Restoration of Motion-Blurred Image by RLS Wiener Smoother and Filter

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Abstract

Hitherto, a variety of image restoration techniques have been applied to the image restoration problems degraded by the blur and the observation noise. This paper examines, for the first time, to restore the degraded image by the RLS Wiener fixed-point smoother and filter. In this paper, it is assumed that the degraded image is generated in terms of blur and additional white observation noise. In the RLS Wiener estimators, the following information is used: (1) the system matrix for the state vector, (2) the variance of the state vector, (3) the variance of the white observation noise, and (4) the observation vector. As is shown in the simulation example, the observation vector is related to the point-spread function (PSF), which is known as a parameter to cause blur in two-dimensional images.

A numerical simulation example shows that the proposed image restoration technique, for the degraded image including the blur, by the RLS Wiener fixed-point smoother and filter is feasible.

Keyword : Image Restoration, Blurred Image, RLS Wiener Fixed-Point Smoother, RLS Wiener Filter, Point-Spread Function (PSF)

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1. Introduction

In the image restoration problem for the blurred image, usually, the estimators, such as the Wiener filter [1] and the Kalman filter [2], are used. It is known that the blurred image is generated by the point-spread function (PSF). In applying the estimators, the information of the PSF is used. In [3], the problem of restoration of images blurred by relative motion between the camera and the object of interest is researched. At first, the PSF of the blur is identified and the motion-blurred image is restored in a straightforward manner. In [4], The maximum of difference ratio between the original image and its two digitally re-blurred images are used to estimate the local blur radius. Then adaptive least mean square filters are applied to restore the image. In [5], a blind deconvolution algorithm for motion blurred images are discussed. The PSF is estimated in terms of the modified discrete Randon transform and the 2-D cepstral analysis.

In this paper, using the information of the PSF, we investigate on the application of the RLS Wiener fixed-point smoother and the filter [6] to the restoration of the degraded image including blur and additional observation noise. The RLS Wiener fixed-point smoother and filter require the following information: (1) The system matrix for the state vector, (2) The variance of the state vector, (3) The observation vector, whose components consist of the PSF coefficients, and (4) The variance of white observation noise. The characteristic of the RLS Wiener estimators is that it does not use the information of the input noise variance or the input matrix in the state equations. This indicates that the estimation accuracy of the RLS Wiener estimators is not influenced by the modeling errors for the input noise variance or the input matrix in the state equations. Also, in this paper, the identification method of the PSF is presented in terms of the auto-covariance function $K_x(k,k)$ of the state vector $x(k)$ and the cross-covariance function $K_{yx}(k,k)$ between the observed value $y(k)$ and the state vector $x(k)$.

A numerical simulation example is shown, in terms of the RLS Wiener fixed-point smoother and filter, for the restoration of the degraded image including blur and additional observation noise. A numerical simulation for the identification of the PSF is also demonstrated.

2. RLS Wiener fixed-point smoother and filter

Let a vector observation equation be given by

$$y(k) = z(k) + v(k), \quad z(k) = Hx(k) \tag{1}$$

in linear discrete-time stochastic systems, where $z(k)$ is an n by 1 signal, and $v(k)$ is white observation noise. It is assumed that the state vector $x(k)$ and observation noise $v(k)$ are mutually independent. Let the variance of $v(k)$ be R.

$$E[v(k)v(s)] = R\delta_K(k-s) \quad (2)$$

Here, $\delta_K(\cdot)$ denotes the Kronecker δ function.

Let a fixed-point smoothing estimate $\hat{z}(k,L)$ of $z(k)$ be given by

$$\hat{x}(k,L) = \sum_{i=1}^{L} h(k,i,L)y(i) \quad (3)$$

as a linear transformation of the set of the observed values $\{y(i),\quad 1 \le i \le L\}$. The impulse response function $h(k,s,L)$, which minimizes the mean-square value of the filtering error $x(k) - \hat{x}(k,L)$,

$$J = E[\| x(k) - \hat{x}(k,L) \|^2], \quad (4)$$

[6] satisfies

$$h(k,s,L)R = K_{xy}(k,s) - \sum_{i-1}^{L} h(k,i,L)K_z(i,s). \quad (5)$$

Here, $K_z(k,s)$ represent the auto-covariance function of the signal $z(k)$. $K_z(k,s)$ is expressed by

$$K_z(k,s) = H\Phi^{k-s}K_{xy}(s,s)1(k-s) + K_{xy}^T(k,k)(\Phi^T)^{s-k}H^T 1(s-k),$$
$$K_{xy}(s,s) = K_x(s,s)H^T. \quad (6)$$

Φ is a stable system matrix concerned with the state vector $x(k)$. $K_x(.,.)$ represents the auto-covariance function of $x(k)$ and $1(k-s)$ denotes the unit step function.

