

Table 1. Comparison of the upper bound solutions from various rigid block mechanisms.

H/D	$\phi'$ (deg.)	Mechanism 1				Mechanism 2				Mechanism 3			
		$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$
1	0	2.83	1.74	0.64	-0.45	2.56	1.40	0.22	-1.00	2.54	1.36	0.13	-1.15
	5	3.76	2.53	1.30	0.07	3.15	1.89	0.62	-0.67	3.10	1.81	0.50	-0.84
	10	5.64	4.13	2.62	1.11	4.04	2.64	1.23	-0.19	3.92	2.49	1.04	-0.43
	15	11.68	9.26	6.84	4.42	5.52	3.89	2.25	0.60	5.23	3.58	1.91	0.23
	20	—	—	—	—	8.39	6.36	4.31	2.24	7.57	5.55	3.52	1.46
2	0	4.90	2.80	0.70	-1.41	3.78	1.56	-0.70	-2.99	3.68	1.40	-0.93	-3.29
	5	8.57	5.80	0.24	0.24	5.10	2.59	0.07	-2.47	4.85	2.30	-0.27	-2.85
	10	35.98	28.16	6.18	12.50	7.53	4.54	1.52	-1.55	6.87	3.89	0.87	-2.18
	15	—	—	—	—	13.14	9.20	5.16	0.93	10.99	7.23	3.37	-0.64
	20	—	—	—	—	33.61	26.95	20.06	12.78	22.07	16.65	11.00	4.92
3	0	6.93	3.82	0.72	-2.38	4.71	1.45	-1.85	-5.17	4.50	1.17	-2.21	-5.62
	5	17.59	12.35	7.10	1.85	6.85	3.06	-0.75	-4.60	6.31	2.50	-1.33	-5.18
	10	—	—	—	—	11.58	6.77	1.83	-3.36	9.88	5.23	0.48	-4.49
	15	—	—	—	—	27.35	19.88	12.05	3.36	19.08	12.70	5.96	-1.68
	20	—	—	—	—	229.19	201.39	172.78	142.98	58.97	47.62	35.57	22.22
H/D	$\phi'$ (deg.)	Mechanism 4				Mechanism 5				Mechanism 6			
		$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$
1	0	2.62	1.49	0.34	-0.82	2.65	1.51	0.35	-0.82	2.55	1.39	0.19	-1.05
	5	3.22	1.99	0.75	-0.50	3.26	2.01	0.77	-0.49	3.22	1.95	0.67	-0.64
	10	4.13	2.75	1.36	-0.02	4.18	2.78	1.39	0.00	4.24	2.80	1.36	-0.09
	15	5.64	4.03	2.41	0.79	5.69	4.08	2.45	0.82	5.89	4.19	2.48	0.77
	20	8.55	6.53	4.50	2.45	8.69	6.71	4.72	2.74	8.95	6.79	4.62	2.42
2	0	3.84	1.65	-0.56	-2.78	3.87	1.67	-0.55	-2.78	3.69	1.43	-0.87	-3.21
	5	5.18	2.70	0.21	-2.28	5.21	2.72	0.22	-2.27	4.98	2.45	-0.09	-2.65
	10	7.63	4.66	1.67	-1.35	7.68	4.71	1.71	-1.32	7.29	4.28	1.24	-1.82
	15	13.28	9.35	5.33	1.15	13.63	9.87	6.11	2.35	12.09	8.19	4.21	0.09
	20	33.86	27.17	20.26	12.98	—	—	—	—	25.41	19.59	13.53	7.04
3	0	4.77	1.54	-1.71	-4.97	4.80	1.55	-1.71	-4.97	4.51	1.20	-2.16	-5.55
	5	6.92	3.16	-0.62	-4.41	6.96	3.18	-0.60	-4.40	6.52	2.69	-1.14	-4.99
	10	11.68	6.88	1.98	-3.13	11.73	6.94	2.03	-3.01	10.50	5.77	0.95	-4.05
	15	27.51	20.03	12.20	3.57	50.39	43.39	36.40	29.40	20.98	14.36	7.38	-0.43
	20	229.73	201.72	172.93	143.03	—	—	—	—	68.25	56.35	43.62	29.51

Table 2. Coefficients of empirical equation.

parameters	smooth interface	rough interface
a	1.492521	1.558360
b	0.070218	0.068660
c	1.040522	1.059821
d	0.472325	0.466462
e	0.015413	0.016675
f	1.317933	1.295656
g	-0.495490	-0.479079
h	0.010923	0.010225
k	0.464155	0.478401
m	-0.012827	-0.013290

Table 3. Comparison of the average values from the numerical limit analysis with those obtained from the empirical equation (smooth interface).

