows.

$$\begin{aligned} S(L) &= \Phi^L r(L)(\Phi^T)^L \\ &= \Phi^L\{r(L-1)+J(L, L)H[K_x(L, L)(\Phi^T)^{-L}-\Phi^L r(L-1)]\}(\Phi^T)^L \\ &= \Phi S(L-1)\Phi^T+\Phi^L J(L, L)H[K_x(L, L)-\Phi S(L-1)\Phi^T] \end{aligned} \quad (25)$$

If we introduce a function $G(L) = \Phi^L J(L, L)$, we obtain (13). The initial condition of (25) for $S(L)$ at $L = 0$ is $S(0) = 0$ from the relationship $S(L) = \Phi^L r(L)(\Phi^T)^L$ and (22). From (23) and (24), $G(L)$ is expressed as follows after some algebraic manipulations.

$$\begin{aligned} G(L) &= [K_x(L, L)H^T-\Phi^L r(L-1)(\Phi^T)^L H^T]/[R+HK_x(L, L)H^T- \\ &\quad H\Phi^L r(L-1)(\Phi^T)^L H^T] \\ &= [K_x(L, L)H^T-\Phi S(L-1)\Phi^T H^T]/[R+HK_x(L, L)H^T-H\Phi S(L-1)\Phi^T H^T] \end{aligned} \quad (26)$$

If we introduce a function

$$P(k, L) = \sum_{i=1}^{L} h(k, i, L)HK_x(i, i)(\Phi^T)^{-i} \quad (27)$$

and use (7), $h(k, L, L)$ in (17) satisfies

$$\begin{aligned} h(k, L, L)R &= K_{xy}(k, L)-\sum_{i=1}^{L} h(k, i, L)K_z(i, L) \\ &= K_x(k, k)(\Phi^T)^{L-k}H^T-\sum_{i=1}^{L} h(k, i, L)HK_x(i, i)(\Phi^T)^{-i}(\Phi^T)^L H^T \\ &= K_x(k, k)(\Phi^T)^{L-k}H^T-P(k, L)(\Phi^T)^L H^T. \end{aligned} \quad (28)$$

If we subtract $P(k, L-1)$ from $P(k, L)$ and use (17), (19) and (22), we have

$$\begin{aligned} P(k, L)-P(k, L-1) &= h(k, L, L)HK_x(L, L)(\Phi^T)^{-L}+ \\ &\sum_{i=1}^{L-1}[h(k, i, L)-h(k, i, L-1)]HK_x(i, i)(\Phi^T)^{-i} \\ &= h(k, L, L)H[K_x(L, L)(\Phi^T)^{-L}-\Phi^L\sum_{i=1}^{L-1} J(i, L-1)HK_x(i, i)(\Phi^T)^{-i}] \\ &= h(k, L, L)H[K_x(L, L)(\Phi^T)^{-L}-\Phi^L r(L-1)]. \end{aligned} \quad (29)$$

Let us introduce a function $q(k, L) = P(k, L)(\Phi^T)^L$. From (29) and the relationship $S(L) = \Phi^L r(L)(\Phi^T)^L$, we obtain recursive equation for $q(k, L)$.

$$\begin{aligned} q(k, L) &= \{P(k, L-1)+h(k, L, L)H[K_x(L, L)(\Phi^T)^{-L}-\Phi^L r(L-1)]\}(\Phi^T)^L \\ &= q(k, L-1)\Phi^T+h(k, L, L)H[K_x(L, L)-\Phi S(L-1)\Phi^T] \end{aligned} \tag{30}$$

Now, we formulate the equation for $P(L, L)$. From (27), we have

$$P(L, L) = \sum_{i=1}^{L} h(L, i, L)HK_x(i, i)(\Phi^T)^{-i}. \tag{31}$$

By the way, $h(L, s, L)$ satisfies

$$h(L, s, L)R = K_{xy}(L, s)-\sum_{i=1}^{L} h(L, i, L)K_z(i, s). \tag{32}$$

Then, from (18), we obtain

$$h(L, s, L) = \Phi^L J(s, L). \tag{33}$$

If we substitute (33) into (31) and use (22), we obtain

$$\begin{aligned} P(L, L) &= \Phi^L \sum_{i=1}^{L} J(i, L)HK_x(i, i)(\Phi^T)^{-i} \\ &= \Phi^L r(L). \end{aligned} \tag{34}$$

The initial condition of the difference equation for $q(k, L)$ at $L = k$ is $q(k, k) = S(k)$ since

$$\begin{aligned} q(k, k) &= P(k, k)(\Phi^T)^k \\ &= \Phi^k r(k)(\Phi^T)^k \end{aligned} \tag{35}$$

from (34).