2. RLS fixed-point smoothing and filtering algorithms

The RLS Wiener fixed-point and filtering algorithms [6], using the covariance information, are shown in Theorem 1.

Theorem 1. Let the observation equation be given by (1). Let the auto-covariance function $K_x(k,s)$ of the state vector and the variance of white observation noise be given. Then the RLS Wiener fixed-point smoothing and filtering equations consist of (7)-(12) in linear discrete-time stochastic systems.

Fixed-point smoothing estimate of $z(k)$: $\hat{z}(k,L)$

$$\hat{x}(k,L) = \hat{x}(k,L-1) + h(k,L,L)(y(L) - H\Phi\hat{x}(L-1,L-1)) \quad (7)$$

$$h(k,L,L) = (K_x(k,k)(\Phi^T)^{L-k}H^T - q(k,L-1)\Phi^T H^T)(R + HK_x(L,L)H^T - H\Phi S(L-1)\Phi^T H^T)^{-1} \quad (8)$$

$$q(k,L) = q(k,L-1)\Phi^T + h(k,L,L)H(K_x(L,L) - \Phi S(L-1)\Phi^T),\ q(L,L) = S(L) \tag{9}$$

Filtering estimate of $x(k)$: $\hat{x}(k,k)$

$$\hat{x}(L,L) = \Phi\hat{x}(L-1,L-1) + G(L)(y(L) - H\Phi\hat{x}(L-1,L-1)),\ \hat{x}(0,0) = 0 \tag{10}$$

Filter gain: $G(L)$

$$G(L) = (K_x(L,L)H^T - \Phi S(L-1)\Phi^T H^T)(R + HK_x(L,L)H^T - H\Phi S(L-1)\Phi^T H^T)^{-1} \tag{11}$$

$$S(L) = \Phi S(L-1)\Phi^T + G(L)H(K_x(L,L) - \Phi S(L-1)\Phi^T),\ S(0) = 0 \tag{12}$$

3. A numerical simulation example

3.1 Restoration of image by RLS Wiener filter and fixed-point smoother

In this numerical simulation example, the original image is “cameraman.tif”, which is shown in Fig.1. The original image consists of pixel levels of $z(i,j)$, $i = 1,\cdots,256, j = 1,\cdots,256.$ The array of the

Fig.1 Original image, “cameraman.tif”.

original image with 256 × 256 pixels is illustrated in **Fig.1**. The average for the pixel levels is

$\bar{z} = 118.7245$. The state vector $x(k) = [x_1(k) \quad x_2(k) \quad x_3(k) \quad \cdots \quad x_{16}(k)]^T$, $k = (i-1) \times 249 + j$, $1 \le i \le 255$, $1 \le j \le 249$, which is estimated by the RLS Wiener fixed-point smoother and filter, are set as follows.

$x_1((i-1) \times 249 + j) = z(i, j) - \bar{z}$,
$x_2((i-1) \times 249 + j) = z(i, j+1) - \bar{z}$,
$x_3((i-1) \times 249 + j) = z(i, j+2) - \bar{z}$,
$x_4((i-1) \times 249 + j) = z(i, j+3) - \bar{z}$,
$x_5((i-1) \times 249 + j) = z(i, j+4) - \bar{z}$,
$x_6((i-1) \times 249 + j) = z(i, j+5) - \bar{z}$,
$x_7((i-1) \times 249 + j) = z(i, j+6) - \bar{z}$,
$x_8((i-1) \times 249 + j) = z(i, j+7) - \bar{z}$,
$x_9((i-1) \times 249 + j) = z(i+1, j) - \bar{z}$,
$x_{10}((i-1) \times 249 + j) = z(i+1, j+1) - \bar{z}$,
$x_{11}((i-1) \times 249 + j) = z(i+1, j+2) - \bar{z}$,
$x_{12}((i-1) \times 249 + j) = z(i+1, j+3) - \bar{z}$,
$x_{13}((i-1) \times 249 + j) = z(i+1, j+4) - \bar{z}$,
$x_{14}((i-1) \times 249 + j) = z(i+1, j+5) - \bar{z}$,
$x_{15}((i-1) \times 249 + j) = z(i+1, j+6) - \bar{z}$,
$x_{16}((i-1) \times 249 + j) = z(i+1, j+7) - \bar{z}$.

