Stability of dual circular tunnels in cohesive-frictional soil subjected to surcharge loading

by

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Abstract

The stability of dual circular tunnels in cohesive-frictional soils subjected to surcharge loading has been investigated theoretically and numerically assuming plane strain conditions. Despite the importance of this problem, previous research on the subject is very limited compared to that on single tunnels. At present, no generally accepted design or analysis method is available to evaluate the stability of dual circular tunnels/openings in cohesive-frictional soils. In the design stage, it is important to consider the interaction effects of dual circular tunnels. Unlike the case of a single tunnel, the centre-to-centre distance appears as a new problem parameter and plays a key role in tunnel stability. In this study, continuous loading is applied to the ground surface, and a smooth interface condition is modelled. For a series of tunnel diameter-to-depth ratios and material properties, rigorous lower- and upper-bound solutions for the ultimate surcharge loading are obtained by applying finite element limit analysis techniques. For practical suitability, the results are presented in the form of dimensionless stability charts and a table with the actual tunnel stability numbers closely bracketed from above and below. As an additional check on the solutions, upper-bound rigid-block mechanisms have been developed, and the predicted collapse loads from these are compared with those from finite element limit analysis. Finally, a discussion is presented regarding the location of the critical tunnel spacing between dual circular tunnels where interaction no longer occurs.

Keywords: dual circular tunnels, stability, rigid-block mechanism, limit analysis, finite elements

1. Introduction

Accurately assessing the stability of circular tunnels, pipelines and disused mine workings in cohesive-frictional soils is an important task due to the ubiquitous construction of buildings and tunnels in urban areas. The design of tunnels for roads and railways often utilises separate tunnels to carry traffic in opposite directions. In addition, in the expansion of pipelines, underground openings and transportation systems, new tunnels/openings have often been constructed near existing tunnels/openings. In practice, it is often observed that the construction of dual circular tunnels is a better option than a single large circular tunnel, due to the soil characteristics and geological conditions, as well as practical and economic concerns. Because many tunnels and pipelines already exist at deep levels, new tunnels and openings are now often constructed at shallow depths. In these cases, it is important to know how the stability and interaction effects of these tunnels/openings are affected by surcharge loading. Because no generally accepted design or analysis method is currently available for this problem, the goal of this study is to equip design engineers with simple design tools to determine the stability of dual circular tunnels in cohesive-frictional soils subjected to surcharge loading. Previously, the authors have investigated the stability of a circular tunnel in cohesive-frictional soil subjected to surcharge loading (Yamamoto et al. [1]). Compared to the case of a single circular tunnel, the effect of centre-to-centre distance is naturally a key factor in the behaviour of dual circular tunnels. In addition, it is assumed that the problem of interaction between dual tunnels is complex, due to the geometry of the tunnels and the properties of the lining and surrounding soils. Drained loading conditions are considered, and a continuous load is applied to the ground surface. For a series of tunnel diameter-to-depth ratios and material properties, rigorous lower- and upper-bound solutions for the ultimate surcharge loading are found by applying recently developed finite element limit analysis techniques [2,3]. The results are presented as dimensionless stability charts for use by practising engineers, and the actual tunnel stability numbers closely bracket the true solution from above and below. As an additional check on the accuracy of the results, a variety of upper-bound rigid-block mechanisms are developed, and the solutions from these are compared with those from finite element limit analysis.

