

# A New Face Recognition Method Using QR Decomposition

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(Received 26 October, 2010)

## Abstract

This paper presents a new face recognition method using the QR decomposition. The face recognition method is compared with the face recognition method by the snapshot Principal Component Analysis (PCA) mainly from the viewpoint of the computation time consumed for the face recognition. The recognition is based on the distance, measured by the  $L_2$  norm, between the vector of the projected test face image and the vectors of the projected training face images. Specifically, each image is stored in a vector of size  $N$ . Instead of an  $N \times N$  covariance matrix in the PCA, as in the snapshot PCA method, Eigenspace is created from a  $P \times P$  covariance matrix, where  $P$  is the number of persons or training images.

Some pattern recognition examples are shown. It is found that the proposed snapshot QR decomposition method is preferable, in face recognition, to the snapshot PCA method.

**Keyword** : Principal Component Analysis, Face recognition, QR decomposition, Pattern recognition, Snapshot PCA method

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## 1. Introduction

The Principal Component Analysis (PCA) is one of the most successful techniques that have been applied to feature extraction, data compression, pattern recognition and face recognition, etc. The PCA is known as a statistical method with a relation to factor analysis. The face recognition can be categorized into face identification, face classification, or sex determination. The most useful applications contain crowd surveillance, video content indexing, personal identification (e.g. driver's license), mug shots matching and entrance security, etc. The main idea of using the PCA for face recognition is to express the large one-dimensional vector of pixels constructed from two-dimensional face image into the compact principal components of the feature space [1], [2]. This can be called eigenspace projection. Eigenspace is calculated by identifying the eigenvectors of the covariance matrix derived from a set of face images (vectors). The method outlined above can lead to extremely large covariance matrices. For example, images of size  $112 \times 92$  combine to create a data matrix of size  $N \times P$ ,  $N = 10304$ , where  $P$  is number of persons or training images to be recognized, and a covariance matrix of size  $10304 \times 10304$  must be used in the calculation for the face recognition. This is a problem because calculating the covariance matrix and the eigenvectors/eigenvalues of the covariance is computationally demanding. Instead of the PCA, in the snapshot PCA, a  $P \times P$  covariance matrix is used to create the eigenspace rather than a  $10304 \times 10304$  covariance matrix.

This paper presents a new face recognition method using the QR decomposition. For the variance matrix  $S_b$  of (1) in [3], the QR decomposition is applied as

$$S_b = H_b H_b^T = Q R R^T Q^T.$$

In the current method, the  $P \times P$  covariance matrix  $\Omega'$  in the snapshot PCA is factorized by the QR decomposition. Specifically, each training image is stored in a vector of size  $N$ . Instead of the  $N \times N$  covariance matrix, as in the snapshot PCA method, eigenspace is created from the  $P \times P$  covariance matrix, where  $P$  is the number of persons or training images. Each of the mean centered vectors ( $\bar{x}^i$ ) of the training images is projected into the eigenspace. The projection of the mean centered training images into the eigenspace is calculated by (14). Each test image is first mean centered by subtracting the mean image, and is then projected into the same eigenspace defined by  $Q$ . The vector of the projected test image is compared with every vector of the projected training image. The vector of the projected training image, that is the closest to the vector of the projected test image, based on the distance measure, is

recognized as the most similar image to the test image. In this paper, as the similarity measure, the  $L_2$  norm is used.

The proposed snapshot QR decomposition method for the face recognition is compared with the snapshot PCA method mainly from the point of the computation time consumed for recognizing the test face image, based on the values of the  $L_2$  norm, between the vector of the projected test face image and the vectors of the projected training face images.

In section 2, at first, the PCA is introduced. Then, the snapshot PCA method is explained from the viewpoint of computer storage memory. Finally, the snapshot recognition method, based on the QR decomposition, is proposed. In Section 3, to compare the proposed snapshot QR decomposition method with the snapshot PCA method, three pattern recognition examples are shown

## 2. Pattern recognition methods

### 2.1 Image recognition by Principal Component Analysis (PCA) [2]

In this section, Principal Component Analysis [2] is introduced, since the PCA is a fundamental technique in pattern recognition. The PCA is implemented, according to the following steps.

