A note on Riemannian manifolds with semi-parallel vector fields : Dedicated to Professor Shoji Tsuboi on the occasion of his sixtieth birthday

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A note on Riemannian manifolds with semi-parallel vector fields

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Abstract

In the present paper, we are concerned with Riemannian manifolds with semi-parallel vector fields. In particular, we shall investigate hyperbolic spaces and give a characterization of them in terms of semi-symmetric metrical connection which is naturally obtained from a semi-parallel vector field.

Key word: Semi-parallel vector field, Semi-symmetric connection.

1 Introduction

Hashiguchi[Ha] studied Riemannian manifolds of negative constant curvature by investigating semi-parallel vector fields and semi-symmetric connections. An interesting results in [Ha] is a characterization of hyperbolic spaces in terms of semi-parallel vector fields and semi-symmetric connections. He proved that for a Riemannian manifold (M, g) to be of constant curvature $K = \epsilon\rho^2$ if and only if the system of equation (2.1) below is integrable and an arbitrary initial value of E may be prescribed to determine the field uniquely (Theorem A in [Ha]). Furthermore he proved that for a Riemannian manifold (M, g) to be of negative constant curvature $K = -1$ if and only if an arbitrary vector E with unit length may be extended to the field E uniquely so that this field is parallel with respect to the associated semi-symmetric metrical connection $\tilde{\nabla}$ (Theorem C in [Ha]).

In the present paper, we also investigate Riemannian manifolds of negative constant curvature, say *hyperbolic spaces*, and we shall give some results obtained after [Ha]. The basic idea in this paper follows to Hashiguchi's idea in [Ha]. The author wishes to express his sincere gratitude to Professor Dr. Masao Hashiguchi for invariable suggestions and encouragement.

2 Semi-parallel vector fields

Let M be a smooth connected manifold of $\dim M \geq 3$ with a Riemannian metric g. We denote by ∇ the Levi-Civita connection of (M, g).

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Definition 2.1 ([Ha], [Fu]) A vector field E on M is said to be *semi-parallel* if

$$\nabla_X E = \rho\,[X + \epsilon g(X,E)E] \tag{2.1}$$

is satisfied for all $X \in \Gamma(TM)$, where ρ is a constant and $\epsilon = \pm 1$.

Let $\beta \in \Gamma(TM^*)$ the dual form of E, i.e., $\beta(X) = g(X,E)$. Then, since ∇ is metrical, (2.1) implies

Proposition 2.1 *Let E be a semi-parallel vector field on a Riemannian manifold (M,g). Then its dual β satisfies*

$$(\nabla_X \beta)(Y) = \rho\,[g(X,Y) + \epsilon\beta(X)\beta(Y)] \tag{2.2}$$

for all $X, Y \in \Gamma(TM)$, and thus $(\nabla_X \beta)(Y) = (\nabla_Y \beta)(X)$ which shows that β is closed, i.e., $d\beta = 0$.

In [Ya1], a 1-form β satisfying (2.2) is called a *torse forming 1-form.* Conversely, any torse forming 1-form β defines a semi-parallel vector filed E by $\beta(X) = g(X,E)$.

The existence of semi-parallel vector field is restrictive. We denote by R the curvature tensor of ∇, i.e., $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$. Since

$$\nabla_X\nabla_Y E = \rho\left\{\nabla_X Y + \epsilon X\beta(Y)E + \epsilon\rho\beta(Y)\,[X + \epsilon\beta(X)E]\right\}$$

and

$$\nabla_{[X,Y]}E = \rho\left\{[X,Y] + \epsilon\beta([X,Y])E\right\},$$

the integrability condition $\nabla_X\nabla_Y E - \nabla_Y\nabla_X E - \nabla_{[X,Y]}E = R(X,Y)E$ (Ricci identity) for (2.1) is given by

$$\begin{aligned} R(X,Y)E &= \epsilon\rho d\beta(X,Y)E + \epsilon\rho^2\,[\beta(Y)X - \beta(X)Y] \\ &= \epsilon\rho^2\,[\beta(Y)X - \beta(X)Y], \end{aligned}$$

