DISCRETE-TIME FILTERING ALGORITHM USING COVARIANCE INFORMATION FOR WHITE GAUSSIAN PLUS COLORED OBSERVATION NOISE

Seiichi NAKAMORI

(Received 1 October, 1999)

Abstract This paper proposes recursive least-squares (RLS) filtering algorithm using the covariance information in the case of white Gaussian plus colored observation noise in linear discrete-time wide-sense stationary systems. Here, it is assumed that the system matrices in the state-space models for both the signal and the colored noise, the crossvariance functions of the state variables for the signal and the colored noise with the observed value, the observation vectors for the signal and the colored noise, the variance of white Gaussian observation noise and the observed value are known.

1. Introduction

In the approach to the estimation theory using the state-space model, the Kalman filter [1],[2] is well-known. Besides the Kalman approach, the estimators using the covariance information are designed. In [3], the estimators use the covariance information in the form of the semi-degenerate kernel function. The semi-degenerate kernel function expresses the covariance information as the finite sum of products of nonrandom functions. In [4],[5],[6], the continuous- time estimators use the observation vector, the system matrix and the crossvariance function of the state variable with the observed value, which constitute the autocovariance function of the stochastic signal.

In the theory of detection and estimation [7], the estimation problem using the covariance information is investigated for both white Gaussian and white Gaussian plus colored observation noises. Correspondingly, the RLS estimation algorithms using the covariance information are derived for white Gaussian plus colored observation noise [3],[6].

Department of Technology, Faculty of Education, Kagoshima University,

1-20-6, Kohrimoto, Kagoshima 890-0065, Japan

This paper proposes the RLS filtering algorithm using the covariance information for white Gaussian plus colored observation noise in wide-sense discrete-time stationary stochastic systems. The filter is derived based on the invariant imbedding method [8]. The filter necessitates the information of the observation vectors, the system matrices for the scalar signal and colored noise in the both state-space models, the crossvariance functions of the state-variables for the signal and the colored noise with the observed value, the variance of white Gaussian noise and the observed value. It is shown that these necessary quantities in the filtering algorithm are obtained from the autocovariance functions of the signal and the colored observation noise.

2. Filtering problems for white Gaussian plus colored noise

In this section, the least-squares filtering problems using the covariance information are introduced for white Gaussian plus colored observation noise in linear discrete-time stochastic systems.

Let a scalar discrete-time observation equation be given by

$$y(k) = z(k) + v(k) + v_c(k), \quad z(k) = Hx(k), \quad v_c(k) = H_c x_c(k) \tag{1}$$

for white Gaussian plus colored observation noise in linear discrete-time wide-sense stationary stochastic systems. Here, $y(k)$ is an observed value, $z(k)$ is a zero-mean signal, H is a $1 \times n$ observation vector for a state variable $x(k)$ concerned with the signal $z(k)$, $v_c(k)$ is zero-mean colored observation noise, H_c is a $1 \times m$ observation vector for a state variable $x_c(k)$ concerned with the colored noise $v_c(k)$ and $v(k)$ is white Gaussian observation noise with the variance R.

$$E[v(k)v(s)] = R\delta_K(k-s) \tag{2}$$

Here, $\delta_K(k-s)$ represents the Kronecker delta function. It is assumed that the signal $z(\cdot)$, the white Gaussian noise $v(\cdot)$ and the colored noise $v_c(\cdot)$ are uncorrelated mutually as written by

$$E[z(k)v(s)] = 0, \quad E[z(k)v_c(s)] = 0, \quad E[v_c(k)v(s)] = 0, \quad 0 \le s, k < \infty. \tag{3}$$

Also, we assume that F and F_c represent the system matrices for $x(k)$ and $x_c(k)$.