The stability of circular tunnels has been extensively studied at Cambridge since the 1970s; see, for example, the work reported by Cairncross [4], Atkinson and Cairncross [5], Mair [6], Seneviratne [7] and Davis et al. [8]. Before the 1990s, most published research on tunnel stability focused on the undrained stability of circular tunnels in clay. Later, theoretical solutions for circular tunnel problems under drained conditions were determined by Muhlhaus [9] and by Leca and Dormieux [10]. All of the theoretical studies mentioned so far have investigated the stability of single tunnels only. It would appear that there is very little information available on the interaction effects between dual tunnels. With respect to the research on dual tunnels, a series of centrifuge model tests and numerical simulations of unlined single and parallel tunnels was conducted under plane strain conditions to investigate tunnel stability, arching effects on the soil mass surrounding tunnels, ground movements and collapse mechanisms induced by tunnelling in clayey soil (Wu and Lee [11]; Lee et al. [12]). Chehade and Shahrour [13] presented an analysis of the interaction between twin tunnels with a particular emphasis on the optimisation of both the relative positions of the twin tunnels and the construction procedure, using the finite element program PLAXIS. Osman [14] investigated the stability number of twin tunnels in an undrained clay layer using upper-bound calculations. He presented a new methodology for calculating an upper bound for twin tunnels based on the superposition of the plastic deformation mechanisms of each individual tunnel. Recently, Mirhabibi and Soroush [15] investigated the effect of surface buildings on the ground settlement of twin tunnels, using field data from the Shiraz metro line 1 and the ABAQUS finite element code. The interaction between buildings and the construction of twin tunnels has been studied less. The studies that have been performed on this subject have focused on developing a feasible methodology for estimating, during preliminary design phases, the settlement of surface buildings due to tunnelling.

The application of finite element limit analysis to the undrained stability of shallow tunnels was first considered by Sloan and Assadi [16], who investigated the case of a plane-strain circular tunnel in a cohesive soil whose shear strength varied linearly with depth using linear programming techniques. Later, Lyamin and Sloan [17] considered the stability of a plane-strain circular tunnel in a cohesive-frictional soil using a more efficient nonlinear programming technique. This method can accommodate large numbers of finite elements, resulting in very accurate solutions. To clarify the effects of interaction between tunnels, Wilson et al. [18] investigated the undrained stability of dual square tunnels using finite element limit analysis and upper-bound rigid-block methods. Stability charts were generated for a variety of tunnel depths, material properties and inter-shaft distances. Recently, Yamamoto et al. [19] studied the stability of dual circular tunnels in cohesive-frictional soils subjected to surcharge loading. Upper-bound rigid-block mechanisms were also developed, and the computed surcharge loads were compared with the results of finite element limit analysis. This paper presents the extension of this research in detail.

2. Problem description

The problem description is given in Figure 1. The ground is modelled as a uniform Mohr-Coulomb material with cohesion c', friction angle ϕ' and unit weight γ, assuming drained loading conditions. The dual circular tunnels are of diameter D, depth H, and centre-to-centre distance S, and deformation takes place under plane strain. The stability of the dual circular tunnels shown in Figure 1 is described conveniently by the dimensionless load parameter σ_s / c', which is a function of ϕ', $\gamma D / c'$, H/D and S/D, as shown in Eq. (1).

$$\sigma_s / c' = f(\phi', \gamma D / c', H / D, S / D) \tag{1}$$

Formulating the problem in this manner permits a compact set of stability charts to be constructed, which are useful for design purposes. The problem parameters considered in this paper are $H / D = 1 - 5$, $\phi' = 0° - 20°$, $\gamma D / c' = 0 - 3$ and tunnel spacing S/D=1.25–12.5. Continuous (flexible) loading is applied to the ground surface and the interface condition between the loading and the soil is modelled by setting the shear stress fixed to zero ($\tau = 0$) along the ground surface in the lower-bound analyses, with no velocity constraints being imposed in the upper-bound analyses.

3. Finite element limit analysis

Finite element limit analysis utilises the power of the lower- and upper-bound theorems of plasticity theory, coupled with finite elements, to provide rigorous bounds on collapse loads from both below and above. The underlying limit theorems assume small deformations and a perfectly plastic material with an associated flow rule. The use of a finite element discretisation of the soil, combined with mathematical optimisation to maximise the lower bound and minimise the upper bound, makes it possible to handle problems with layered soils, complex geometries and complicated loading conditions. The formulations of the finite element limit analysis used in this paper originate from those given by Sloan [20,21] and Sloan and Kleeman [22], who employed active-set linear programming and discontinuous stress and velocity fields to solve a variety of practical stability problems. Finite element limit analysis has since evolved significantly with the advent of very fast nonlinear optimisation solvers. The techniques used in this paper are those described by Lyamin and Sloan [2] and Krabbenhøft et al. [3].