and thus the integrability condition for (2.1) is given by

$$R(X,Y)E = \epsilon\rho^2\,[g(Y,E)X - g(X,E)Y] \tag{2.3}$$

for all $X, Y \in \Gamma(TM)$. Then, the sectional curvature $K(X \wedge E)$ of the 2-plane $X \wedge E$ is given by

$$K(X \wedge E) = \frac{g\,(R(X,E)E, X)}{\|X \wedge E\|^2} = \frac{g\,(R(X,E)E, X)}{\|X\|^2\,\|E\|^2 - g(X,E)^2} = \epsilon\rho^2$$

for all $X \in \Gamma(TM)$. In particular, if (M,g) is a Riemannian manifold of constant curvature $\epsilon\rho^2$, i.e., if its curvature R satisfies

$$R(X,Y)Z = \epsilon\rho^2\,[g(Y,Z)X - g(X,Z)Y] \tag{2.4}$$

for all $X, Y, Z \in \Gamma(TM)$, then the integrability condition (2.3) is satisfied.

Proposition 2.2 *Let E be a semi-parallel vector field on (M, g) satisfying* (2.1). *The sectional curvature $K(X \wedge E)$ of the 2-plane $X \wedge E$ is the constant $\epsilon\rho^2$ for all $X \in \Gamma(TM)$. In particular, if (M, g) is a Riemannian manifold of constant curvature $K = \epsilon\rho^2$, then M admits a semi-parallel vector field E satisfying* (2.1).

It is reasonable to assume that the parallel displacement with respect to ∇ preserves the length $\|E\|$ of semi-parallel E. Because of

$$\begin{aligned}\nabla_X \|E\|^2 &= 2g(\nabla_X E, E)\\ &= 2\rho\beta(X)\left(1 + \epsilon \|E\|^2\right)\end{aligned}$$

for all $X \in \Gamma(TM)$, if we assume that E has unit length, the constant ϵ must to be $\epsilon = -1$. Hence, in the sequel, we assume that the semi-parallel vector field E has a unit length, i.e., E satisfies

$$\nabla_X E = \rho\left[X - \beta(X)E\right]. \tag{2.5}$$

Then, since $\nabla_E E = \rho\left[E - g(E, E)E\right] = 0$, we have

Proposition 2.3 *Let E be a semi-parallel vector field on a Riemannian manifold (M, g). Then the integral curve of E is a geodesic.*

Example 2.1 Let $\mathbb{H}^2 = \{(x, y) \in \mathbb{R}^2;\ y > 0\}$ be the upper half plane in $\mathbb{R}^2$ with Poincaré metric

$$ds^2 = \frac{1}{y^2}\left(dx \otimes dx + dy \otimes dy\right), \quad g = \frac{1}{y^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The Christoffel symbols Γ^i_{jk} are given by

$$\Gamma^1_{11} = \Gamma^1_{22} = \Gamma^2_{12} = \Gamma^2_{21} = 0, \quad \Gamma^1_{12} = \Gamma^1_{21} = \Gamma^2_{22} = -\Gamma^2_{11} = -\frac{1}{y},$$

and $(\mathbb{H}^2, ds^2)$ is a space of constant curvature $K = -1$. The vector field

$$E(x, y) = \begin{pmatrix} 0 \\ -y \end{pmatrix} = -y\frac{\partial}{\partial y}$$

in $(\mathbb{H}^2, ds^2)$ has the unit length. The dual 1-form β is given by $\beta = -d\log y$. By direct calculations, we have

$$\nabla_X E = \begin{pmatrix} X^1 \\ 0 \end{pmatrix}$$

and

$$X - g(X, E)E = \begin{pmatrix} X^1 \\ 0 \end{pmatrix}$$

for all $X = {}^t(X^1, X^2)$, and thus E satisfies (2.5) for $\rho = 1$. The integral curve of E is a line orthogonal to the x ray which is a geodesic in $(\mathbb{H}^2, ds^2)$. □

For the simplicity for computations, we assume $\rho = 1$ in the sequel if we treat a semi-parallel vector field E of unit length on a Riemannian manifold (M, g).