#### Step (1)

Let each image be stored in an  $N \times 1$  column vector as

$$x^i = [x_1^i \ \cdots \ x_N^i]^T, \quad i=1, 2, \dots, P. \quad (1)$$

Here,  $P$  denotes the number of persons or training images. Let  $m$  be the mean vector of the  $P$  number of persons or training image vectors  $x^1, x^2, \dots, x^P$ .

$$m = \frac{1}{P} \sum_{i=1}^P x^i. \quad (2)$$

Let  $\bar{x}^i$  represent the mean centered image vectors, which are calculated by subtracting the mean vector  $m$  from the training image vectors  $x^i$  as

$$\bar{x}^i = x^i - m. \quad (3)$$

#### Step (2)

Let us introduce  $N \times P$  matrix  $\bar{X}$ , which consists of the mean centered vectors  $\bar{x}^1, \bar{x}^2, \dots, \bar{x}^P$ ,

$$\bar{X} = [\bar{x}^1 \quad \bar{x}^2 \quad \cdots \quad \bar{x}^p]. \quad (4)$$

**Step (3)**

Let  $\Omega$  be the covariance matrix given by

$$\Omega = \overline{XX^T}. \quad (5)$$

**Step (4)**

Let  $V$  consist of eigenvectors corresponding to eigenvalues, which are the diagonal elements of a matrix  $\Lambda$ .

$$\Omega V = \Lambda V \quad (6)$$

Let us introduce an  $N \times P$  matrix  $\hat{V}$ , where the eigenvectors  $v_i \in V$ , corresponding to non-zero eigenvalues  $\lambda_i \in \Lambda$ , are ordered from their large to small values, e.g.

$$\hat{V} = [v_1 \quad v_2 \quad \cdots \quad v_p]. \quad (7)$$

**Step (5)**

Let us project each of the mean centered training vectors  $\bar{x}^i$ ,  $i = 1, 2, \dots, P$ , into the eigenspace  $\hat{V}$ . This projection is calculated by the dot product of the mean centered training images with each of the ordered eigenvectors.

$$\tilde{x}^i = \hat{V}^T \bar{x}^i, \quad i = 1, 2, \dots, P \quad (8)$$

Henceforth, the dot product of the mean centered training images  $\bar{x}^i$  and the first eigenvectors generates the first component values in the vectors  $\tilde{x}^i$ .

**Step (6)**

Let  $y^j$  be an  $N \times 1$  vector, which corresponds to the test image. Let  $\bar{y}^j$  be the mean centered vector obtained by subtracting the mean vector  $m$  of the training image vectors from  $y^j$ .

$$\bar{y}^j = y^j - m, \quad \text{where } m = \frac{1}{P} \sum_{i=1}^P x^i \quad (9)$$

Let us project the vector  $\bar{y}^j$  into the eigenspace given by  $\hat{V}$  as

$$\tilde{y}^j = \hat{V}^T \bar{y}^j. \quad (10)$$

The projected test vector  $\tilde{y}^j$  is compared with the projected training vectors  $\tilde{x}^i$ ,  $i = 1, 2, \dots, P$ , respectively. The test image with the closest measure to the training image is found to be the face of the most similar person of the training images, and the unknown test image is identified. In this paper, as similarity between the training images and the test image, the measure by the  $L_2$  norm is used.

However, the PCA method, described above, leads to the covariance matrices with extremely large dimensions. For example, for the  $112 \times 92$  dimensional matrix of the training image, the dimensions of the matrix  $\bar{X}$  of (4) become  $10304 \times P$ . The size of the covariance matrix  $\Omega$  of (5) is  $10304 \times 10304$ . In the PCA method, the calculation of the covariance matrix might be difficult, because, due to the limited main memory, it is impossible for the personal computer to storage all the matrix elements of the covariance matrix. Hence, the face recognition, based on the PCA, could not be proceeded to the next step of calculating the eigenvalues and the eigenvectors further.

## 2.2 Image recognition by snapshot PCA method [2]

It is known that, for an  $N \times P$  matrix, the maximum number of non-zero eigenvectors of the matrix is the smaller value of  $N-1$  and  $P-1$  [5], [6]. From this fact, in the snapshot PCA method [2], instead of the covariance matrix  $\Omega$  of (5), the  $P \times P$  covariance matrix  $\Omega' = \bar{X}^T \bar{X}$  is used. In face recognition,  $P$  is the number of persons or training images, so, usually,  $P$  might be smaller than the number of the image pixels,  $N$ .

**Step (1):** same as **Step (1)** in the PCA method.

**Step (2):** same as **Step (2)** in the PCA method.

**Step (3)**

Let  $\Omega'$  be the  $P \times P$  covariance matrix given by

$$\Omega' = \bar{X}^T \bar{X}. \quad (11)$$

**Step (4)**

Let  $P \times P$  matrix  $V'$  consist of eigenvectors corresponding to eigenvalues, which are the diagonal elements of the matrix  $\Lambda'$ .

$$\Omega'V' = \Lambda'V' \quad (12)$$

### Step (5)

Let us order the eigenvectors  $v_i \in V$ , corresponding to non-zero eigenvalues

$\lambda_i \in \Lambda$  from their large to small values, e.g.

$$V' = [v_1 \quad v_2 \quad \cdots \quad v_p]. \quad (13)$$

Let us project each of the mean centered training vectors  $\bar{x}^i$ ,  $i = 1, 2, \dots, P$ , into the eigenspace. This projection is calculated by

$$\hat{V} = \bar{X}V', \quad \hat{V} = [\hat{v}_1 \quad \hat{v}_2 \quad \cdots \quad \hat{v}_p]. \quad (14)$$