Briefly, these formulations use linear-stress (lower-bound) and linear-velocity (upper-bound) triangular finite elements to discretise the soil mass. In contrast to conventional displacement finite element analysis, each node in the limit analysis mesh is unique to a particular element so that statically admissible stress and kinematically admissible velocity discontinuities are permitted in the lower-bound and upper-bound formulations, respectively. The objective of the lower-bound analysis is to maximise the load multiplier subject to the element equilibrium, stress boundary conditions and yield constraints. For the upper-bound analysis, the internal power dissipation minus the rate of work done by the prescribed external forces is minimised with respect to any kinematically admissible failure mechanism (velocity field). A kinematically admissible velocity field is one which satisfies the velocity boundary conditions, compatibility and flow rule constraints. Both formulations result in convex mathematical programs, which (after considering the dual to the upper-bound optimisation problem) can be cast in the following form:

$$\begin{array}{ll} \text{Maximise} & \lambda \\ \text{subject to} & \mathbf{A}\boldsymbol{\sigma} = \mathbf{p}_0 + \lambda\mathbf{p} \\ & f_i(\boldsymbol{\sigma}) \leq 0, \quad i=\{1,\ldots,N\} \end{array} \tag{2}$$

where λ is a load multiplier, $\boldsymbol{\sigma}$ is a vector of stress variables, $\mathbf{A}$ is a matrix of equality constraint coefficients, $\mathbf{p}_0$ and $\mathbf{p}$ are vectors of prescribed and optimisable forces, f_i is the yield function for the stress set i, and N is the number of stress nodes.

The solutions to problem (2) can be found very efficiently by solving the system of non-linear equations that define its Kuhn-Tucker optimality conditions. The interior-point procedure used is based on an efficient conic programming implementation that usually requires 30–50 iterations, regardless of the problem size, and is many times faster than previously employed linear programming schemes. The solutions of the lower- and upper-bound computations bracket the actual collapse load from below and above and thus give a clear indication of the accuracy of the results.

Figure 2 shows the lower-bound/upper-bound half-meshes for *H/D*=1 and *S/D*=2 with smooth interfaces. The meshes are symmetric, and similar meshes are used for the lower- and upper-bound analyses. The boundary conditions for the lower-bound solutions are the normal stress, σ_n, and the shear stress, τ, while the upper-bound solutions require kinematic constraints on the horizontal velocity, u and the vertical velocity, v. The lower-bound/upper-bound mesh has 7,680 triangular elements and 11,412 stress/velocity discontinuities. In the lower-bound analysis, extension elements are included along the soil domain boundaries to represent a semi-infinite material. This feature is necessary to guarantee that the lower bounds are rigorous and is a convenient means of extending the stress field throughout a semi-infinite domain in a manner that satisfies equilibrium, the stress boundary conditions and the yield criterion. The surcharge loading is applied to the full extent of the surface of the soil domain, and thus corresponds to loading over an infinite width. The size of the soil domain for each of the tunnel geometries considered is chosen such that the plasticity zone at failure lies well within the domain. Careful mesh refinement is required to obtain accurate solutions, with the mesh density being high around the tunnel face and transitioning smoothly to larger elements near the boundary of the mesh. Furthermore, the difference between the two bounds then provides an exact measure of the discretisation error in the solution, and can be used to refine the meshes until a suitably accurate estimate of the collapse load is found.