Let us assume that the filtering estimate $\hat{x}(k,k)$ of the state variable $x(k)$ is given by

$$\hat{x}(k,k)=\sum_{i=1}^{k}h(k,i)y(i), \tag{4}$$

where $h(k,s)$ represents an $n\times 1$ impulse response function. Minimizing the mean-square value of the filtering error $x(k)-\hat{x}(k,k)$

$$J=E[(x(k)-\hat{x}(k,k))^{T}(x(k)-\hat{x}(k,k))], \tag{5}$$

we obtain the Wiener-Hopf equation [2]:

$$E[x(k)y(s)]=\sum_{i=1}^{k}h(k,i)E[y(i)y(s)]. \tag{6}$$

Let $K_{xy}(k,s)$ represent the crosscovariance function of the state variable $x(k)$ with the observed value $y(s)$ and let $K_{cy}(k,s)$ represent the crosscovariance function of the state variable $x_c(k)$ with the observed value $y(s)$. Substituting (1) into (6), and using (2) and (3), we obtain

$$h(k,s)R=K_{xy}(k,s)-\sum_{i=1}^{k}h(k,i)(HK_{xy}(i,s)+H_cK_{cy}(i,s)). \tag{7}$$

(7) is the equation which the optimal impulse response function $h(k,s)$ satisfies in linear least-squares filtering problem for white Gaussian plus colored observation noise.

Let $K_x(k,s)$ represent the autocovariance function of the state variable $x(k)$. The autocovariance function $K_z(k,s)$ of the signal $z(k)$ is given by

$$K_z(k,s)=HA(k)B^{T}(s)H^{T}1(k-s)+HB(k)A^{T}(s)H^{T}1(s-k),$$
$$A(k)=F^{k},\quad B^{T}(s)=F^{-s}K_x(s,s), \tag{8}$$

where F is the system matrix in the state-space model for the state variable x(k) and 1(k-s) represents the unit step function. Let $K_c(k,s)$ represent the autocovariance function of the state variable $x_c(k)$ for the colored noise $v_c(k)$. The autocovariance function of the colored noise $v_c(k)$ is given by

$$K_c(k,s)=H_cC(k)D^{T}(s)H_c^{T}1(k-s)+H_cD(k)C^{T}(s)H_c^{T}1(s-k),$$
$$C(k)=F_c^{k},\quad D^{T}(s)=F_c^{-s}K_c(s,s), \tag{9}$$

where F_c is the system matrix in the state-space model for the state variable $x_c(k)$.

The RLS filtering algorithm proposed in section 3 uses the information of the observation vector H for the state variable x(k), the system matrix F for x(k), the crossvariance function of x(k) with y(k), $K_{xy}(k,k)$, the observation vector H_c for the state variable $x_c(k)$, the system matrix F_c for $x_c(k)$, the crossvariance function of the colored noise $v_c(k)$ with y(k) , $K_{cy}(k,k)$, and the observed value y(k).

3. Derivation of filtering algorithm for white Gaussian plus colored observation noise

In [Theorem 1], the RLS filtering algorithm using the covariance information is derived based on the invariant imbedding method [8].

[Theorem 1]

Let F and F_c be square system matrices of orders n and m respectively in the state-space models for the signal $z(k)$ and the colored observation noise $v_c(k)$. Let H and H_c be the observation vectors concerned with $z(k)$ and $v_c(k)$. Let $K_{xy}(k,k)$ be the crossvariance function of the state variable $x(k)$ with the observed value $y(k)$. Let $K_{cy}(k,k)$ be the crossvariance function of the colored noise $v_c(k)$ with the observed value $y(k)$. Let $\hat{z}(k,k)$ and $\hat{v}_c(k,k)$ represent the filtering estimates of $z(k)$ and $v_c(k)$ respectively. Then the RLS algorithm for the filtering estimates $\hat{z}(k,k)$ and $\hat{v}_c(k,k)$ consists of the following equations (10)-(17) for white Gaussian plus colored observation noise.

Filtering estimate of signal $z(k)$: $\hat{z}(k,k)=H\hat{x}(k,k)$

$$\hat{x}(k,k)=F\hat{x}(k-1,k-1)+h(k,k)(y(k)-HF\hat{x}(k-1,k-1)-H_cF_cf(k-1)),\quad \hat{x}(0,0)=0 \tag{10}$$

Filtering estimate of colored noise $v_c(k)$: $\hat{v}_c(k,k)=H_cf(k)$

$$f(k)=F_cf(k-1)+\Phi(k,k)(y(k)-HF\hat{x}(k-1,k-1)-H_cF_cf(k-1)),\quad f(0)=0 \tag{11}$$