Divide the eigenvectors by each norm.

$$\check{v}_i = \frac{\hat{v}_i}{\|\hat{v}_i\|}, \quad i = 1, 2, \dots, P, \quad \check{V} = [\check{v}_1 \quad \check{v}_2 \quad \cdots \quad \check{v}_p] \quad (15)$$

Let us project each of the vectors of the mean centered training images,  $\bar{x}^i$ ,  $i = 1, 2, \dots, P$ , into the eigenspace. This projection is calculated by the dot product of the vectors of the mean centered training images and the matrix  $\check{V}$  with the eigenvectors ordered, from their large to small values of the corresponding non-zero eigenvalues.

$$\tilde{x}^i = \check{V}^T \bar{x}^i, \quad i = 1, 2, \dots, P \quad (16)$$

Henceforth, the dot product of the mean centered training images and the first eigenvectors generates the first values in the vectors  $\tilde{x}^i$ .

### Step (6)

Let  $y^j$  be an  $N \times 1$  vector corresponding to the test image. Let  $\bar{y}^j$  be the vector given by (9). Let us project the vector  $\bar{y}^j$  into the eigenspace defined by  $\check{V}$  as

$$\tilde{y}^j = \check{V}^T \bar{y}^j. \quad (17)$$

The projected test vector  $\tilde{y}^j$  of (17) is compared with the projected training vectors  $\tilde{x}^i$ ,  $i = 1, 2, \dots, P$ , respectively in terms of the  $L_2$  norm. The test image with the closest measure to the training image is found to be the face of the same person of the training image.

#### 2.4 Snapshot QR decomposition method

In this section, the snapshot  $QR$  decomposition method for the face recognition is presented. As in the snapshot PCA, the snapshot  $QR$  decomposition method is calculated based on the  $P \times P$  covariance matrix  $\Omega' = \bar{X}^T \bar{X}$  in (11). The  $QR$  decomposition of the matrix  $\Omega'$  is calculated as shown in the following **Step (4)**. The calculation steps of the snapshot QR decomposition method are as follows.

**Step (1):** same as **Step (1)** in the PCA method.

**Step (2):** same as **Step (2)** in the PCA method.

**Step (3):** same as **Step (3)** in the snapshot PCA method.

**Step (4)**

Let  $P \times P$  covariance matrix  $\Omega' = \bar{X}^T \bar{X}$  in (11) be factorized, in terms of the  $QR$  decomposition, as

$$\Omega' = QR \quad (18)$$

$Q$ : orthonormal matrix,  $R$ : upper triangular matrix

**Step (5)**

Each of the mean centered training image vectors  $\bar{x}^i$ ,  $i = 1, 2, \dots, P$ , is projected into the eigenspace defined by  $Q$  as

$$\tilde{x}^i = Q^T \bar{x}^i. \quad (19)$$

**Step (6)**

Let  $y^j$  be an  $N \times 1$  vector corresponding to the test image. Let  $\bar{y}^j$  be the vector obtained by subtracting the mean vector  $m$  of the training image vectors from  $y^j$ .

$$\bar{y}^j = y^j - m, \text{ where } m = \frac{1}{P} \sum_{i=1}^P x^i \quad (20)$$

Let us project the vector  $\bar{y}^j$  into the eigenspace defined by  $Q$  as

$$\tilde{y}^j = Q^T \bar{y}^j. \quad (21)$$

The projected test vector is compared with the projected training vectors respectively using the  $L_2$  norm. The test image with the closest measure to the training image is found to be the face of the same person of the training image.

In the PCA and the snapshot PCA, the calculation step to order the eigenvectors, corresponding to non-zero eigenvalues, from their large to small values, is included. In the snapshot QR decomposition method, proposed in this paper, it is not necessary to order the eigenvectors constituting the orthonormal matrix  $Q$ .

In section 3, numerical experiments are demonstrated, concerning the snapshot PCA method and the snapshot QR decomposition method. In sections 3.2 and 3.3, as the human face database, the dataset ORL [4] is used. The ORL face database consists of face images by 40 persons. The 10 face images are stored for each person, and there are 400 face images in total.

### 3. Pattern recognition examples

#### 3.1 Recognition using 4 monochrome training images with $3 \times 3$ gray levels

In this section, an example of the pattern recognition, through eigenspace projection, is shown. That is, the vector of the projected test image is compared to every vector of the projected training images. As a result, the closest image of the 4 training images to the test image is found.