4. Upper-bound rigid-block analysis

Semi-analytical rigid-block methods were used to find upper-bound solutions for the cases considered. These provided an additional check on the limit analysis results. Three types of rigid-block mechanisms were constructed, as shown in Fig. 3. In this figure, A_i is the area of the rigid block i; V_i is the kinematically admissible velocity of block i; V_{ij} is the velocity jump along the discontinuity between blocks i and j; l_{ij} is the distance between points i and j; w is the width of the ground surface subjected to surcharge, σ_s; α, β, γ, δ, ε, λ and ω are the angular parameters that determine the geometry of the rigid-block mechanisms for mechanism 1 (θ is a dependent parameter for mechanism 1); and ϕ' is the dilatancy angle. The compatible velocity diagrams using V_i and V_{ij} are given to the right and left sides of the block mechanisms. All velocities can be obtained using the geometry of these diagrams. With an associated flow rule, we assume the dilatancy angle is equal to the friction angle. Although it is well known that the use of an associated flow rule predicts excessive dilation during shear failure of frictional soils, it is unlikely that this feature will have a major impact on the predicted limit loads for cases with low to moderate friction angles. Generally speaking, any inaccuracy caused by an associated flow rule will be most pronounced for soils with very high friction angles and/or problems that are subject to high degrees of kinematic constraint (which is not the case for the tunnel considered here). Mechanism 1 is a simple roof and side collapse mechanism typical of shallow tunnels, while mechanisms 2 and 3 are more complex and are characterised by collapse of both the roof and side of the tunnel. The total numbers of unknown angular parameters in each of the mechanisms are 7, 8 and 12, respectively. The behaviour of the soil mass was assumed to be governed by the Mohr-Coulomb failure criterion and an associated flow rule. The geometry of the blocks is allowed to vary while being constrained such that their areas and boundary segment lengths remain non-negative. The details of rigid-block analysis can be found in Chen [23]. The upper-bound solutions derived from mechanisms 1–3 are given as follows:

Mechanism 1

$$\sigma_s \le \frac{c'(V_1 l_{10}\cos\phi' + V_{21} l_{12}\cos\phi' + V_2 l_{20}\cos\phi' + V_{31} l_{13}\cos\phi' + V_3 l_{30}\cos\phi') - \gamma(A_1 V_1 + A_2 V_2 \sin(\varepsilon - \phi') + A_3 V_3 \sin(\omega - \phi'))}{V_1 w} \tag{3}$$

Mechanism 2

$$\sigma_s \le \frac{c'(V_{21} l_{12}\cos\phi' + V_2 l_{20}\cos\phi' + V_{31} l_{13}\cos\phi' + V_3 l_{30}\cos\phi' + V_{43} l_{34}\cos\phi' + V_4 l_{40}\cos\phi' + V_{51} l_{15}\cos\phi' + V_5 l_{50}\cos\phi')}{w V_2 \sin(\lambda - \phi')}$$
$$\frac{-\gamma(A_1 V_1 + A_2 V_2 \sin(\lambda - \phi') + A_3 V_3 \sin(\varepsilon - \phi') + A_4 V_4 \sin(\gamma - \phi') + A_5 V_5 \sin(\omega - \phi'))}{w V_2 \sin(\lambda - \phi')} \tag{4}$$

Mechanism 3

$$\sigma_s \le \frac{c'(V_{21} l_{12}\cos\phi' + V_2 l_{20}\cos\phi' + V_{31} l_{13}\cos\phi' + V_3 l_{30}\cos\phi' + V_{43} l_{34}\cos\phi' + V_4 l_{40}\cos\phi' + V_{51} l_{15}\cos\phi' + V_5 l_{50}\cos\phi' + V_{65} l_{56}\cos\phi' + V_6 l_{60}\cos\phi')}{w V_2 \sin(\lambda - \phi')}$$

$$\frac{-\gamma(A_1 V_1 + A_2 V_2 \sin(\lambda - \phi') + A_3 V_3 \sin(\varepsilon - \phi') + A_4 V_4 \sin(\lambda - \phi') + A_5 V_5 \sin(\omega - \phi') + A_6 V_6 \sin(\psi - \phi'))}{w V_2 \sin(\lambda - \phi')} \tag{5}$$

The minimum upper-bound solution for each mechanism was obtained by optimising its geometry using the Hooke and Jeeves algorithm with discrete steps (Bunday [24]). This method works by performing two different types of searches: an exploratory search and a pattern search. The rigid-block analyses are fast and, provided an appropriate mechanism is chosen, can give useful upper bounds on the true collapse load. These solutions can also be used to check the finite element limit analysis solutions.

Mechanism 1 tends to give the best upper-bound solution for smaller values of *H/D* and ϕ'. When *H/D* or ϕ' increases, the best upper-bound solution is given by Mechanism 3, which has the largest number of variables, with 6 rigid blocks and 12 angular parameters.

