Estimation Technique Using Covariance Information with Relation to Wavelet Transformation in Linear Discrete Stochastic Systems

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Abstract

This paper proposes an estimation technique in terms of the recursive least-squares (RLS) Wiener filter by applying the wavelet transformation to the state vector generating a signal in linear discrete-time stochastic systems. The RLS Wiener filter uses the factorized covariance information of the signal and the variance of observation noise. Here, it is assumed that the observed value vector consists of subsequent scalar observed values on the time axis. This paper also examines an estimation technique in terms of the RLS Wiener filter by operating an identity matrix transformation to the state vector. It is advantageous that the estimation accuracy by the proposed estimation method is superior to the standard RLS Wiener filter and the estimation procedure with the identity transformation matrix.

Keywords: Wiener-Hopf equation, linear discrete-time systems, recursive estimation, covariance information, wavelet transformation, autoregressive model, filtering, stochastic signal, estimation

1. Introduction

In [1], an estimation technique for a scalar signal is explored as a combination of the standard Kalman filter and the wavelet transformation in linear discrete-time stochastic systems. This filter is called the wavelet Kalman filter [1]. In this approach, the wavelet transformation is applied to the state variables in the state equations

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generating the stochastic signal. The filter bank is composed of decomposition and composition in the octave division, where the number of the sub-band decomposition is chosen for a data block with length $n=2^2=4$. In the wavelet Kalman filter, the scaling coefficients and the wavelet coefficients, which are given as the outputs in the sub-band decomposition, are estimated by the Kalman filter. The filtering estimate of the signal is obtained by composing the scaling and wavelet coefficients. In the simulation example of [1], a scalar first-order stochastic signal process known as the Brownian random walk is estimated. The estimation accuracy by the estimation method in [1] is superior to the standard Kalman filter. However, even for the first-order model, the derivation of the variance of the input vector in a dynamical system in a data-block form is not straightforward. For the autoregressive (AR) model with the order $n \geq 2$, it is seen that its derivation might be difficult. From this viewpoint, the wavelet Kalman filter might not be suitable for being applied to the estimation of the stochastic signal modeled by the AR model with the order higher than one.

In [2], [3], the estimation algorithms using the covariance information of the signal and observation noise are studied in linear stochastic systems. This kind of recursive filter is called the recursive Wiener filter. In [3], the continuous-time RLS Wiener filter is shown. This paper proposes an estimation technique in terms of the recursive least-squares (RLS) Wiener filter by operating the wavelet transformation to the state vector generating a signal in linear discrete-time stochastic systems. The RLS Wiener filter uses the factorized covariance information of the signal and the variance of observation noise. Here, it is assumed that the observed value vector consists of subsequent scalar observed values on the time axis. This paper also examines an estimation technique in terms of the RLS Wiener filter by operating an n-by-n identity matrix transformation to the state vector. In the simulation example, a speech signal is estimated. It is advantageous that the estimation accuracy by the proposed estimation method is superior to the standard RLS Wiener filter and the estimation procedure with the identity transformation matrix.

2. Wavelet Wiener filtering problem

Let a scalar observation equation be given by

$$y(k) = z(k) + v(k), \quad z(k) = Hx(k) \tag{1}$$

in linear discrete-time stochastic systems, where $z(k)$ is a signal, $x(k)$ is an n-dimensional state vector and $v(k)$ is white observation noise. It is assumed that the signal and the observation noise are zero-mean and are mutually independent. Let the variance of $v(k)$ be R.

$$E[v(k)v(s)] = R\delta_k(k-s) \tag{2}$$

Here, $\delta_k(\cdot)$ denotes the Kronecker δ function.

Let a filtering estimate $\hat{z}(k,k)$ of z(k) be given by

$$\hat{z}(k,k) = \sum_{i=1}^{k} h(k,i)y(i) \tag{3}$$

as a linear transformation of the set of the observed value $\{y(i),\ 1 \le i \le k\}$. The impulse response function $h(k,s)$, which minimizes the mean-square value of the filtering error $z(k)-\hat{z}(k,k)$,

$$J = E\left[\|z(k)-z(k,k)\|^2\right], \tag{4}$$

satisfies

$$h(k,s)R = K_{xy}(k,s) - \sum_{i=1}^{k} h(k,i)HK_{xy}(i,s). \tag{5}$$

Let $K_z(k,s)$ represent the autocovariance function of the signal $z(k)$. $K_z(k,s)$. is expressed as

$$K_z(k,s) = H\Phi^{k-s}K_{xy}(s,s)1(k-s) + K_{xy}^T(k,k)(\Phi^T)^{s-k}H^T 1(s-k),\quad K_{xy}(s,s) = K_x(s,s)H_,^T \tag{6}$$

where $K_x(s,s)$ represents an autovariance function of the state vector $x(s)$, $K_{xy}(k,k)$ represents a crossvariance function of $x(k)$ with $y(k)$ and $1(k\text{-}s)$ denotes the unit step function.