##### 3.1.1 Images with $3 \times 3$ gray levels

Fig.1 shows the test image and 4 training images [2]. Here, simple pattern recognition is performed using the images with  $3 \times 3$  gray levels. The snapshot PCA method and the snapshot QR decomposition method are applied to find out the closest image of the 4 training images to the test image. The similarity between the test image and the training images is measured with the  $L_2$  norm. The training image with the smallest value of the  $L_2$  norm, between the vector of the projected test image and the vectors of the projected training images, are recognized as the closest to the test image.



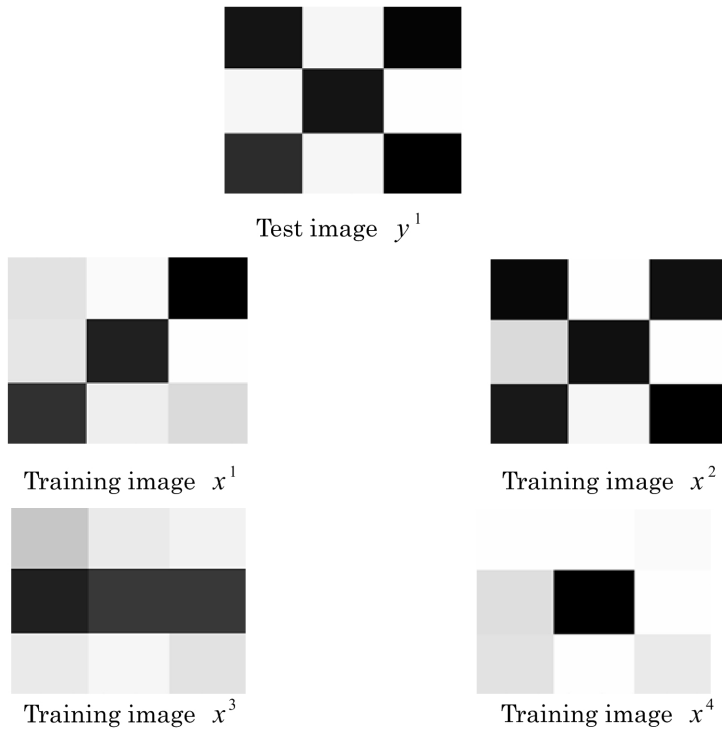


Fig. 1 Test image and 4 training images.

$$x^1 = \begin{bmatrix} 225 \\ 229 \\ 48 \\ 251 \\ 33 \\ 238 \\ 0 \\ 255 \\ 217 \end{bmatrix}, \quad
 x^2 = \begin{bmatrix} 10 \\ 219 \\ 24 \\ 255 \\ 18 \\ 247 \\ 17 \\ 255 \\ 2 \end{bmatrix}, \quad
 x^3 = \begin{bmatrix} 196 \\ 35 \\ 234 \\ 232 \\ 59 \\ 244 \\ 243 \\ 57 \\ 226 \end{bmatrix}, \quad
 x^4 = \begin{bmatrix} 255 \\ 223 \\ 224 \\ 255 \\ 0 \\ 255 \\ 249 \\ 255 \\ 235 \end{bmatrix}, \quad
 y^1 = \begin{bmatrix} 20 \\ 244 \\ 44 \\ 246 \\ 21 \\ 244 \\ 4 \\ 255 \\ 2 \end{bmatrix}$$

Fig. 2 Pixel levels in the pattern of the training images and the test image.

(As the value of the level becomes small, the corresponding pattern becomes dark, whereas, as it becomes large, the corresponding pattern becomes white.)

### 3.1.2 Recognition results

#### (1) Snapshot PCA method

Here, the snapshot PCA method is applied to the current pattern

recognition problem. It is a task to find out the most similar image of the 4 training images,  $x^1 \sim x^4$ , to the test image  $y^1$ . The values of the  $L_2$  norm between the vector of the projected test image  $y^1$  and the vectors of the projected training images,  $x^1$ ,  $x^2$ ,  $x^3$ ,  $x^4$ , are shown in Table 1.

**Table 1** Values of the  $L_2$  norm between the vector of the projected test image and the vectors of the projected training images in terms of the orthonormal matrix  $\tilde{V}$  (see (15)).

$x^1$	$x^2$	$x^3$	$x^4$
296.04	17.84	507.76	449.39

By comparing the values of the  $L_2$  norm, the second training image  $x^2$  is found to be the closest to the test image  $y^1$ . Therefore, the test image  $y^1$  is recognized as belonging to the same class of the second training image  $x^2$ . By viewing the original images, it is seen that image  $y^1$  is very similar to the training image  $x^2$ .

## (2) Snapshot QR decomposition method

By the snapshot QR decomposition method, the values of the  $L_2$  norms, between the vector of the projected test image and the vectors of the 4 projected training images, are shown in Table 2.

**Table 2** Values of the  $L_2$  norm between the vector of the projected test image and the vectors of the projected training images in terms of the orthonormal matrix  $Q$ .