5. Results and discussion

Figures 4-7(a), (b) and (c) show the plastic multiplier field, power dissipations(velocity plots) and rigid-block mechanisms for various cases. Note that velocity plot is shown only in Figs. 5(b) and 8(b) to indicate a better appreciation for the velocity distribution, instead of the power dissipation. The plastic multiplier field and power dissipation(velocity plot) are obtained from the finite element lower-bound and upper-bound analyses, and the optimum rigid-block mechanism is obtained from the upper-bound rigid-block analysis. The intensity of the plastic multiplier field and the power dissipation is shown by the grey shading. Lower- and upper-bound estimates of the dimensionless load parameter, σ_s / c', obtained from finite element limit analysis and rigid-block analysis, are included in each figure. In this paper, Eq. (6) is used to measure the gap between the bounds and is thus a direct estimate of the error in the solutions.

$$Error(\%) = \pm 100 \times \left(UB - LB\right) / \left(UB + LB\right) \tag{6}$$

Figs. 4(a) and (b) show that for tunnels of relatively shallow depth, small friction angles and close centre-to-centre spacing, a small slip surface originates between dual circular tunnels and a large slip surface originates at the top on the outside of the tunnels. The large slip surface curves toward the ground surface. As these figures show, the failure mechanisms of the rigid-block technique agree well with those observed in the plastic zones and power dissipations. In addition, the upper-bound solution (Fig. 4(c)) obtained from the rigid-block technique is in very good agreement with that from the finite element upper-bound analysis (Fig. 4(b)). Figure 5 shows the case for a shallow depth, moderate friction angles and close centre-to-centre spacing. A clear “cross”-shaped slip surface between dual circular tunnels originates from the top and bottom of the tunnels, and a large slip surface originates around the middle of the outer side of the tunnels. Compared with the case shown in Fig. 4, the large slip surface curves noticeably toward the ground surface, and both the stability numbers, σ_s / c', from the finite element analysis increase due to the moderate friction angles. The upper-bound solution (Fig. 5(c)) obtained from the rigid-block technique is slightly greater than that from the finite element upper-bound analysis (Fig. 5(b)). For the case of moderate depth, small friction angles and close centre-to-centre spacing presented in Figs. 6(a) and (b), a small slip surface between dual circular tunnels enlarges to cover the top and bottom of the tunnels and a large slip surface originates around the outer side of the pair of tunnels. The ultimate surcharge loading in this case is greater than that shown in Fig. 4, due to the increase in *H/D* and *S/D*. The upper-bound solution (Fig. 6(c)) obtained using the rigid-block technique is in good agreement with that from the finite element upper-bound analysis (Fig. 6(b)). Figure 7 shows the case with a moderate depth, small friction angle and moderate centre-to-centre distance. The slip surface between dual circular tunnels enlarges to encompass the entire tunnel, and a large slip surface originates around the bottom of the tunnel. Figures 7(a) and (b) show that plastic zones have developed around the entire tunnel. In this case, the rigid-block result (Fig. 7(c)) tends to be larger than the result from the finite element upper-bound analysis (Fig. 7(b)), due to the moderate depth and centre-to-centre distance.