$\phi'$ (deg.)	H/D	(LB+UB)/2				Empirical equation			
		$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$
0	1	2.44	1.26	0.02	-1.28	2.49	1.26	0.02	-1.21
	2	3.46	1.18	-1.16	-3.59	3.46	1.09	-1.27	-3.63
	3	4.13	0.80	-2.60	-6.08	4.19	0.73	-2.72	-6.18
	4	4.64	0.27	-4.17	-8.68	4.80	0.27	-4.25	-8.77
	5	5.04	-0.35	-5.81	-11.34	5.33	-0.24	-5.82	-11.39
5	1	2.94	1.68	0.38	-0.95	2.95	1.65	0.35	-0.95
	2	4.42	1.94	-0.57	-3.12	4.28	1.79	-0.70	-3.19
	3	5.47	1.81	-1.88	-5.62	5.32	1.68	-1.96	-5.60
	4	6.30	1.48	-3.39	-8.33	6.21	1.44	-3.32	-8.09
	5	6.99	1.01	-5.04	-11.21	7.00	1.12	-4.75	-10.62
10	1	3.66	2.27	0.87	-0.55	3.65	2.24	0.83	-0.58
	2	5.89	3.11	0.30	-2.55	5.74	2.94	0.14	-2.65
	3	7.64	3.48	-0.76	-5.13	7.47	3.30	-0.88	-5.05
	4	9.12	3.58	-2.13	—	9.01	3.46	-2.08	-7.62
	5	10.42	3.49	-3.76	—	10.41	3.51	-3.40	-10.31
15	1	4.70	3.15	1.60	0.02	4.73	3.16	1.59	0.02
	2	8.32	5.09	1.81	-1.57	8.33	5.00	1.66	-1.68
	3	11.51	6.58	1.44	—	11.60	6.41	1.23	-3.96
	4	14.44	7.77	0.66	—	14.67	7.58	0.49	-6.60
	5	17.18	8.74	-0.48	—	17.60	8.56	-0.48	-9.52
20	1	6.38	4.59	2.79	0.96	6.37	4.61	2.84	1.08
	2	12.77	8.84	4.78	0.49	13.20	8.98	4.75	0.53
	3	19.26	13.06	6.42	-0.96	20.21	13.18	6.14	-0.89
	4	25.84	17.27	7.80	—	27.34	17.24	7.14	-2.96
	5	32.50	21.48	8.95	—	34.55	21.18	7.81	-5.56
25	1	9.28	7.11	4.91	2.68	8.88	6.91	4.93	2.96
	2	22.01	16.86	11.41	5.58	23.12	17.44	11.76	6.08
	3	37.45	28.89	19.47	8.85	40.45	29.92	19.39	8.85
	4	55.15	42.92	28.91	12.47	60.17	43.84	27.52	11.19
	5	74.93	58.79	39.71	16.52	81.87	58.93	35.99	13.05
30	1	14.98	12.15	9.27	6.30	12.71	10.58	8.45	6.31
	2	44.73	37.14	28.96	20.05	45.73	37.62	29.51	21.41
	3	89.58	75.91	60.37	42.59	96.68	78.98	61.28	43.58
	4	150.65	129.01	105.14	75.00	164.46	133.66	102.85	72.05
	5	228.03	197.16	161.85	118.46	248.33	200.99	153.64	106.30
35	1	28.04	23.91	19.66	15.26	18.58	16.48	14.38	12.27
	2	115.57	101.99	86.99	70.42	105.89	93.78	81.67	69.56
	3	289.34	261.33	228.42	190.11	293.05	259.33	225.60	191.88
	4	581.10	533.68	474.96	398.03	603.41	533.66	463.92	394.17
	5	1013.81	942.38	853.55	739.79	1056.60	934.06	811.51	688.96

Table 4. Comparison of the average values from the numerical limit analysis with those obtained from the empirical equation (rough interface).