Filter gain for the filtering estimate of $x(k)$: $h(k,k)$

$$\begin{aligned}h(k,k)=&\{K_{xy}(k,k)-FG(k-1)F^TH^T-FJ(k-1)F_c^TH_c^T\}/\\&\{R+HK_{xy}(k,k)-HFG(k-1)F^TH^T-H_cF_cL(k-1)F^TH^T+\\&H_cK_{cy}(k,k)-HFJ(k-1)F_c^TH_c^T-H_cF_cM(k-1)F_c^TH_c^T\}\end{aligned} \tag{12}$$

Filter gain for the function $f(k)$: $\Phi(k,k)$

$$\begin{aligned}\Phi(k,k)=&\{K_{cy}(k,k)-F_cL(k-1)F^TH^T-F_cM(k-1)F_c^TH_c^T\}/\\&\{R+HK_{xy}(k,k)-HFG(k-1)F^TH^T-H_cF_cL(k-1)F^TH^T+\\&H_cK_{cy}(k,k)-HFJ(k-1)F_c^TH_c^T-H_cF_cM(k-1)F_c^TH_c^T\}\end{aligned} \tag{13}$$

$$G(k)=FG(k-1)F^{T}+h(k,k)(K_{xy}^{T}(k,k)-HFG(k-1)F^{T}-H_cF_cL(k-1)F^{T}),\quad G(0)=0 \tag{14}$$

$$J(k)=FJ(k-1)F_c^{T}+h(k,k)(K_{cy}^{T}(k,k)-HFJ(k-1)F_c^{T}-H_cF_cM(k-1)F_c^{T}),\quad J(0)=0 \tag{15}$$

$$L(k)=F_cL(k-1)F^{T}+\Phi(k,k)(K_{xy}^{T}(k,k)-HFG(k-1)F^{T}-H_cF_cL(k-1)F^{T}),\quad L(0)=0 \tag{16}$$

$$M(k)=F_cM(k-1)F_c^{T}+\Phi(k,k)(K_{cy}^{T}(k,k)-HFJ(k-1)F_c^{T}-H_cF_cM(k-1)F_c^{T}),\quad M(0)=0 \tag{17}$$

Proof

Subtracting the equation obtained by putting k→k-1 in (7) from (7), we have

$$(h(k,s)-h(k-1,s))R=(F-I)K_{xy}(k-1,s)-h(k,k)(HK_{xy}(k,s)+H_cK_{cy}(k,s))-\sum_{i=1}^{k}(h(k,i)-h(k-1,i))(HK_{xy}(i,s)+H_cK_{cy}(i,s)). \tag{18}$$

Let us introduce a function $\Phi(k,s)$ which satisfies

$$\Phi(k,s)R=K_{cy}(k,s)-\sum_{i=1}^{k}\Phi(k,i)(HK_{xy}(i,s)+H_cK_{cy}(i,s)). \tag{19}$$

From (7), (18) and (19), we have a difference equation for h(k,s).

$$h(k,s)=Fh(k-1,s)-h(k,k)(HFh(k-1,s)+H_cF_c\Phi(k-1,s)) \tag{20}$$

Subtracting the equation obtained by putting k→k-1 in (19) from (19), we have

$$(\Phi(k,s)-\Phi(k-1,s))R=(F-I)K_{cy}(k-1,s)-\Phi(k,k)(HK_{xy}(k,s)+H_cK_{cy}(k,s))-\sum_{i=1}^{k}(\Phi(k,i)-\Phi(k-1,i))(HK_{xy}(i,s)+H_cK_{cy}(i,s)). \tag{21}$$

From (19) and (21), we obtain a difference equation for $\Phi(k,s)$.

$$\Phi(k,s)=F_c\Phi(k-1,s)-\Phi(k,k)(HFh(k-1,s)+H_cF_c\Phi(k-1,s)) \tag{22}$$

Substituting (20) into (4), we have

$$\hat{x}(k,k)=F\hat{x}(k-1,k-1)+h(k,k)(y(k)-HF\hat{x}(k-1,k-1)-H_cF_c\sum_{i=1}^{k-1}\Phi(k-1,i)y(i)). \tag{23}$$

Introducing a function f(k) given by

$$f(k)=\sum_{i=1}^{k}\Phi(k,i)y(i), \tag{24}$$

we obtain the recursive equation for the filtering estimate $\hat{x}(k,k)$.

$$\hat{x}(k,k)=F\hat{x}(k-1,k-1)+h(k,k)(y(k)-HF\hat{x}(k-1,k-1)-H_cF_cf(k-1)) \tag{25}$$

The initial condition on (25) at k=0 is $\hat{x}(0,0)=0$ from (4).