$x^1$	$x^2$	$x^3$	$x^4$
296.04	17.84	507.76	449.39

### 3.1.3 Review of the results

As a result, the values of the  $L_2$  norm by the snapshot PCA method and that by the snapshot QR decomposition method, between the vector of the projected test image and the vectors of the projected training images, are same. Namely, there are no differences in the values of the  $L_2$  norm calculated by the both methods.

### 3.2. Face recognition using face images of 9 persons

In section 3.1, the simulation is implemented by using the simple pattern images with the  $3 \times 3$  gray levels. In section 3.2, the simulation, using the human face images, is demonstrated actually. Let us investigate if a difference can be found between the values of the  $L_2$  norm by the snapshot PCA method and those by the snapshot QR decomposition method.

#### 3.2.1 Face images used in the recognition

The training face images of 9 persons are shown in Fig.3. The respective face image consists of  $112 \times 92$  pixels.

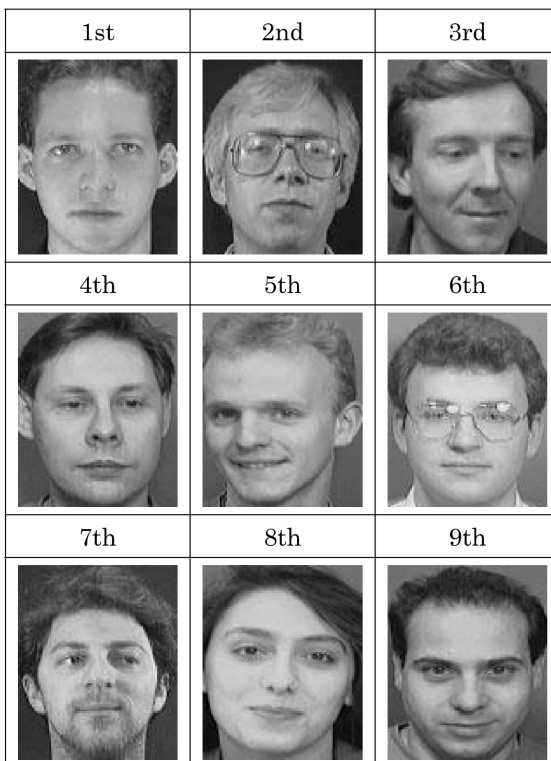


Fig.3 Training face images of 9 persons.

The first person is located on the left side of the top and the second person is on the right of the first person. The 9th person is located on the right side of the lowest.

### 3.2.2 Test face image and average face image

As a test face image, a different face image for the 4th person is used.

The test face image is shown on left side in Fig.4. Also, the average face image of the 9 persons is shown on the right side in Fig.4.



(a) Different face image of the 4th person.



(b) Average face image

Fig.4 Test face image (left) and average face image.

The values of the  $L_2$  norm, between the vector of the projected test face image (different face image of the 4th person) and the vectors of the projected training face images, are evaluated by the snapshot PCA method and the snapshot QR decomposition method.

### 3.2.3 Face recognition results

#### (1) Snapshot PCA method

The values of the  $L_2$  norm, calculated by the snapshot PCA method, are shown in Table 3.

Table 3 Values of the  $L_2$  norm by the snapshot PCA method.

1st	2nd	3rd	4th	5th
1.6904 $\times 10^7$	2.1784 $\times 10^7$	1.1704 $\times 10^7$	4.8297 $\times 10^6$	1.4266 $\times 10^7$
6th	7th	8th	9th	
3.4602 $\times 10^7$	2.4910 $\times 10^7$	1.4633 $\times 10^7$	1.1314 $\times 10^7$	

#### (2) Snapshot QR decomposition method

The values of the  $L_2$  norm, calculated by the snapshot QR decomposition method, are

shown in Table 4.

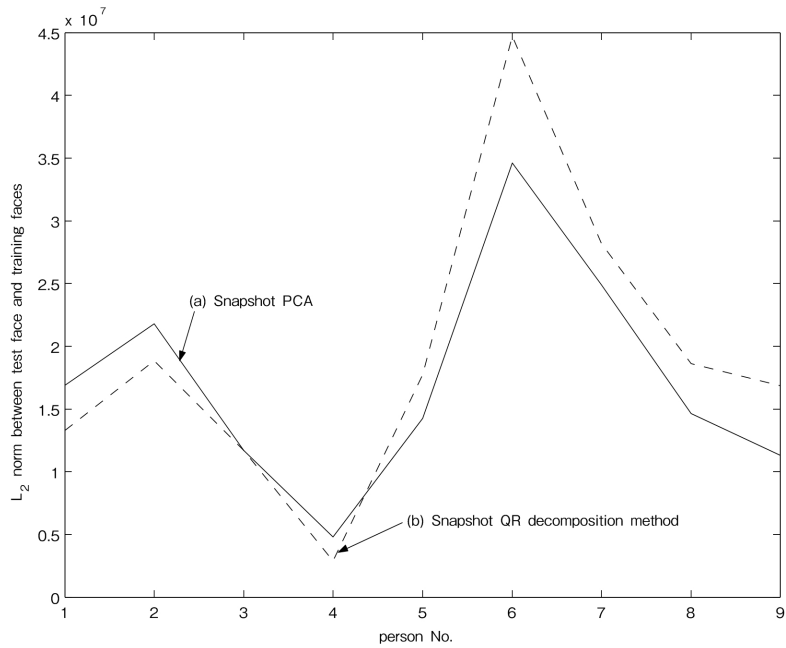
Table 4 Values of the  $L_2$  norm by the snapshot QR decomposition method.