When *H/D* and *S/D* increase, as shown in Figures 4–7, the failure mechanism slowly extends vertically and transversely and eventually encompasses the entire tunnel. These deeper and wider collapse mechanisms are more complex than those for shallow and narrow tunnels. Figures 4–7 show that the *S/D* parameter plays a key role in the behaviour of the failure mechanism and the increase in bearing capacity due to the effects of interaction. Of the developed rigid-block mechanisms shown in Figure 3, the best upper-bound solutions were almost always obtained from mechanisms 1 and 3, which consist of three and six rigid blocks, respectively. In general, mechanism 1 is suitable for shallow tunnels with low friction angles, and mechanism 3 is suitable for deeper tunnels and high friction angles. In the case of moderate *H/D* and ϕ', when *S/D* is increased, the best upper-bound solutions begin to be produced from mechanism 3. For the case of $\phi' = 20^{\circ}$, all of the best upper-bound solutions were obtained from mechanism 3. The best upper-bound solutions for the cases shown in Figs. 4–7 were given by mechanism 1, mechanism 3, mechanism 1 and mechanism 3, respectively. Additionally, the upper-bound solutions obtained from the rigid block and finite element limit analyses are in good agreement (Figs. 4 and 6), with the rigid block results tending to be larger than the limit analysis results when *H/D* or ϕ' or *S/D* increases (Figs. 5 and 7). This is due to the assumption that the failure mechanism extends from the ground surface into the soil mass with an inclination angle equal to the friction angle ϕ'. In addition, tunnels with a deeper and wider centre-to-centre configuration have a more complex collapse pattern; therefore, the simple rigid-block mechanisms proposed in this study are generally less accurate for deeper and moderate centre-to-centre-distance tunnels than for shallow and close-distance ones. Furthermore, it is more difficult to propose an efficient rigid-block mechanism for cohesive-frictional soils than for purely cohesive materials. As Fig. 7 shows, the collapse mechanism for moderate depth and centre-to-centre distance tunnels is quite wide at the surface and extends further around the bottom of the tunnel. Even using mechanism 3, feasible solutions could not be easily obtained for high values of *H/D* or ϕ' or *S/D*. The numerical results from the finite element limit analysis are shown in Fig. 8. This case is moderate depth, small friction angles and wide centre-to-centre distance (*S/D* is two times greater than that shown in Fig. 7). The plots of plastic multiplier fields and velocity vectors show no interaction between dual circular tunnels, and the failure mechanism becomes that of two individual single tunnels failing independently. As discussed later, the interaction tends to disappear when the centre-to-centre distance exceeds a certain value. These points are regarded as the no-interaction points for dual circular tunnels, and beyond these points, the stability number obtained tends to become constant in the dimensionless stability charts. The errors calculated from Eq. (6) in the cases shown in Figs. 4, 5, 6, 7 and 8 are 5.3%, 3.9%, 14.6%, 6.8% and 4.7%, respectively. The high value of error in the case of Fig. 6 is explained by the fact that the only stability number obtained from the finite element lower-bound analysis is close to zero for the problem parameters considered therein, making it difficult to maintain the relative error at low level. Therefore, the rigorous lower- and upper-bound solutions bracket the true ultimate surcharge load quite accurately for soils with moderate frictional angles.

Figures 9 and 10 compare the stability numbers obtained from both rigid-block and finite element limit analyses for a smooth interface condition. As expected, the stability numbers decrease when $\gamma D / c'$ increases. The ultimate surcharge load increases monotonically with increasing *S/D*, except for the cases of moderate depth and close proximity, $S / D \leq 1.5 - 2.0$ (Figs. 10(a), (b), (c) and (d)). In the cases of moderate depth and close proximity, the ultimate surcharge load exhibits a slight decrease. This is because the extra resistance gained by increasing the width of the pillar is not sufficient to counterbalance the extra soil mass that needs to be supported. For the cases of a) $\phi' = 5^{\circ}, S/D \leq 3.0$, b) $\phi' = 10^{\circ}, S/D \leq 2.5$, c) $\phi' = 15^{\circ}, S/D \leq 2.0$ and d) $\phi' = 20^{\circ}, S/D \leq 1.5$ shown in Fig. 9 and a) $\phi' = 5^{\circ}, S/D \leq 4.0$, b) $\phi' = 10^{\circ}, S/D \leq 3.0$ and c) $\phi' = 15^{\circ}, S/D \leq 2.0$ shown in Fig. 10, the upper-bound solutions from the rigid-block method yield relatively good agreement with those obtained from the finite element limit analysis. However, with increasing *S/D*, the accuracy of the rigid-block mechanisms considered becomes poor. In most cases of $\gamma D/c'=3$, $S/D \le 4.0$ (Figs. 10(b), (c) and (d)), the feasible solutions from the finite element limit analysis could not be obtained because the tunnel collapses under the weight of the soil. It is important to mention the sign convention used for the presentation of stability numbers. A positive stability number implies that a compressive normal stress can be applied to the ground surface up to this value, while a negative stability number means no bearing capacity in the normal sense. The negative range of stability numbers is likely to be of less practical interest than the positive numbers. When *S/D* increases further, the lower- and upper-bound solutions of the finite element limit analysis tend to become constant at a certain point, e.g., at $S/D=3.5$ for $\gamma D/c'=0-3$ (Fig. 9(b)) and at $S/D=8.0$, 7.0, 6.0 and 5.0 for $\gamma D/c'=0$, 1, 2 and 3, respectively (Fig. 10(b)). At such points, the plots of the plastic multiplier fields and power dissipations show no interaction between dual circular tunnels, and the failure mechanism becomes that of two individual single tunnels failing independently. Thus, these points are regarded as the no-interaction points for dual circular tunnels. This information is important for engineering practice. In addition, the no-interaction points are found to decrease when $\gamma D/c'$ increases for each *H/D* and ϕ'. Figure 11 shows the stability numbers for deeper cases, *H/D*=5. When *S/D* increases from 1.25 to approximately 3.0, the stability numbers exhibit a slight decrease, as in Fig. 10. In Figs. 11(a) and (b), the stability numbers obtained from the rigid-block method are included because those can be compared with the finite element limit analysis results. For the cases of a) $\phi'=5^{\circ}, S/D \le 4.0$ and b) $\phi'=10^{\circ}, S/D \le 3.0$, the upper-bound solutions from the rigid-block method agree well with those obtained from the finite element limit analysis. In most cases of $\gamma D/c'=3$, feasible solutions from the finite element limit analysis could not be obtained due to the deeper tunnels. Figs. 11(c) and (d) show that the gap between the lower- and upper-bound solutions became large when *S/D* increases. This tendency would be obvious for the cases of deeper tunnels and high friction angles.