3 Semi-symmetric connections

We suppose that a Riemannian manifold (M, g) admits a vector field E of unit length. We denote by β the dual 1-form of E, i.e., $\beta(X) = g(X, E)$. We define a linear connection $\tilde{\nabla}$ by

$$\tilde{\nabla}_X Y = \nabla_X Y + [g(X, Y)E - \beta(Y)X] \tag{3.1}$$

for all $X, Y \in \Gamma(TM)$, where ∇ is the Levi-Civita connection of (M, g). The torsion tensor $\tilde{T}$ is given by

$$\tilde{T}(X, Y) = \beta(X)Y - \beta(Y)X,$$

i.e., ∇ is *semi-symmetric*. Then we have (Theorem B in [Ha])

Proposition 3.1 *The semi-symmetric connection* $\tilde{\nabla}$ *defined by* (3.1) *satisfies*

$$\tilde{\nabla} g \equiv 0. \tag{3.2}$$

Moreover, the unit vector field E *is semi-parallel if and only if* E *is parallel with respect to* $\tilde{\nabla}$, *i.e.,*

$$\tilde{\nabla} E \equiv 0. \tag{3.3}$$

By Theorem A in [Ha], if (M, g) is a space of constant curvature $K = -1$, then M admits a unit semi-parallel vector field E, i.e., a vector field E satisfying (2.5) for the constant $\rho = 1$. Now we shall show a characterization of hyperbolic spaces in terms of semi-symmetric connection $\tilde{\nabla}$.

Lemma 3.1 *Let* (M, g) *be a Riemannian manifold with Levi-Civita connection* ∇. *Suppose that* M *admits a semi-parallel vector filed* E *of unit length. The curvature* $\tilde{R}$ *of the semi-symmetric connection* $\tilde{\nabla}$ *defined by* (3.1) *is given by*

$$\tilde{R}(X, Y)Z = R(X, Y)Z + [g(Y, Z)X - g(X, Z)Y] \tag{3.4}$$

for all $X, Y, Z \in \Gamma(TM)$.

Proof. By definition (3.1), we have

$$\begin{aligned}\tilde{\nabla}_X \left(\tilde{\nabla}_Y Z\right) &= \nabla_X (\nabla_Y Z) \\ &\quad + \{g(X, \nabla_Y Z) - \beta(\nabla_Y Z)X - X\beta(Z)Y + Xg(Y, Z)E - \beta(Z)\nabla_X Y\} \\ &\quad - \beta(Z)\{g(X, Y)E - \beta(Y)X\}\end{aligned}$$

and

$$\tilde{\nabla}_{[X,Y]}Z = \nabla_{[X,Y]}Z + \{g([X,Y],Z)E - \beta(Z)[X,Y]\}.$$

Thus we obtain

$$\begin{aligned}\tilde{R}(X,Y)Z &= \tilde{\nabla}_X\tilde{\nabla}_Y Z - \tilde{\nabla}_Y\tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]}Z \\ &= R(X,Y)Z \\ &+ \{g(X,\nabla_Y Z) + Xg(Y,Z) - Yg(X,Z) - g(Y,\nabla_X Z) - g([X,Y],Z)\}\, E \\ &+ \{Y\beta(Z) - \beta(\nabla_Y Z)\}\, X - \{X\beta(Z) - \beta(\nabla_X Z)\}\, Y \\ &+ \{\beta(Y)X - \beta(X)Y\}\, \beta(Z)\end{aligned}$$

Since ∇ is metrical and symmetric, we have

$$\begin{aligned}&g(X,\nabla_Y Z) + Xg(Y,Z) - Yg(X,Z) - g(Y,\nabla_X Z) - g([X,Y],Z) \\ &= g(\nabla_X Y, Z) - g(\nabla_Y X, Z) - g([X,Y],Z) \\ &= g(T(X,Y),Z) \\ &= 0.\end{aligned}$$