2. RLS filtering equation

The filtering equations using the covariance information are shown in Theorem 1.

THEOREM 1. [2] Let the observation equation be given by (1). Let the autocovariance function $K_z(k,s)$ of the signal $z(k)$ be expressed by (6) in the factorized functional form.

Then the RLS filtering equations consist of (7)-(9) in linear discrete-time stochastic systems.

Filtering estimate of $x(k)$: $\hat{x}(k,k)$

$$\hat{x}(k,k)=\Phi\hat{x}(k-1,k-1)+G(k)(y(k)-H\Phi\hat{x}(k-1,k-1)),\quad \hat{x}(0,0)=0 \tag{7}$$

Filter gain: $G(k)$

$$G(k)=(K_{xy}(k,k)-\Phi S(k-1)\Phi^T H^T)(R+HK_{xy}(k,k)-H\Phi S(k-1)\Phi^T H^T)^{-1} \tag{8}$$

$$S(k)=\Phi S(k-1)\Phi^T+G(k)(K_{xy}^T(k,k)-H\Phi S(k-1)\Phi^T),\quad S(0)=0 \tag{9}$$

3. Discrete wavelet transformation, filter banks and estimation technique

Let us apply the filtering algorithm in Theorem 1 to the estimation of the signal in relation with the wavelet transformation.

Let us introduce a signal vector $Z(k)$ consisting of components, $z_1(k),\ z_2(k),\ \cdots,\ z_n(k)$, as

$$Z(k)=\begin{bmatrix} z_1(k) \\ z_2(k) \\ \vdots \\ z_n(k) \end{bmatrix},\quad z_1(k)=z(k),\quad z_2(k)=z(k+1),\quad z_3(k)=z_2(k+1),\cdots,\quad z_n(k)=z_{n-1}(k+1). \tag{10}$$

Let $Z(k)=x(k)$ be valid and the state vector $x(k)$ be given by

$$x(k)=\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix},\quad x_2(k)=x_1(k+1),\quad x_3(k)=x_2(k+1),\cdots,\quad x_n(k)=x_{n-1}(k+1). \tag{11}$$

Also,let the signal $z(k)=Hx_1(k)$ be observed by

$$y(k)=H\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}+v_1(k),\quad H=[10\cdots0]. \tag{12}$$

Let the signal process $z(k)$ be generated by the AR model of order n.

$$z(k) = -a_1 z(k-1) - a_2 z(k-2) - \cdots - a_n z(k-n) + e(k), \quad e(k) = u(k-n). \tag{13}$$

In this case, the state variables $x_i(k)$, i=1, 2, ⋯ , n, are generated by the stochastic system of order n,

$$x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(k),$$

$$E[u(k)u(s)] = \sigma^2 \delta_K (k-s). \tag{14}$$

Let $K_z(i)$, i=1, 2, ⋯ , n, represent the autocovariance function of $z(k)$. Then the autovariance function of the state vector $x(k)$ is calculated as

$$K_x(k,k) = E[x(k)x^T(k)]$$

$$= \begin{bmatrix} K_z(0) & K_z(1) & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & K_z(0) \end{bmatrix} \tag{15}$$

In (13), the AR parameters a_i, i =1, 2,⋯, n, are calculated by the Yule-Walker equations [2]

$$\begin{bmatrix} K_z(0) & K_z(1) & \cdots & K_z(n-1) \\ K_z(1) & K_z(0) & \cdots & K_z(n-2) \\ \vdots & \vdots & \vdots & \vdots \\ K_z(n-2) & \cdots & \cdots & K_z(1) \\ K_z(n-1) & K_z(n-2) & \cdots & K_z(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} -K_z(1) \\ -K_z(2) \\ \vdots \\ -K_z(n) \end{bmatrix}. \tag{16}$$

Therefore, the system matrix Φ in (14) is obtained by the autocovariance data $K_z(i)$, i= 0, 1, ⋯, n.