If we apply the relationship $q(k, L) = P(k, L)(\Phi^T)^L$ to (28), we have

$$h(k, L, L) = [K_x(k, k)(\Phi^T)^{L-k}H^T-q(k, L)H^T]/R. \tag{36}$$

From (30) and (36), we obtain (10) for $h(k, L, L)$ after some algebraic manipulations.

If we introduce a function

$$O(L) = \sum_{i=1}^{L} J(i, L)y(i) \tag{37}$$

and use (4) and (33), we obtain the equation for the filtering estimate $\hat{x}(L, L)$ as

$$\begin{aligned} \hat{x}(L, L) &= \sum_{k=1}^{L} h(L, i, L)y(i) \\ &= \Phi^L O(L). \end{aligned} \tag{38}$$

If we subtract $O(L-1)$ from $O(L)$, we have

$$\begin{aligned} O(L)-O(L-1) &= J(L, L)y(L)+\sum_{i=1}^{L-1}[J(i, L)-J(i, L-1)]y(i) \\ &= J(L, L)y(L)-J(L, L)H\Phi^L \sum_{i=1}^{L-1} J(i, L-1)y(i) \end{aligned}$$

$$= J(L, L)(y(L) - H\Phi^L O(L-1)) \tag{39}$$

from (21) and (37). If we substitute (39) into (38), we obtain

$$\begin{aligned}\hat{x}(L, L) &= \Phi^L[O(L-1) + J(L, L)(y(L) - H\Phi^L O(L-1))] \\ &= \Phi\hat{x}(L-1, L-1) + G(L)(y(L) - H\Phi\hat{x}(L-1, L-1))\end{aligned} \tag{40}$$

from the relationship $G(L) = \Phi^L J(L, L)$ and (38). The initial condition of (40) for $\hat{x}(L, L)$ at $L = 0$ is $\hat{x}(0, 0) = 0$ from (37) and (38).

Finally, if we subtract $\hat{x}(k, L-1)$ from $\hat{x}(k, L)$, we obtain

$$\begin{aligned}\hat{x}(k, L) - \hat{x}(k, L-1) &= h(k, L, L)[y(L) - H\Phi^L \sum_{i=1}^{L-1} J(i, L-1)y(i)] \\ &= h(k, L, L)[y(L) - H\Phi\hat{x}(L-1, L-1)]\end{aligned} \tag{41}$$

from (4), (17), (19), (37) and (38).

4. Factorization technique of autocovariance function of signal

The filtering and fixed-point smoothing algorithms of [Theorem 1] calculate the estimates recursively by use of the observation vector H, the system matrix Φ, the autovariance function $K_x(k, k)$ of $x(k)$, the variance R of the observation noise $v(k)$ and the observed value $y(k)$. We consider the problem which determines the parameters H and Φ in the state-space model and $K_x(k, k)$, being given the autocovariance data of $z(k)$. Here, we assume that the signal process is stationary and the wide-sense stationarity for the autocovariances is valid as $K_z(k, s) = K_z(k-s)$ and $K_x(k, s) = K_x(k-s)$.