Moreover, we have

$$\begin{aligned}\{Y\beta(Z) - \beta(\nabla_Y Z)\}\, X &= \{Yg(E,Z) - g(E,\nabla_Y Z)\}\, X \\ &= g(Z,\nabla_Y E)X \\ &= g\,(Z, Y - \beta(Y)E)\, X \\ &= \{g(Z,Y)X - \beta(Y)\beta(Z)\}\, X\end{aligned}$$

and similarly

$$\{X\beta(Z) - \beta(\nabla_X Z)\}\, Y = \{g(Z,X)Y - \beta(X)\beta(Z)\}\, Y.$$

These equations imply the identity (3.4). □

Lemma 2.1 above implies

Theorem 3.1 *A Riemannian manifold (M,g) is a space of constant curvature $K = -1$ if and only if the following conditions are satisfied.*

(1) *(M,g) admits a semi-parallel vector field E of unit length.*

(2) *The semi-symmetric connection $\tilde{\nabla}$ defined by (3.1) has zero curvature.*

Proof. We suppose that (M,g) satisfies the conditions (1) and (2). Then, the curvature $\tilde{R}$ of the connection $\tilde{\nabla}$ defined by (3.1) vanishes identically. Hence (3.4) implies

$$K(X \wedge Y) = \frac{g\,(R(X,Y)Y,X)}{\|X \wedge Y\|^2} = \frac{g\,(R(X,Y)Y,X)}{\|X\|^2\,\|Y\|^2 - g(X,Y)^2} = -1$$

for all vector fields X, Y on M, and thus (M, g) is a space of constant curvature $K = -1$.

Conversely, we suppose that (M, g) is a space of constant curvature $K = -1$. Then there exists a semi-parallel vector field E of unit length by Hashiguchi's theorem (Theorem C in [Ha]), i.e., E satisfies (2.5) for the constant $\rho = 1$. The semi-symmetric connection $\tilde{\nabla}$ defined by (3.1) satisfies the relation (3.4), and this relation implies $\tilde{R} \equiv 0$. □

Remark 3.1 As is well-known (cf. [Ya2]), a Riemannian manifold is conformally flat if and only if it admits a semi-symmetric metrical connection $\tilde{\nabla}$ whose curvature vanishes identically. Theorem 3.1 shows that a conformally flat Riemannian manifold is reduced a hyperbolic space if such a connection $\tilde{\nabla}$ is defined by (3.1) for a semi-parallel vector field E under the assumption $\dim M \geq 3$. □

4 Some remarks

4.1 Weyl structures

We suppose that a Riemannian manifold (M, g) admits a semi-parallel vector field with unit length. Then, by Proposition 2.1, the dual 1-form β for E satisfies (2.2) for the constant $\rho = 1$ and $\epsilon = -1$, i.e.,

$$(\nabla_X \beta)(Y) = g(X, Y) - \beta(X)\beta(Y) \tag{4.1}$$

for all $X, Y \in \Gamma(TM)$. Then, for the semi-symmetric connection $\tilde{\nabla}$ defined by (3.1), we define a symmetric connection D by

$$\begin{aligned} D_X Y &= \tilde{\nabla}_X Y - \beta(X)Y \\ &= \nabla_X Y + \{g(X, Y)E - \beta(X)Y - \beta(Y)X\}. \end{aligned}$$

Since $\tilde{\nabla}$ is metrical, i.e., $\tilde{\nabla}$ satisfies (3.2), we have

$$Dg = 2\beta \otimes g, \tag{4.2}$$

and thus D is the so-called *Weyl connection* of the conformal $C = [g]$, and the 1-form β is the *Lee form* of (M, C). Moreover, since the dual form β is closed, (D, β) defines a closed *Weyl structure* on M. The curvature R^D of D is given by

$$R^D = \tilde{R} - d\beta \otimes Id = \tilde{R}, \tag{4.3}$$

since β is closed. Thus, if (M, g) admits a semi-parallel vector field E of unit length, then the conformal manifold $(M, C = [g])$ admits a closed Weyl structure.