Subtracting the equation obtained by putting k→k-1 in (24) from (24), we have

$$f(k)-f(k-1)=\Phi(k,k)y(k)+\sum_{i=1}^{k-1}(\Phi(k,i)-\Phi(k-1,i))y(i). \tag{26}$$

Substituting (22) into (26) and using (4) and (24), we obtain the difference equation for f(k)

$$f(k)=F_cf(k-1)+\Phi(k,k)(y(k)-HF\hat{x}(k-1,k-1)-H_cF_cf(k-1)). \tag{27}$$

The initial condition on (27) at k=0 is f(0)=0 from (24).

Let us introduce a function T(k) given by

$$T(k)=\sum_{i=1}^{k}h(k,i)HB(i). \tag{28}$$

Subtracting the equation obtained by putting k→k-1 in (28) from (28), we have

$$T(k)-T(k-1)=h(k,k)HB(k)+\sum_{i=1}^{k-1}(h(k,i)-h(k-1,i))HB(i). \tag{29}$$

Substituting (20) into (29) and introducing a function W(k) given by

$$W(k)=\sum_{i=1}^{k}\Phi(k,i)HB(i), \tag{30}$$

we obtain a difference equation for T(k)

$$T(k)=FT(k-1)+h(k,k)(HB(k)-HFT(k-1)-H_cF_cW(k-1)). \tag{31}$$

The initial condition on (31) at k=0 is T(0)=0 from (28).

Subtracting the equation obtained by putting k→k-1 in (30) from (30), we have

$$W(k)-W(k-1)=\Phi(k,k)HB(k)+\sum_{i=1}^{k-1}(\Phi(k,i)-\Phi(k-1,i))HB(i). \tag{32}$$

Substituting (22) into (32) and using (28) and (30), we obtain

$$W(k)=F_cW(k-1)+\Phi(k,k)(HB(k)-HFT(k-1)-H_cF_cW(k-1)). \tag{33}$$

The initial condition on (32) at k=0 is W(0)=0 from (30).

Let us introduce a function V(k) given by

$$V(k)=\sum_{i=1}^{k}h(k,i)H_cD(i). \tag{34}$$

Subtracting the equation obtained by putting k→k-1 in (34) from (34), we have

$$V(k)-V(k-1)=h(k,k)H_cD(k)+\sum_{i=1}^{k-1}(h(k,i)-h(k-1,i))H_cD(i). \tag{35}$$

Substituting (20) into (35) and introducing the function U(k) given by

$$U(k)=\sum_{i=1}^{k}\Phi(k,i)H_cD(i), \tag{36}$$

we obtain a difference equation for V(k)

$$V(k)=FV(k-1)+h(k,k)(H_cD(k)-HFV(k-1)-H_cF_cU(k-1)). \tag{37}$$

The initial condition on (37) at k=0 is V(0)=0 from (34).

Subtracting the equation obtained by putting k→k-1 in (36) from (36), we have

$$U(k)-U(k-1)=\Phi(k,k)H_cD(k)+\sum_{i=1}^{k-1}(\Phi(k,i)-\Phi(k-1,i))H_cD(i). \tag{38}$$

Substituting (22) into (38) and using (34) and (36), we obtain

$$U(k)=F_cU(k-1)+\Phi(k,k)(H_cD(k)-HFV(k-1)-H_cF_cU(k-1)). \tag{39}$$

The initial condition on (39) at k=0 is U(0)=0 from (36).

Let G(k), L(k), J(k) and M(k) be given by

$$G(k)=T(k)A^T(k),\ L(k)=W(k)A^T(k),\ J(k)=V(k)C^T(k),\ \ M(k)=U(k)C^T(k). \tag{40}$$

Substituting (31) into $G(k)=T(k)A^T(k)(=T(k)(F^k)^T)$, and using (40) and the relationship $K_{xy}^T(k,k)=HB(k)A^T(k)$ from (8), we obtain the difference equation (14) for G(k). The initial value on (14) at k=0 is G(0)=0 from (28).