The observation equation is given by

$$y(k) = H\begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdot \\ \cdot \\ x_n(k) \end{bmatrix} + v(k),\ H = [1\ 0\ 0\ \cdots 0],\ z(k) = x_1(k) \tag{42}$$

in terms of the components of the state vector $x(k)$. We assume that the processes of $x_i(k)$, $i = 1, 2, \cdots, n$, are generated by the stochastic system of order n

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \cdot \\ \cdot \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 & 0 \\ 0 & 0 & 1 \cdots & 0 & 0 \\ \cdot & \cdot\cdot & \cdots\cdots & \cdot\cdot & \cdot \\ 0 & 0 & \cdots\cdot & 0 & 1 \\ -a_n & -a_{n-1} & \cdots\cdot & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdot \\ \cdot \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} u(k),$$

$$E[u(k)u(s)] = \sigma^2 \delta_D(k-s). \tag{43}$$

For the signal process of $z(k)(=x_1(k))$ with the wide-sense stationarity, we note that $K_z(k, k)=K_z(k-k)(=K_z(0))$. Then

$$K_x(k, k)=E[x(k)x^T(k)]=\begin{bmatrix} K_z(0) & K_z(1) & \cdots\cdot & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots\cdot & K_z(n-2) \\ \cdots\cdot & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ K_z(n-2) & \cdot\cdot & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdot\cdot & K_z(0) \end{bmatrix}. \quad (44)$$

From (43), the signal process of $z(k)$ is generated by the AR model of order n

$$z(k)=-a_1 z(k-1)-a_2 z(k-2)-\cdots-a_n z(k-n)+e(k),\ e(k)=u(k-n). \quad (45)$$

The AR parameters a_i, $i=1, \cdots, n$, are calculated by the Yule-Walker equations

$$\begin{bmatrix} K_z(0) & K_z(1) & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & K_z(n-2) \\ \cdots & \cdots & \cdots & \cdots \\ K_z(n-2) & \cdots & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & K_z(0) \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_n \end{bmatrix}=\begin{bmatrix} -K_z(1) \\ -K_z(2) \\ \cdot \\ \cdot \\ -K_z(n) \end{bmatrix} \quad (46)$$

[4]. Therefore, the system matrix in (43) is obtained by the autocovariance data $K_z(i)$, $i=0, \cdots n$. From (44), the matrix elements of $K_x(k, k)$ consist of $K_z(i)$, $i=0, \cdots, n-1$. Hence, by use of the obtained H, Φ and $K_x(L, L)$, from $K_z(i)$, $i=0, \cdots, n$, with R and $y(L)$, we calculate the filtering and fixed-point smoothing estimates in [Theorem 1].

In section 5, we attempt to apply the estimation algorithms of [Theorem 1] to the estimation of a voice signal based on the factorization method which represents the autocovariance function of the signal in terms of H, Φ and $K_x(k, k)$.

5. A digital simulation example

We consider to estimate the utterance "ih" spoken by a male. Its phonetic transcription is written as "i:". The samling frequency of the voice signal is $51.2(kHz)$. In the simulation, the autocovariance data of the signal are obtained by use of the $N(=1,000)$ sampled signal data. The signal sequence of the vowel sound is modeled in terms of the AR process of order n. The optimum order of the AR process is selected such that an information-theoretic criterion (AIC(n)) is minimized.

$$AIC(n)=N\cdot ln(2\pi)+N\cdot ln(\sigma^2)+N+2(n+1),\ \sigma^2=K_z(0)-a_1K_z(1)-a_2K_z(2)-\cdots-a_nK_z(n) \quad (47)$$

Here, σ^2 denotes the variance of $e(k)$. In the calculation of AIC(n), the term $N\cdot ln(2\pi)+N$ is omitted. Fig.1 shows AIC(n) vs. the order n of the AR model. We find that the optimum order of the AR model is 26. We determine H, Φ and $K_x(k, k)$ by use of the autocovariance data of the signal, $K_z(i)$, $i=0, \cdots, n$. If we substitute H, Φ, K_x