Conversely, if a closed Weyl structure D satisfies

$$D_X E = -\beta(X)E \tag{4.4}$$

for every $X \in \Gamma(TM)$, then E is a semi-parallel vector field.

Proposition 4.1 *Suppose that a (D, β) a closed Weyl structure is given on a conformal manifold (M, C). The vector field E dual to the Lee form β is semi-parallel if and only if β satisfies* (4.4).

We note that a Weyl structure (D, β) is closed if and only if (i) the Ricci curvature Ric^D of D is symmetric or (ii) D is locally the Levi-Civita connection of a local metric in the class C. Thus a Riemannian manifold (M, g) is conformally flat if and only if the conformal manifold (M, C) with $C = [g]$ admits a closed Weyl structure whose curvature vanishes identically ([Ai]). Consequently, if the curvature R^D of a closed Weyl connection D satisfying the condition (4.4) vanishes identically, then the space (M, g) is reduced a hyperbolic space.

Proposition 4.2 *A Riemannian manifold (M, g) is a space of negative constant curvature $K = -1$ if and only if the conformal manifold (M, C) with $C = [g]$ admits a closed Weyl structure (D, β) satisfying* (4.4) *and $R^D \equiv 0$.*

4.2 The triplet (ϕ, E, β)

In this section, we shall consider a smooth manifold of $\dim M = n$ with a vector field E, 1-form β and an endmorphism $\phi : TM \to TM$ satisfying

$$\beta(E) = 1 \tag{4.5}$$

and

$$\phi^2 = I - \beta \otimes E. \tag{4.6}$$

The endmorphism ϕ may be considered as a deformation of an *almost product structure* on M. An almost product structure on M is the reduction of the structure group $GL(n, \mathbb{R})$ to the sub group $GL(n_1, \mathbb{R}) \times GL(n_2, \mathbb{R})$, where $n_1 + n_2 = n$. A smooth manifold M admits an almost product structure if and only if there exists a section $\psi(\neq Id) \in \Gamma(\mathrm{End}(TM))$ satisfying $\psi^2 = I$. With respect to an adapted frame of $GL(n_1, \mathbb{R}) \times GL(n_2, \mathbb{R})$-structure, the endmorphism ψ is written as

$$\psi = \begin{pmatrix} I_{n_1} & O \\ O & -I_{n_2} \end{pmatrix}.$$

The triplet (ϕ, E, β) exists in a hyperbolic space.

Proposition 4.3 *We suppose that a Riemannian manifold (M, g) admits a semi-parallel vector field E satisfying* (2.5). *Then M admits a triplet (ϕ, E, β) satisfying* (4.5) *and* (4.6). *Especially any hyperbolic space admits such a triplet (ϕ, E, β).*

Proof. Let E be a semi-parallel vector field E of unit length. Then, E satisfies (2.5) for $\rho = 1$. If we denote by β its dual 1-form, the condition (4.5) is satisfied. Then, we shall define an endmorphism ϕ by

$$\phi(X) = \nabla_X E = X - \beta(X)E.$$

By this definition, we have $\phi(E) = E - \beta(E)E = 0$, and

$$\beta(\phi(X)) = g(E, \phi(X)) = g(E, X - \beta(X)E) = 0$$

for an arbitrary $X \in \Gamma(TM)$, and thus we get (2.4). Moreover,

$$\begin{aligned}\phi^2(X) &= \phi(\phi(X)) \\ &= \phi(X) - \beta(\phi(X))E \\ &= \phi(X) \\ &= X - \beta(X)E,\end{aligned}$$

and thus the triplet (ϕ, E, β) satisfies (4.6). □

The proof of the following is the same as the one of Theorem 4.1 in [Bl]

Proposition 4.4 *Suppose that M admits a triplet (ϕ, E, β) satisfying* (4.5) *and* (4.6). *Then the identities*

$$\phi(E) = 0 \tag{4.7}$$

and

$$\beta \circ \phi = 0 \tag{4.8}$$

are satisfied.