Substituting (37) into $J(k)=V(k)C^T(k)(=V(k)(F_c^k)^T)$, and using (40) and the relationship $K_{cy}^T(k,k)=H_cD(k)C^T(k)$ from (9), we obtain the difference equation (15) for J(k). The initial value on (15) at k=0 is J(0)=0 from (34).

Substituting (33) into $L(k)=W(k)A^T(k)(=W(k)(F^k)^T)$, and using (40) and the relationship $K_{xy}^T(k,k)=HB(k)A^T(k)$ from (8), we obtain the difference equation (16) for L(k). The initial value on (16) at k=0 is L(0)=0 from (30).

Substituting (39) into $M(k)=U(k)C^T(k)(=U(k)(F_c^k)^T)$, and using (40) and the relationship $K_{cy}^T(k,k)=H_cD(k)C^T(k)$ from (9), we obtain the difference equation (17) for M(k). The initial value on (17) at k=0 is M(0)=0 from (36). □

We readily notice that $H_cf(k)$ represents the filtering estimate of colored noise $v_c(k)$ and $h(k,k)$ the filter gain for the filtering estimate $\hat{x}(k,k)$ of the state variable $x(k)$. Here, we should note that (10) and (11) are the innovations state-space models for the filtering estimates $\hat{x}(k,k)$ and $f(k)$ of the state variables $x(k)$ and $x_c(k)$.

Let $P_{\tilde{x}}(k,k)$ represent the autovariance function of the filtering error $x(k)-\hat{x}(k,k)$. Let $P_{\hat{x}}(k,k)$ represent the autovariance function of the filtering estimate $\hat{x}(k,k)$. There exists a relationship $P_{\tilde{x}}(k,k)=K_x(k,k)-P_{\hat{x}}(k,k)$. $P_{\tilde{x}}(k,k)$ and $P_{\hat{x}}(k,k)$ are the positive-semidefinite matrices. Also, from (12), we find that $P_{\hat{x}}(k,k)=FG(k)F^T$. Hence, the condition on the existence of the filtering estimate $\hat{x}(k,k)$ in [Theorem 1] is that $P_{\hat{x}}(k,k)(=FG(k)F^T)$ is upper bounded by the autovariance function $K_x(k,k)$ and lower bounded by the zero matrix [5].

$$0\le P_{\hat{x}}(k,k)\le K_x(k,k) \tag{41}$$

A digital simulation example is demonstrated in section 5 to examine the validity of the filtering algorithm in [Theorem 1].

4. Realization of H, F and $K_{xy}(k,k)$ from $K_z(k,s)$

The filtering algorithm of [Theorem 1] uses the information of the observation vectors H and H_c, the system matrices F and F_c, the crossvariance functions $K_{xy}(k,k)$ and $K_{cy}(k,k)$ and the observed value y(k). This section shows the estimation technique for H, F and $K_{xy}(k,k)$ from the autocovariance function $K_z(k,s)$ of the signal z(k). Here, we assume that the signal is the wide-sense stationary stochastic process. That is, $K_z(k,s)=K_z(k-s)$.

The autocovariance function $K_z(k,s)$ of the signal $z(k)$ is represented as

$$K_z(k,s)=H\Phi(k,s)K_{xy}(s,s)1(k-s)+K_{xy}^T(k,k)\Phi^T(s,k)H^T1(s-k),$$

$$\Phi(k,s)=F^{k-s},\ \Phi(k+1,s)=F\Phi(k,s). \tag{42}$$

Here, $\Phi(k,s)$ is called the state-transition function. In terms of the autocovariance function $K_z(k-s)(=K_z(k,s))$ of the wide-sense stationary signal z(k), H, F and $K_{xy}(k,s)$ are estimated by

$$H=[K_z(0)\quad K_z(1)\quad \cdots \quad K_z(n-1)] \times \begin{bmatrix} K_z(0) & K_z(1) & \cdots & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & \cdots & K_z(0) \end{bmatrix}^{-1} \tag{43}$$

$$F=\begin{bmatrix} K_z(1) & K_z(0) & \cdots & \cdots & K_z(n-2) \\ K_z(2) & K_z(1) & \cdots & \cdots & K_z(n-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_z(n-1) & \cdots & \cdots & K_z(1) & K_z(0) \\ K_z(n) & K_z(n-1) & \cdots & \cdots & K_z(1) \end{bmatrix} \times \begin{bmatrix} K_z(0) & K_z(1) & \cdots & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(0) & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & \cdots & K_z(0) \end{bmatrix}^{-1} \tag{44}$$

and

$$K_{xy}(k,k)=K_{xz}(k,k)=[K_z(0)\quad K_z(1)\quad \cdots \quad K_z(n-2)\quad K_z(n-1)]^T. \tag{45}$$