Let us introduce an observed value vector $Y(k)$ consisting of observed values as a sum of state vector and observation noise vector as follows

$$Y(k)=\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_n(k) \end{bmatrix}=\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}+\begin{bmatrix} v(k) \\ v(k+1) \\ \vdots \\ v(k+n-1) \end{bmatrix},\ y_1(k)=y(k),\ y_2(k)=y_1(k+1),\ y_3(k)=y_2(k+1),\cdots,$$

$$y_n(k)=y_{n-1}(k+1). \tag{17}$$

For the wavelet transformation of a two-channel filter bank, the value of n is 4. The discrete wavelet transformation can be implemented by an octave-band filter bank as shown in Fig.1. The input to the filter bank is $x(k)=\left[x_1(k)\ x_2(k)\ x_3(k)\ x_4(k)\right]^T$. Here, the scaling coefficient $s_2(k)$ and the wavelet coefficients $w_2(k)$, $w_{1,2}(k)$ and $w_{1,2}(k)$ at time k are obtained as the outputs in Fig.1.

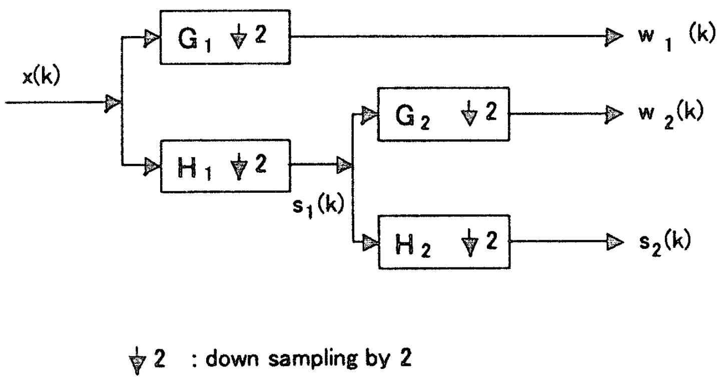

Fig.1 A two-channel filter bank.

In the Haar transformation for n=2, G_1 is a high pass filter [4]- [7] represented by

$$G_1=\begin{bmatrix} 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix}, \tag{18}$$

which plays a role of down sampling simultaneously. Output of G_1 is the wavelet coefficient $w_1(k)=\left[w_{1,1}(k)\ \ w_{1,2}(k)\right]^T$. Here, $w_{1,1}(k)$ and $w_{1,2}(k)$ are introduced as vector components of $w_1(k)$. H_1 is a low pass filter denoted by

$$H_1=\begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}, \tag{19}$$

which also plays a role of down sampling at the same time. Output of H_1 is the scaling coefficient $s_1(k)$. Output of a high pass filter G_2 = [0.5 -0.5] is a wavelet coefficient $w_2(k)$. Output of a low pass filter $H_2 = [0.5 \quad 0.5]$ is a scaling coefficient $s_2(k)$.

As a result, the wavelet transformation is reduced to

$W(k) = Tx(k)$,

$$W(k) = \begin{bmatrix} s_2(k) \\ w_2(k) \\ w_{1,1}(k) \\ w_{1,2}(k) \end{bmatrix}, \quad T = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix}, \quad x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}. \tag{20}$$

In the wavelet filtering technique, the transformed process $W(k)$ of the state vector is estimated by (21)-(23) which are obtained as an extention of Theorem 1 to the vector observation equation (17).

Filtering estimate of $W(k)$: $\hat{W}(k,k)$

$$\hat{W}(k,k) = \Phi\hat{W}(k-1,k-1) + G_W(k)(Y(k) - \Phi\hat{W}(k-1,K-1)), \quad \hat{W}(0,0) = 0 \tag{21}$$

Filter gain: $G_W(k)$

$$G_W(k) = (TK_x(k,k)T^T - \Phi S(k-1)\Phi^T)(R + TK_x(k,k)T^T - \Phi S(k-1)\Phi^T)^{-1} \tag{22}$$

$$S(k) = \Phi S(k-1)\Phi^T + G_W(k)(TK_x(k,k)T^T - \Phi S(k-1)\Phi^T), \quad S(0) = 0 \tag{23}$$