Let us consider the PSF p given by the 2 by 8 matrix

$$p = \begin{bmatrix} 0.13 & 0.1 & 0.1 & 0.1 & 0.1 & 0.01 & 0.01 & 0.01 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}.$$

In this simulation, the observed values are calculated by

$$y(k) = Hx(k) + v(k),\ k = 1, \quad \cdots, \quad 63495,$$

$$H = [p(2,8) \quad p(2,7) \quad \cdots \quad p(2,2) \quad p(2,1) \quad p(1,8) \quad p(1,7) \quad \cdots \quad p(1,2) \quad p(1,1)]. \quad (13)$$

Fig.2 shows the observed imeage for SNR=10 [dB]. **Fig.3** shows the restorated image by the RLS Wiener filter for SNR=10 [dB]. **Fig.4** shows the restorated image by the RLS Wiener fixed-point smoother for SNR=10 [dB] and Lag=3. **Fig.5** shows the observed image for SNR=20 [dB]. **Fig.6** shows the restorated image by the RLS Wiener filter for SNR=20 [dB]. **Fig.7** shows the restorated image by the RLS Wiener fixed-point smoother for SNR=20 [dB] and Lag=3. **Fig.8** shows the observed image for SNR=30 [dB]. **Fig.9** shows the restorated image by the RLS Wiener filter for SNR=30 [dB]. **Fig.10** shows the restorated image by the RLS Wiener fixed-point smoother for SNR=30 [dB] and Lag=3. **Fig.11** shows the observed image for SNR=40 [dB]. **Fig.12** shows the restorated image by the RLS Wiener filter for SNR=40 [dB]. **Fig.13** shows the restorated image by the RLS Wiener fixed-point smoother for SNR=40 [dB] and Lag=3.

In the simulation, to extract the estimate of the pixel with regard to the state vector, the 16th component of the estimated state vector corresponds to the filtering or the fixed-point smoothing estimates in accordance with the proceeding the RLS Wiener filter and the fixed-point smoother recursively.

Fig.2 Observed image for SNR=10 [dB].

Fig.3 Restorated image by the RLS Wiener filter for SNR=10 [dB].

Fig.4 Restorated image by the RLS Wiener fixed-point smoother for SNR=10 [dB] and Lag=3.

Fig.5 Observed image for SNR=20 [dB].

Fig.6 Restorated image by the RLS Wiener filter for SNR=20 [dB].

Fig.7 Restorated image by the RLS Wiener fixed-point smoother for SNR=20 [dB] and Lag=3.

Fig.8 Observed image for SNR=30 [dB].

Fig.9 Restorated image by the RLS Wiener filter for SNR=30 [dB].

Fig.10 Restorated image by the RLS Wiener fixed-point smoother for SNR=30 [dB] and Lag=3.

Fig.11 Observed image for SNR=40 [dB].

Fig.12 Restorated image by the RLS Wiener filter for SNR=40 [dB].

Fig.13 Restorated image by the RLS Wiener fixed-point smoother for SNR=40 [dB] and Lag=3.

Table 1 shows the mean-square values (MSVs) of the filtering errors $z(k)-\hat{z}(k,k)$ and the fixed-point smoothing errors $z(k)-\hat{z}(k,k+Lag)$, $Lag=1,\ 2,\ 3$, for the values of the signal to noise ratio (SNR), $SNR=10,\ 20,\ 30,\ 40$ [dB]. The MSVs of the filtering errors and fixed-point smoothing errors are evaluated in terms of 63495 and 63495$\times Lag$ number of estimation errors respectively.

Table 1 MSVs of filtering and fixed-point smoothing errors.

SNR	MSV of filtering errors $z(k)-\hat{z}(k,k)$	MSV of fixed-point smoothing errors $z(k)-\hat{z}(k,k+Lag)$		
		Lag=1 $\hat{z}(k,k+1)$	Lag=2 $\hat{z}(k,k+2)$	Lag=3 $\hat{z}(k,k+3)$
10[dB]	34.2967	30.8085	29.3185	28.1788
20[dB]	25.5281	20.8055	19.8341	18.9677
30[dB]	17.4893	14.1139	14.2847	13.8033
40[dB]	12.5813	11.6733	11.6729	11.8550

From Table 1, for SNR=10 [dB], since the degradation of the image is influenced by white observation noise to a large extent in comparison with that by the blur, by the RLS Wiener filter and fixed-point smoother, the degradation might be somewhat suppressed. For SNR=40 [dB], the observed image is influenced mainly by the blur. By applying the RLS Wiener filter and fixed-point smoother, it is shown that the blur itself is mainly restorated, since the influence of white observation noise to the degradation is relatively small. From Table 1, the MSV of the RLS Wiener fixed-point smoothing errors is less that that of the RLS Wiener filter. Also, Among the MSVs of the filtering errors and the fixed-point smoothing errors, the MSV of fixed-point smoothing errors for Lag=3 is the smallest for each SNR. This indicates, in addition to the restoration of the image by the RLS Wiener filter, the image restoration by the fixed-point smoother is further effective in improving the estimation accuracy.

In addition to the above properties of the RLS Wiener estimators in the image restoration, both the filter and the fixed-point smoother have the estimation characteristics that the estimation accuracy of the RLS Wiener filter and the fixed-point smoother is improved as the SNR increases.