Here, $K_{xz}(k,k)$ represents the crossvariance function of the state variable x(k) with the signal z(k). The necessary and sufficient condition, that the dimension of the state vector x(k) is n, is that the rank of the Hankel matrix Γ

$$\Gamma=\begin{bmatrix} K_z(0) & K_z(1) & K_z(2) & \cdots \\ K_z(1) & K_z(2) & K_z(3) & \cdots \\ K_z(2) & K_z(3) & K_z(4) & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{46}$$

is n [9].

The estimation technique for H_c, F_c and $K_{cy}(k,k)$ from $K_c(k,s)$ obeys the above technique quite similarly with that for H, F and $K_{xy}(k,k)$ from $K_z(k,s)$.

5. A numerical simulation example

Let the observation equation be given by

$$y(k) = Hx(k) + v_c(k) + v(k), \;\; z(k) = x_1(k), \;\; x(k) = [x_1(k) \quad x_2(k)]^T. \tag{47}$$

Let the autocovariance function of the signal z(k) be given by

$K_z(k,s) = HF^{k-s}K_{xy}(s,s),\;\; 0 \le s \le k$. Here, let the observation vector H, the system matrix F for the state variable x(k) and the crossvariance function $K_{xy}(k,k)$ of the state variable x(k) with the observed value y(k) be given by

$$H=[1 \;\; 0], \tag{48}$$

$$F = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}, \;\; a_1 = -0.1, \quad a_2 = -0.8, \quad \sigma = 0.5 \tag{49}$$

and

$$K_{xy}(k,k) = \begin{bmatrix} \dfrac{1+a_2}{(1-a_1^2-a_2^2)(1+a_2)+2a_1^2a_2}\sigma^2 \\ -\dfrac{a_1}{(1-a_1^2-a_2^2)(1+a_2)+2a_1^2a_2}\sigma^2 \end{bmatrix}. \tag{50}$$

Let the autocovariance function of the colored noise $v_c(k)$ be given by

$K_c(k,s) = H_cF_c^{k-s}K_{cy}(s,s),\;\; 0 \le s \le k$. Here, let the observation function H_c, the system matrix F_c for the state variable $x_c(k)$ and the crossvariance function $K_{cy}(k,k)$ of the state variable $v_c(k)$ with the observed value y(k) be given by

$$H_c = 1, \tag{51}$$

$$F_c = 0.9 \tag{52}$$

and

$$K_{cy}(k,k) = \frac{\sigma_c^2}{0.19}, \;\; \sigma_c = 0.01. \tag{53}$$

If we substitute the quantities H, F, $K_{xy}(k,k)$, H_c, F_c and $K_{cy}(k,k)$ into the estimation algorithms of [Theorem 1], the filtering estimate of $z(k)$ is calculated. Fig.1 illustrates the colored noise process for $v_c(0)=0.7$. Fig.2 illustrates the signal $z(k)(=x_1(k))$ (solid line) and the filtering estimate $\hat{z}(k,k)(=\hat{x}_1(k,k))$ (notation by "+—+") for white Gaussian observation noise $N(0,0.2^2)$. Here, $\hat{x}_1(k,k)$ represents the filtering estimate of $x_1(k)$. Fig.3 illustrates the mean-square values of the filtering error $z(k)-\hat{z}(k,k)$ vs. k for white Gaussian observation noises $N(0,0.1^2)$ and $N(0,0.2^2)$. Solid line in Fig.3 depicts the MSV for observation noise $N(0,0.1^2)$. Notation by "+—+" in Fig.3 depicts the MSV for observation noise $N(0,0.2^2)$. The mean-square value is calculated by $\dfrac{\sum_{k=1}^{100}(z(k)-\hat{z}(k,k))^2}{100}$.