1st	2nd	3rd	4th	5th
1.3307 $\times 10^7$	1.8870 $\times 10^7$	1.1681 $\times 10^7$	2.8977 $\times 10^6$	1.7752 $\times 10^7$
6th	7th	8th	9th	
4.4732 $\times 10^7$	2.8175 $\times 10^7$	1.8631 $\times 10^7$	1.6848 $\times 10^7$	

### 3.2.4 Review of the results

In this section, the recognition experiment is implemented by using 9 human face images. As a result, it is shown that there are differences in the values of the  $L_2$  norm, between the vector of the projected test face image and the vectors of the projected training face images, by the snapshot PCA method, when compared with those by the snapshot QR decomposition method. From Table 3 and Table 4, Fig.5 is illustrated. Fig.5 shows that the values of the  $L_2$  norm by the snapshot QR decomposition method take wider range than those by the snapshot PCA method. The minimum value of the  $L_2$  norm by the snapshot QR decomposition method for the 4th person is smaller than that by the snapshot PCA method.

Also, in the case of the snapshot PCA method, the computation time for the face recognition is 22.3900 seconds, whereas it takes 23.3120 seconds when the snapshot QR decomposition is used. This shows that the snapshot PCA method is slightly shorter than the time consumed by the snapshot QR decomposition method.



**Fig.5** Values of the  $L_2$  norm, between the vector of the projected test face image and the vectors of the 9 projected training face images, vs. person No. by the snapshot PCA method and the snapshot  $QR$  decomposition method.

### 3.3. Face recognition using 40 face images

In this section, the number of the training face images, used for the face recognition, is increased from 9 to 40. The aim of this section is to investigate the differences on the values of the  $L_2$  norms and the computation times between the snapshot QR decomposition method and the snapshot PCA method.

#### 3.3.1 Face images used in the recognition

The training face images of 40 persons are shown in Fig.6.