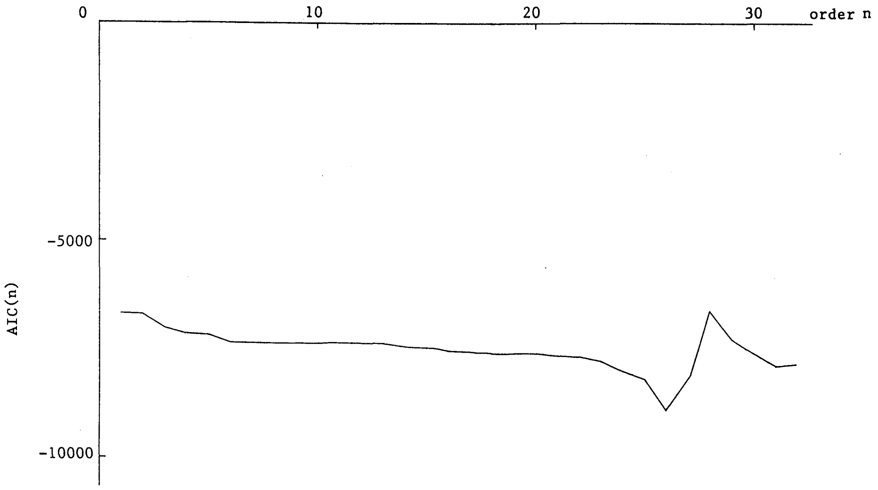

Fig. 1 AIC(n) vs. the model order n.

$(k, k)(=K_x(0))$, the variance R of the observation noise and the observed value into [Theorem 1], we can calculate the filtering estimate $\hat{z}(k, k)$ and the fixed-point smoothing estimate $\hat{z}(k, L)$ recursively. Fig. 2 shows the signal $z(k)$ (graph(a)) and its filtering estimate $\hat{z}(k, k)$ (graph(b)) vs. k when the signal process is fitted to the AR model of the *10th* order and the variance R of the observation noise is 0.1^2. Table 1 summarizes the M. S. V. (mean-square values) of the filtering and fixed-point smoothing errors for white Gaussian observation noises $N(0, 0.1^2)$ and $N(0, 0.3^2)$. Here, the M. S. V. are calculated by $\sum_{k=1}^{100}(z(k)-\hat{z}(k, k))^2/100$ for the filter and $\sum_{k=1}^{100}\sum_{j=1}^{20}(z(k)-\hat{z}(k, k+j))^2/2000$ for the fixed-point smoother. Table 1 indicates that the estimation accuracy of the signal for the reduced

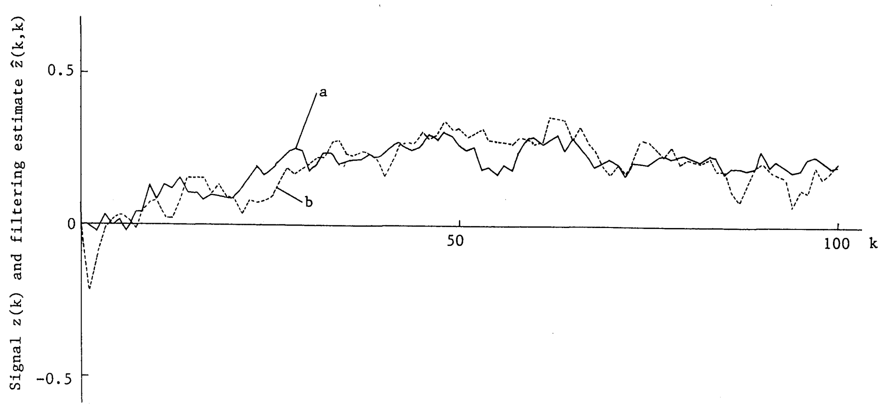

Fig. 2 Signal process $z(k)$ and its filtering estimate $\hat{z}(k, k)$ vs. k.
(a)…Signal process.
(b)…Filtering estimate for white Gaussian observation noise $N(0, 0.1^2)$.

Table 1 M. S. V. of the filtering and fixed-point smoothing errors for white Gaussian observation noises $N(0, 0.1^2)$ and $N(0, 0.3^2)$.