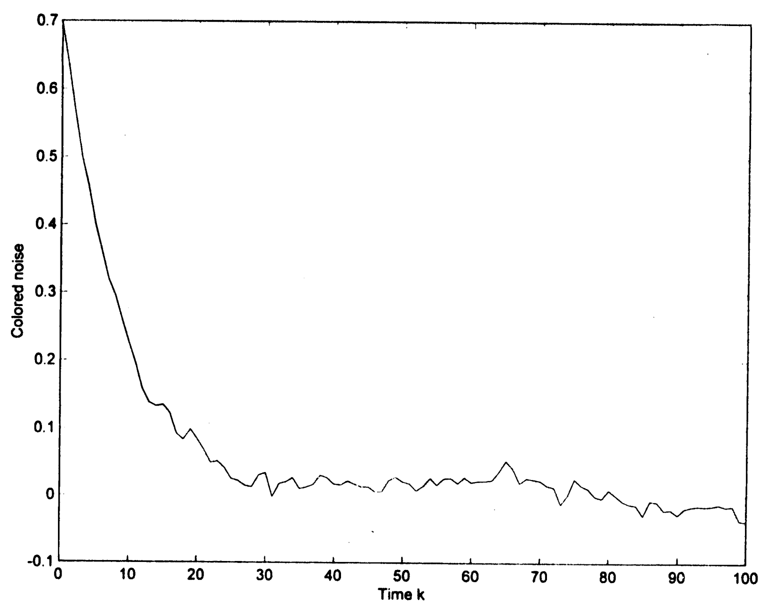

Fig.1 Colored noise process $v_c(k)$ vs. k for $v_c(0)=0.7$.

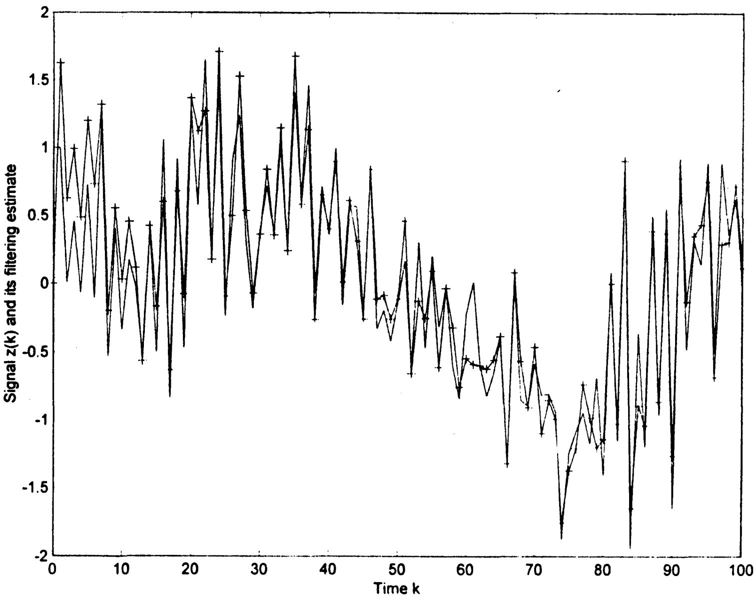

Fig.2 Signal process $z(k)(=x_1(k))$ (solid line) and the filtering estimate $\hat{z}(k,k)(=\hat{x}_1(k,k))$ (notation by "+—+") calculated by the filtering algorithm of [Theorem 1] for white Gaussian observation noise $N(0,0.2^2)$.