$\phi'$ (deg.)	H/D	(LB+UB)/2				Empirical equation			
		$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$	$\gamma D/c'=0$	$\gamma D/c'=1$	$\gamma D/c'=2$	$\gamma D/c'=3$
0	1	2.51	1.33	0.10	-1.22	2.56	1.33	0.11	-1.12
	2	3.53	1.26	-1.09	-3.53	3.53	1.18	-1.18	-3.54
	3	4.21	0.88	-2.53	-6.01	4.27	0.81	-2.65	-6.11
	4	4.71	0.35	-4.09	-8.61	4.88	0.34	-4.20	-8.74
	5	5.11	-0.28	-5.74	-11.27	5.42	-0.19	-5.80	-11.40
5	1	3.04	1.77	0.46	-0.88	3.02	1.73	0.43	-0.86
	2	4.52	2.03	-0.50	-3.07	4.37	1.88	-0.61	-3.09
	3	5.58	1.90	-1.82	-5.57	5.42	1.77	-1.88	-5.52
	4	6.42	1.56	-3.34	-8.30	6.32	1.53	-3.26	-8.05
	5	7.12	1.10	-4.99	-11.20	7.12	1.21	-4.71	-10.62
10	1	3.79	2.39	0.97	-0.48	3.76	2.34	0.93	-0.49
	2	6.05	3.23	0.39	-2.50	5.89	3.07	0.25	-2.57
	3	7.83	3.62	-0.68	-5.11	7.66	3.44	-0.78	-5.00
	4	9.34	3.72	-2.07	—	9.23	3.62	-2.00	-7.62
	5	10.66	3.63	-3.71	—	10.67	3.66	-3.35	-10.36
15	1	4.90	3.31	1.72	0.11	4.92	3.31	1.71	0.11
	2	8.60	5.30	1.94	-1.51	8.64	5.23	1.81	-1.60
	3	11.87	6.82	1.58	—	12.02	6.70	1.38	-3.94
	4	14.87	8.05	0.78	—	15.20	7.91	0.62	-6.67
	5	17.68	9.05	-0.38	—	18.22	8.92	-0.38	-9.68
20	1	6.69	4.83	2.98	1.09	6.73	4.88	3.03	1.19
	2	13.31	9.23	5.03	0.62	13.90	9.48	5.06	0.63
	3	20.02	13.60	6.75	-0.86	21.25	13.88	6.51	-0.86
	4	26.84	17.97	8.19	—	28.73	18.14	7.54	-3.05
	5	33.74	22.34	9.39	—	36.29	22.26	8.23	-5.80
25	1	9.78	7.52	5.23	2.90	9.57	7.43	5.29	3.16
	2	23.09	17.70	12.03	5.96	24.79	18.66	12.53	6.40
	3	39.27	30.31	20.46	9.39	43.26	31.91	20.56	9.21
	4	57.84	45.02	30.37	13.19	64.22	46.65	29.07	11.50
	5	78.60	61.67	41.69	17.43	87.25	62.58	37.92	13.25
30	1	15.92	12.94	9.90	6.79	14.04	11.62	9.20	6.78
	2	47.43	39.41	30.72	21.28	49.96	40.86	31.77	22.67
	3	95.21	80.67	64.14	45.29	104.97	85.23	65.48	45.74
	4	159.33	137.23	110.96	79.78	177.75	143.54	109.33	75.12
	5	240.57	209.98	172.28	125.96	267.46	215.06	162.66	110.26
35	1	30.06	25.61	21.11	16.41	21.10	18.54	15.99	13.43
	2	123.85	109.31	93.36	75.73	117.49	103.12	88.75	74.38
	3	312.86	282.38	246.78	205.33	320.79	281.34	241.89	202.44
	4	622.25	571.22	508.04	431.13	654.20	573.44	492.68	411.92
	5	1076.84	999.53	903.00	781.57	1137.03	996.25	855.47	714.70