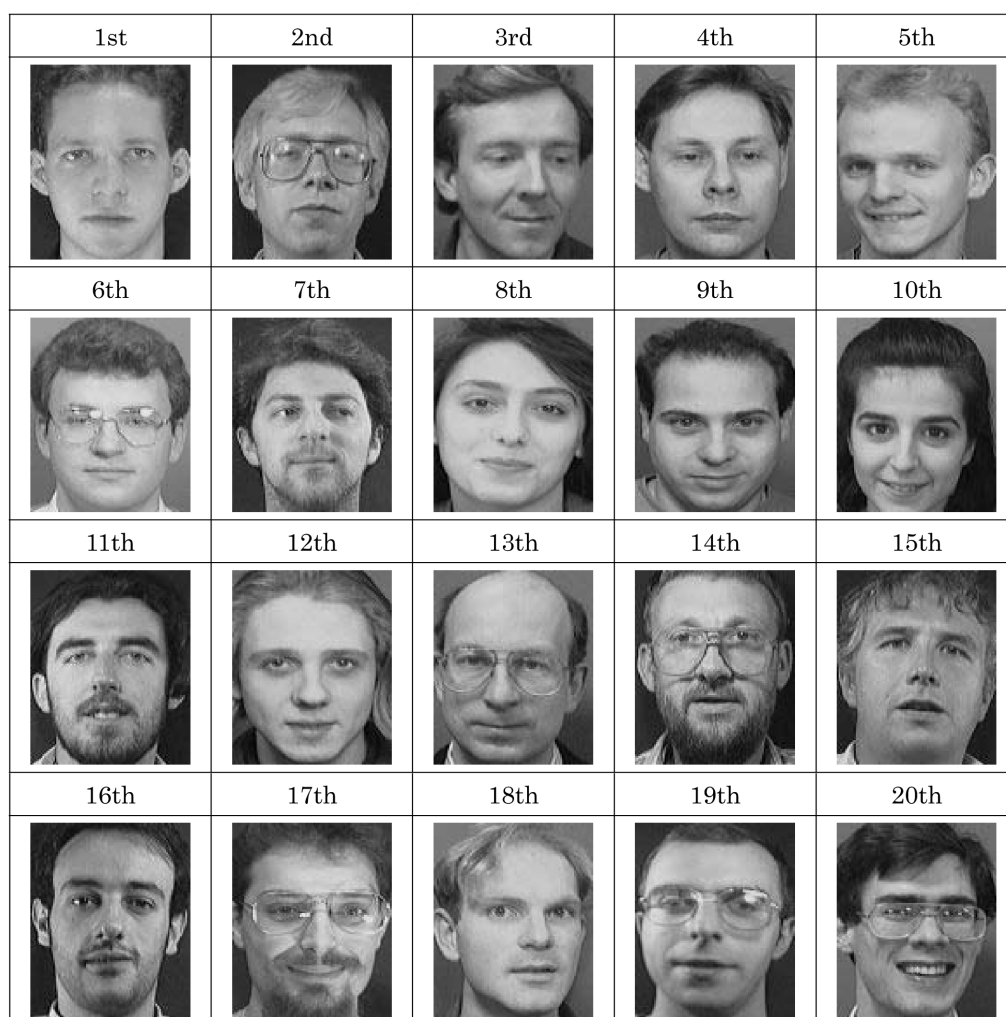




Fig.6 Training face images of 40 persons.

### 3.3.2 Test image and average face image

As a test image, a different face image of the 20th person is used.



(a) Different face image of the 20th person



(b) Average face image of 40 persons

Fig.7 Test face image (left side) and average image (right side).



### 3.3.3 Face recognition results

#### (1) Snapshot PCA method

The values of the  $L_2$  norm calculated by the snapshot PCA method are shown in Table 5.

Table 5 Values of  $L_2$  norm by the snapshot PCA method.

1st	2nd	3rd	4th	5th	6th	7th	8th
3.3973 $\times 10^7$	3.1396 $\times 10^7$	1.7561 $\times 10^7$	2.0183 $\times 10^7$	3.2407 $\times 10^7$	2.3841 $\times 10^7$	2.1201 $\times 10^7$	3.6513 $\times 10^7$
9th	10th	11th	12th	13th	14th	15th	16th
1.5834 $\times 10^7$	2.8016 $\times 10^7$	2.6591 $\times 10^7$	4.1492 $\times 10^7$	4.1972 $\times 10^7$	5.3080 $\times 10^7$	2.5246 $\times 10^7$	3.0797 $\times 10^7$
17th	18th	19th	20th	21th	22th	23th	24th
3.0524 $\times 10^7$	3.7019 $\times 10^7$	4.1264 $\times 10^7$	2.2763 $\times 10^6$	1.3410 $\times 10^7$	1.8146 $\times 10^7$	1.9012 $\times 10^7$	1.8559 $\times 10^7$
25th	26th	27th	28th	29th	30th	31th	32th
2.3854 $\times 10^7$	3.6568 $\times 10^7$	4.0635 $\times 10^7$	4.2308 $\times 10^7$	1.1059 $\times 10^7$	1.2135 $\times 10^7$	1.5985 $\times 10^7$	4.3251 $\times 10^7$
33th	34th	35th	36th	37th	38th	39th	40th
1.4340 $\times 10^7$	3.0112 $\times 10^7$	3.0013 $\times 10^7$	3.4373 $\times 10^7$	3.9686 $\times 10^7$	1.8774 $\times 10^7$	1.9782 $\times 10^7$	2.9509 $\times 10^7$

#### (2) Snapshot QR decomposition method

The values of the  $L_2$  norm calculated by the snapshot QR decomposition method are shown in Table 6.