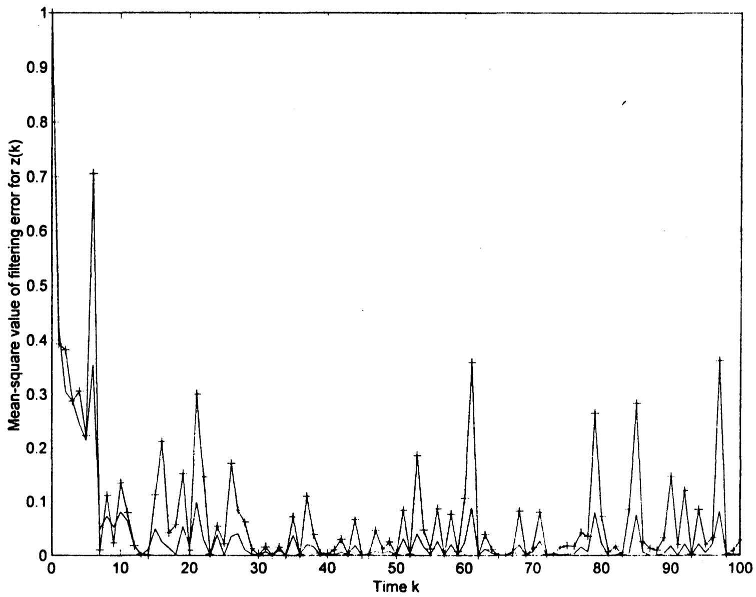

Fig.3 Mean-square values of the filtering error $z(k)-\hat{z}(k,k)$ vs. k for white Gaussian observation noises $N(0,0.1^2)$ and $N(0,0.2^2)$.

Solid line………MSV for the observation noise $N(0,0.1^2)$.

"+—+" ………MSV for the observation noise $N(0,0.2^2)$.

Table 1 MSV of filtering error $z(k)-\hat{z}(k,k)$ for observation noises $N(0,0.1^2)$, $N(0,0.2^2)$, $N(0,0.4^2)$, $N(0,0.7^2)$ and $N(0,1)$.

White Gaussian observation noise	Mean-square value of filtering error $z(k)-\hat{z}(k,k)$
$N(0,0.1^2)$	0.0349223
$N(0,0.2^2)$	0.0755164
$N(0,0.4^2)$	0.187773
$N(0,0.7^2)$	0.315724
$N(0,1)$	0.384757

Table 1 shows the MSV of filtering error $z(k)-\hat{z}(k,k)$ for observation noises $N(0,0.1^2)$, $N(0,0.2^2)$, $N(0,0.4^2)$, $N(0,0.7^2)$ and $N(0,1)$. The MSV decreases as the variance of white Gaussian noise becomes small.

For references, the stochastic processes z(k), $x_1(k)$ and $x_2(k)$ are generated by

$$z(k)(=x_1(k))=[1 \quad 0]\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \tag{54}$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k+1), \quad E[u(k)u(s)]=\sigma^2\delta_K(k-s). \tag{55}$$

Namely,

$$z(k+2)=-a_1z(k+1)-a_2z(k)+u(k+1). \tag{56}$$

Also, the colored noise process is generated by

$$v_c(k+1)=0.9v_c(k)+u_c(k+1), \quad E[u_c(k)u_c(s)]=\sigma_c^2\delta_K(k-s). \tag{57}$$

6. Conclusions

A numerical simulation example in section 5 has shown that the proposed filtering algorithm in [Theorem 1] is feasible.

In this paper, the filtering algorithm using the covariance information has been devised for white Gaussian plus colored observation noise in linear discrete-time wide-sense stationary stochastic systems. The proposed filter is suitable for recursive calculation of the filtering estimate. Also, the condition on the existence of the filtering estimate is given.

References

[1] R.E. Kalman, “A new approach to linear filtering and prediction problems,” Trans. ASME, J. Basic Eng., vol.82D, pp.34-45, 1960.

[2] A.P. Sage and J.L. Melsa, Estimation Theory with Applications to Communications and Control, McGraw-Hill, New York, 1971.

[3] S. Nakamori, “New design of linear least-squares fixed-point smoother using covariance information in continuous systems,” International Journal of Systems Sciences, vol.21, pp.527-536, 1990.

[4] S. Nakamori, “Design of recursive Wiener smoother given covariance information,” IEICE Trans. Fundamentals of Electronics, Communications and Computer Sciences, vol.E79-A, pp.864-872, 1996.

[5] T. Kailath, Lectures on Wiener and Kalman Filtering, Springer-Verlag, 1981.

[6] S. Nakamori, “A new prediction algorithm using covariance information in linear continuous systems,” Automatica, vol.27, pp.1055-1058, 1991.

[7] H. L. Trees, Detection, Estimation and Modulation Theory, Part 1, Wiley, 1968.

[8] H. Kagiwada and R. Kalaba, “An initial value theory for Fedholm integral equation with semi-degenerate kernels,” Journal of Association for Computing Machinery, vol.1, pp.412-417, 1970.

[9] L. M. Silverman, “Realization of linear dynamical systems,” IEEE Trans. Automatic Control, vol.AC-16, pp.554-567, 1971.