3.2 Estimation of PSF

As mentioned in "1.Introduction" , the methods to estimate the parameters in the PSF have been researched. The method to estimate the length and the angle of the PSF [3] and the blind estimation technique [5] for estimating the parameters in the PSF are also studied. In this subsection, a simple method to identify the PSF is implemented in terms of the covariance functions.

Let us consider the observation equation (13). The problem is to estimate the PSF and the parameters in the PSF are the components of the observation vector H . It is noticed, in terms of the un-correlation property between the state vector $x(k)$ and the observation noise $v(k)$, that the observation vector can be estimated by the relationship $K_{yx}(k,k) = HK_x(k,k)$ as

$$H = K_{yx}(k,k)K_x^{-1}(k,k) \tag{14}$$

where $K_{yx}(k,k)$ represents the cross-variance function of $y(k)$ with $x(k)$.

The estimation results of the PSF are summarized in **Table 2**. In **Table 2**, the estimates of the PSF and the absolute values of the estimation errors for each parameter of the PSF are shown for SNR=10 [dB], 20 [dB], 30 [dB] and 40 [dB]. As a whole results for the estimated values of the PSF, the estimates are calculated relatively with high accuracy. From **Table 2**, it is noted that, as the SNR becomes large, the estimation accuracy for the PSF is improved.

Table 2 Estimated results of the PSF parameters.

Values of the PSF		The SNR							
		10[dB]		20[dB]		30[dB]		40[dB]	
		Estimate	Absolute value of estimation error	Estimate	Absolute value of estimation error	Estimate	Absolute value of estimation error	Estimate	Absolute value of estimation error
PSF(1)	0.13	0.1448	0.0148	0.1357	0.0057	0.1291	0.0009	0.1305	0.0005
PSF(2)	0.1	0.0805	0.0195	0.0964	0.0036	0.1004	0.0004	0.0994	0.0006
PSF(3)	0.1	0.0976	0.0024	0.1058	0.0058	0.1012	0.0012	0.0999	0.0001
PSF(4)	0.1	0.0953	0.0047	0.0887	0.0113	0.0952	0.0048	0.0999	0.0001
PSF(5)	0.1	0.0964	0.0036	0.1021	0.0021	0.1015	0.0015	0.0999	0.0001
PSF(6)	0.01	0.0112	0.0012	0.0091	0.0009	0.0070	0.0030	0.0100	0.0000
PSF(7)	0.01	0.0265	0.0165	0.0158	0.0058	0.0143	0.0043	0.0105	0.0005
PSF(8)	0.01	0.0110	0.0010	0.0130	0.0030	0.0070	0.0030	0.0100	0.0000
PSF(9)	0.1	0.0861	0.0139	0.0967	0.0033	0.1006	0.0006	0.0996	0.0004
PSF(10)	0.1	0.1199	0.0199	0.0992	0.0008	0.0993	0.0007	0.1006	0.0006
PSF(11)	0.1	0.0990	0.0010	0.0974	0.0026	0.1000	0.0000	0.1000	0.0000
PSF(12)	0.1	0.1020	0.0020	0.1085	0.0085	0.1022	0.0022	0.1001	0.0001
PSF(13)	0.01	0.0040	0.0060	0.0086	0.0014	0.0110	0.0010	0.0098	0.0002
PSF(14)	0.01	0.0174	0.0074	0.0077	0.0023	0.0107	0.0007	0.0102	0.0002
PSF(15)	0.01	-0.0018	0.0118	0.0054	0.0046	0.0074	0.0026	0.0096	0.0004
PSF(16)	0.01	0.0086	0.0014	0.0109	0.0009	0.0129	0.0029	0.0100	0.0000

5. Conclusions

In this paper, the RLS Wiener filter and the fixed-point smoother have been applied to the image restoration problems for the degraded image including blur and white observation noise.

In the numerical simulation example, for the blur by the 2×8 PSF, in the restorated image, the suppression of the blur is ascertained (see **Fig.9**, **Fig.10** for SNR=30 [dB] and **Fig.12** and **Fig.13** for SNR=40 [dB]) relatively to the influence from white observation noise. In the fixed-point smoother for Lag=3, in the case of SNR=30 [dB] and SNR=40 [dB], it is visually grasped that the estimation accuracy of the RLS Wiener fixed-point smoother is better than the that of the RLS Wiener filter. For the case where the SNR is small, such as SNR=10 [dB] and SNR=20 [dB], as the influence of white observation noise is large, it is seen that the degradation by observation noise, in comparison with that by the blur, is improved. From Table 1, as the SNR becomes large, the MSVs of the filtering errors and the fixed-point smoothing errors decrease. Also, the estimation accuracy of the RLS Wiener fixed-point smoother is superior to that of the RLS Wiener filter.

From **Table 2**, as the SNR becomes large, the estimation accuracy for the PSF is improved.

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