We suppose that a triplet (ϕ, E, β) satisfying (4.5) and (4.6) on a manifold M. For any Riemannian metric $\hat{g}$ on M, if we put

$$g(X, Y) = \frac{1}{2}\left[\hat{g}(X, Y) + \hat{g}(\phi X, \phi Y) + \beta(X)\beta(Y)\right],$$

we have

$$g(\phi(X), \phi(Y)) = g(X, Y) - \beta(X)\beta(Y). \tag{4.9}$$

We call a Riemannian metric g satisfying this condition a *compatible metric* of (ϕ, E, β)-structure. In such a case, since $\phi(E) = 0$, we have $\beta(X) = g(X, E)$, i.e., β is the dual form of E.

Proposition 4.5 *If M is a manifold with a triplet (ϕ, E, β) satisfying (4.7) and (4.8), then M admits a compatible metric g of the given triplet (ϕ, E, β). Moreover*

$$g(X, \phi(Y)) = g(\phi(X), Y) \tag{4.10}$$

is satisfied for all section X and Y on M, i.e., ϕ is symmetric.

Proof. By the identities (4.4) and (4.5), we have

$$g\left(\phi^2(X), \phi(Y)\right) = g\left(\phi(X), Y\right).$$

The left hand side of this equation is written as

$$\begin{aligned} g\left(\phi^2(X), \phi(Y)\right) &= g\left(X - \beta(X)E, \phi(Y)\right) \\ &= g\left(X, \phi(Y)\right) - \beta(X)g\left(E, \phi(Y)\right) \\ &= g\left(X, \phi(Y)\right) - \beta(X)\beta\left(\phi(Y)\right) \\ &= g\left(X, \phi(Y)\right), \end{aligned}$$

and thus we obtain (4.10). □

From (4.7), we have $\mathrm{rank}(\phi) \leq n-1$. If X satisfies $\phi(X) = 0$, then (4.6) implies $0 = \phi^2(X) = X - \beta(X)E$, and $X = \beta(X)E$. Thus we have $\mathrm{rank}(\phi) = n-1$ and $L = \ker \phi$ is a line bundle spanned by E. We put

$$N = TM/L.$$

We note that the restriction $\phi|_N = \phi_N$ satisfies $\phi_N^2 = I$, and thus ϕ_N is an almost product structure on the bundle N. Thus ϕ_N has eigenvalues ± 1, and, with respect to a suitable local frame field $\{E, X_i\}$ of $TM = L \oplus N$, the endmorphism ϕ is of the form

$$\phi = \begin{pmatrix} 0 & \mathrm{O} \\ \mathrm{O} & \phi_N \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & I_{n_1} & \mathrm{O} \\ 0 & \mathrm{O} & -I_{n_2} \end{pmatrix}. \tag{4.11}$$

Then, the structure group of $TM = L \oplus N$ is reducible to $1 \times O(n_1) \times O(n_2)$, where n_1 and n_2 are non-negative integer satisfying $n_1 + n_2 = \dim M - 1$.

Conversely, if M admits a $1 \times O(n_1) \times O(n_2)$-structure, then M admits a (ϕ, E, β)-structure. In fact, with respect to the adapted frame $\{E, X_i\}$, we shall define an endmorphism ϕ by (4.11). Moreover, for the unit vector field $E = {}^t(1, 0, \cdots, 0)$ and its dual $\beta = (1, 0, \cdots, 0)$, we have

$$\phi^2 + \beta \otimes E = I. \tag{4.12}$$

Theorem 4.1 *A smooth manifold M admits a triplet (ϕ, E, β) satisfying (4.5) and (4.6) if and only if the structure group $GL(n, \mathbb{R})$ of TM is reducible to $1 \times O(n_1) \times O(n_2)$, where $n_1 \geq 0$ and $n_2 \geq 0$ are integers satisfying $n_1 + n_2 = n - 1$.*

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