Table 1 provides the stability bounds of dual circular tunnels for *H/D*=1, 3 and 5 and $\phi' = 5^\circ, 10^\circ, 15^\circ$ and 20°. This table is included to provide the numerical values illustrated in Figs. 9, 10 and 11. As this table shows, the rigorous lower- and upper-bound solutions closely bracket the true stability number. The numbers highlighted in bold show the stability bounds at the no-interaction points for dual circular tunnels; beyond these points, the bound solutions obtained tend to become constant in the table. Finally, Figure 12 shows the relationship between the critical tunnel spacing *S/D* (no-interaction point) and the dimensionless tunnel depth *H/D* for each frictional angle considered. When *S/D* does not exceed the value of the no-interaction point, the interaction between dual circular tunnels causes reduced stability. On the other hand, when *S/D* is greater than that the no-interaction point, the interaction cannot be observed and the failure mechanism is that of two individual single tunnels failing independently. In general, the critical tunnel spacing *S/D* tends to decrease as $\gamma D / c'$ increases. For example, Figs. 12(a) and (b) show that the critical tunnel spacings *S/D* are 9.5 and 8.0, respectively, when *H/D*=5, $\phi' = 5^\circ$ and 10°, and $\gamma D / c' = 2$. In these cases, when *S/D* is greater than 9.5 and 8.0, there is no interaction for dual circular tunnels.

6. Conclusions

The stability of plane strain dual circular tunnels in a cohesive-frictional soil subjected to surcharge loading has been investigated using upper-bound rigid-block analysis and finite element limit analysis. The results of these analyses have been presented in the form of dimensionless stability charts and a table. The lower and upper bounds obtained using finite element limit analysis bracket the actual ultimate surcharge load quite accurately for soils with moderate frictional angles. As an additional check of the validity of the finite element limit analysis results, three upper-bound rigid-block mechanisms have been developed. A comparison of the upper-bound solutions obtained from the rigid-block analysis with those from finite element limit analysis shows good agreement when *H/D*, ϕ' and *S/D* are small.

For the cases in which the dual circular tunnels are deeper and in close proximity, it has been confirmed that the stability number σ_s / c' exhibits a slight decrease due to the interaction between the pair of circular tunnels. The interaction tends to disappear when the centre-to-centre distance exceeds a certain value (the no-interaction point). Finally, information regarding the critical tunnel spacing between dual circular tunnels (the no-interaction point) has been summarised as a chart. In future work, rigid-block mechanisms that are suitable for practical use and also efficient for high frictional angles and moderate centre-to-centre distances should be developed.

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