**Table 6** Values of  $L_2$  norm by the snapshot QR decomposition method.

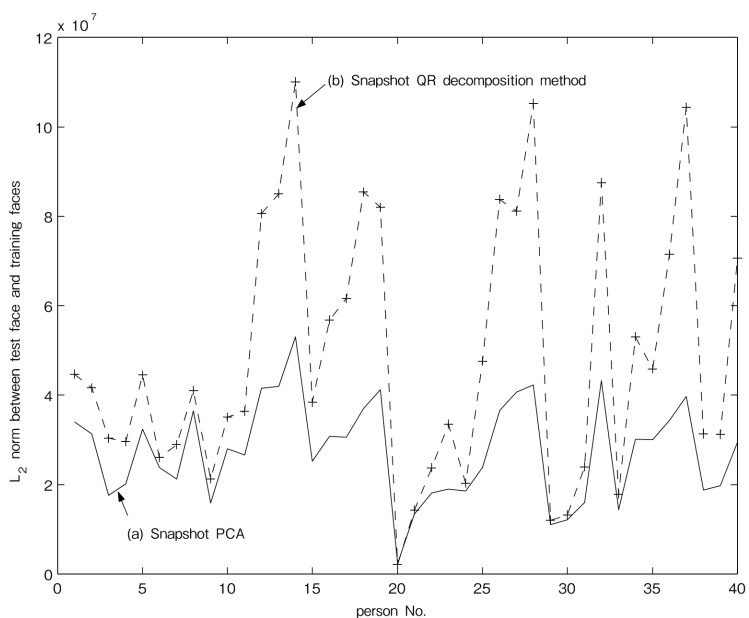
1st	2nd	3rd	4th	5th	6th	7th	8th
4.4664 $\times 10^7$	4.1635 $\times 10^7$	3.0336 $\times 10^7$	2.9595 $\times 10^7$	4.4504 $\times 10^7$	2.6020 $\times 10^7$	2.8927 $\times 10^7$	4.1009 $\times 10^7$
9th	10th	11th	12th	13th	14th	15th	16th
2.1258 $\times 10^7$	3.5083 $\times 10^7$	3.6364 $\times 10^7$	8.0604 $\times 10^7$	8.5035 $\times 10^7$	1.1007 $\times 10^8$	3.8379 $\times 10^7$	5.6797 $\times 10^7$
17th	18th	19th	20th	21th	22th	23th	24th
6.1591 $\times 10^7$	8.5407 $\times 10^7$	8.2003 $\times 10^7$	2.1657 $\times 10^6$	1.4268 $\times 10^7$	2.3748 $\times 10^7$	3.3507 $\times 10^7$	2.0323 $\times 10^7$
25th	26th	27th	28th	29th	30th	31th	32th
4.7561 $\times 10^7$	8.3773 $\times 10^7$	8.1160 $\times 10^7$	1.0526 $\times 10^8$	1.2043 $\times 10^7$	1.3225 $\times 10^7$	2.3946 $\times 10^7$	8.7529 $\times 10^7$
33th	34th	35th	36th	37th	38th	39th	40th
1.7828 $\times 10^7$	5.3056 $\times 10^7$	4.5828 $\times 10^7$	7.1514 $\times 10^7$	1.0439 $\times 10^8$	3.1338 $\times 10^7$	3.1198 $\times 10^7$	7.0621 $\times 10^7$

### 3.3.4 Review of the results

In this section, the face recognition experiment has been implemented using 40 training face images. From Table 5 and Table 6, Fig.8 is illustrated. Fig.8 shows that the values of the  $L_2$  norm by the snapshot QR decomposition method take wider range than those by the snapshot PCA method. The minimum value of the  $L_2$  norm by the QR decomposition method for the 20th person is smaller than that by the snapshot PCA.

It is also interesting, in comparison with the case of 9 training face images, to investigate on the computation time consumed for the case of the 40 training face images. In the case of snapshot PCA method, the computation time for the face recognition is 140.9220 seconds, whereas it takes 112.9840 seconds when the snapshot QR decomposition method is used. Since the computation time by the snapshot QR decomposition method is fairly shorter than the snapshot PCA method, the snapshot QR decomposition method is much preferable, in face recognition, to the snapshot PCA method.

Throughout the simulation, MATLAB program is implemented by the personal computer (AMD Athlon™ 64×2 Dual Core Processor 4200+, 2.21GHz, 896 MB RAM).



**Fig.8** Values of  $L_2$  norm, between the vector of the projected test face image and the vectors of the 40 projected training face images, vs. person No. by the snapshot PCA method and snapshot *QR* decomposition method.

#### 4. Conclusions

In this paper, the snapshot QR decomposition method is presented. The recognition method is compared with the snapshot PCA method in section 3. As a result, in the case of the simulation for the image with  $3 \times 3$  gray levels, there are no differences on the values of the  $L_2$  norm by the both methods. In contrast, when the human face images are used, the differences appear in the values of the  $L_2$  norm between the two methods. This result might be caused by using the face training images with 256 of pixel levels in contrast to the case of using four simple training images with  $3 \times 3$  gray levels.

In the snapshot PCA method and the snapshot QR decomposition method, the test face image is correctly recognized for the both cases using the 40 training face images and 9 training face images from the minimum values of the  $L_2$  norm between the vector of the projected test image and the vectors of the projected training images.

By the snapshot PCA method with 9 training face images, it takes 22.3900 seconds, which might be compared with 23.3120 seconds by the snapshot QR decomposition method. In the face recognition, the snapshot PCA method is slightly faster than the snapshot QR decomposition method. Meanwhile, the computation times, in the face recognition using the 40 training face images, are 140.9220 seconds by the snapshot PCA method and 112.9840 seconds by the snapshot QR decomposition method. This indicates, in the case of 40 training face images, the computation time by the snapshot QR decomposition method is considerably reduced from that by the snapshot PCA method. The face recognition results are correct and values of the  $L_2$  norm in the snapshot QR decomposition method take the wider range than in the snapshot PCA method. On the basis of these considerations, the snapshot QR decomposition method, proposed in this paper, can be applied more efficiently to the face recognition problem than the snapshot PCA method.

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