Order of the AR model	M. S. V. of filtering error for $N(0, 0.1^2)$	M. S. V. of smoothing error for $N(0, 0.1^2)$	M. S. V. of filtering error for $N(0, 0.3^2)$	M. S. V. of smoothing error for $N(0, 0.3^2)$
1	2.55493×10^{-3}	2.06072×10^{-3}	6.64871×10^{-3}	5.62511×10^{-3}
2	2.63142×10^{-3}	2.10749×10^{-3}	6.94994×10^{-3}	5.87954×10^{-3}
3	3.15227×10^{-3}	2.26045×10^{-3}	0.010043	7.91305×10^{-3}
4	2.89816×10^{-3}	2.27297×10^{-3}	8.17498×10^{-3}	6.90183×10^{-3}
5	2.85184×10^{-3}	2.2721×10^{-3}	7.87695×10^{-3}	6.72151×10^{-3}
6	2.54871×10^{-3}	2.21279×10^{-3}	6.33387×10^{-3}	5.6054×10^{-3}
7	2.4812×10^{-3}	2.18177×10^{-3}	6.06144×10^{-3}	5.36519×10^{-3}
8	2.5038×10^{-3}	2.19115×10^{-3}	6.15071×10^{-3}	5.44209×10^{-3}
9	2.54045×10^{-3}	2.2068×10^{-3}	6.30385×10^{-3}	5.56454×10^{-3}
10	2.58764×10^{-3}	2.2191×10^{-3}	6.51658×10^{-3}	5.71424×10^{-3}
11	2.53233×10^{-3}	2.20664×10^{-3}	6.25199×10^{-3}	5.54249×10^{-3}
12	2.60663×10^{-3}	2.21043×10^{-3}	6.6322×10^{-3}	5.7582×10^{-3}
13	2.62416×10^{-3}	2.20873×10^{-3}	6.73469×10^{-3}	5.80448×10^{-3}
14	2.67966×10^{-3}	2.15791×10^{-3}	7.25523×10^{-3}	5.93431×10^{-3}
15	2.68124×10^{-3}	2.11239×10^{-3}	7.48971×10^{-3}	5.92239×10^{-3}
16	2.64713×10^{-3}	2.04868×10^{-3}	7.65872×10^{-3}	5.8602×10^{-3}
17	2.63624×10^{-3}	2.03202×10^{-3}	7.71648×10^{-3}	5.83814×10^{-3}
18	2.62213×10^{-3}	2.01457×10^{-3}	7.7708×10^{-3}	5.81305×10^{-3}
19	2.62736×10^{-3}	2.02028×10^{-3}	7.75466×10^{-3}	5.82275×10^{-3}
20	2.61025×10^{-3}	2.0026×10^{-3}	7.79407×10^{-3}	5.79317×10^{-3}
21	2.59418×10^{-3}	1.9843×10^{-3}	7.83958×10^{-3}	5.76433×10^{-3}
22	2.57956×10^{-3}	1.93345×10^{-3}	7.94519×10^{-3}	5.72051×10^{-3}
23	2.57956×10^{-3}	1.93345×10^{-3}	8.08171×10^{-3}	5.71293×10^{-3}
24	2.6497×10^{-3}	1.92628×10^{-3}	8.40942×10^{-3}	5.8105×10^{-3}
25	2.832×10^{-3}	1.97852×10^{-3}	8.83587×10^{-3}	6.0777×10^{-3}
26	3.15394×10^{-3}	2.15635×10^{-3}	9.16928×10^{-3}	6.36881×10^{-3}

orders, $n = 1, 2, \cdots, 25$, is almost same with that for the optimum order $n(=26)$. Also, the M. S. V. for the fixed-point smoothing estimate are slightly less than those for the filtering estimate. This shows that the estimation accuracy of the fixed-point smoothing estimate is preferable to that of the filtering estimate, whether the model order is reduced or optimum.

6. Conclusions

This paper has proposed new estimation technique by use of finite number of autocovariance data of the signal process, the variance of white Gaussian observation noise and the observed value. The estimation technique requires finite autocovariance data $K_z(i)$, $i = 0, 1, 2, \cdots, n$, when the signal sequence is modeled by the AR process of order n.

From the numerical example, we have confirmed that the estimation accuracy of the current estimators is not degraded by fitting the signal process to the AR model of the reduced-order. Also, the estimation accuracy for the fixed-point smoothing estimate is better than that for the filtering estimate, whether the order of the AR model is reduced or optimum.

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