The filtering estimate of the state vector

$$x(k) = [x_1(k) \;\; x_2(k) \;\; x_3(k) \;\; x_4(k)]^T = [z(k) \;\; z(k+1) \;\; z(k+2) \;\; z(k+3)]^T$$

is composed by $\hat{x}(k,k) = T^{-1}\hat{W}(k,k)$

For the wavelet transformation of the four-channel filter bank of Fig.2,

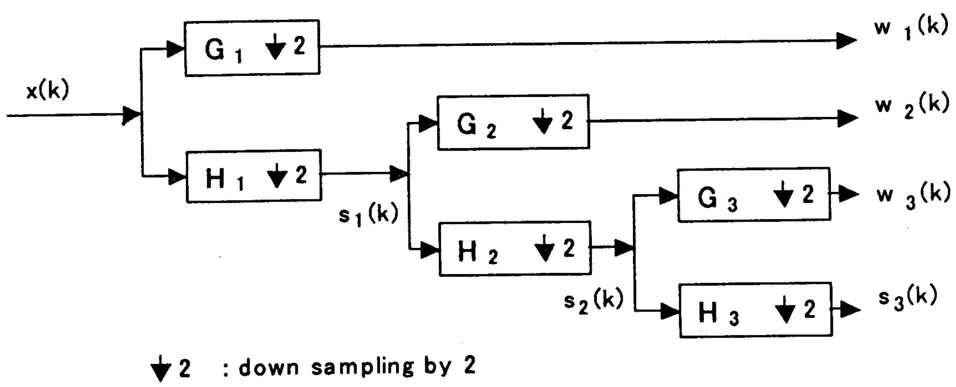

Fig.2 A four-channel filter bank

the value of n is 8. The input to the filter bank is $x(k)=\left[x_1(k)\ x_2(k)\ \cdots\ x_8(k)\right]^T$.
Here, the scaling coefficient $s_3(k)$ and the wavelet coefficients

$w_1(k)$, $w_2(k)=\left[w_{2,1}(k)\ w_{2,2}(k)\right]$ and $w_3(k)$ at time k are obtained as the outputs of the four-channel filter bank. In the Haar transformation for n=4, G_1 is a high pass filter denoted by

$$G_1=\begin{bmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 \end{bmatrix}, \tag{24}$$

which plays a role of down sampling simultaneously. Output of G_1 is the wavelet coefficient $w_1(k)=\left[w_{1,1}(k)\ w_{1,2}(k)\ w_{1,3}(k)\ w_{1,4}(k)\right]^T$. Here, $w_{1,1}(k)$, $w_{1,2}(k)$, $w_{1,3}(k)$ and $w_{1,4}(k)$ are introduced as vector components of $w_1(k)$. H_1 is a low pass filter denoted by

$$H_1=\begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \end{bmatrix}, \tag{25}$$

which also plays a role of down sampling at the same time. Output of H_1 is the scaling coefficient $s_1(k)$. Output of a high pass filter

$$G_2=\begin{bmatrix} 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix} \tag{26}$$

is a wavelet coefficient $w_2(k) = [w_{2,1}(k)\ \ w_{2,2}(k)]^T$. Output of a low pass filter

$$H_2=\begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix} \tag{27}$$

is a scaling coefficient $s_2(k)$. Outputs of a high pass filter G_3 = [0.5 -0.5] and a low pass filter H_3 = [0.5 0.5] are $w_3(k)$ and $s_3(k)$.

As a result, the wavelet transformation is reduced to $W(k)=Tx(k)$,

$$W(k)=\begin{bmatrix} s_3(k) \\ w_3(k) \\ w_{2,1}(k) \\ w_{2,2}(k) \\ w_{1,1}(k) \\ w_{1,2}(k) \\ w_{1,3}(k) \\ w_{1,4}(k) \end{bmatrix}, \quad T=\begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 & -0.125 & -0.125 & -0.125 & -0.125 \\ 0.25 & 0.25 & -0.25 & -0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.25 & -0.25 & -0.25 \\ 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 \end{bmatrix}. \quad (28)$$

4. A Numerical Simulation Example

Let us consider estimating a vowel signal pronounced by the author. Its phonetic symbol is written as " /a:/" . The sampling frequency f_s for the continuous voice signal is 11.025 [kHz]. For the sampling frequency, the sampling period is $T_s = 1/f_s$(sec). The sampled discrete-time signal sequence of the vowel sound is modeled in terms of the AR process of order n. Based on the technique in section 3, $K_z(k,k)$ and the AR parameters, a_i, $i = 1, 2, \cdots, n$, are estimated by the autocovariance data $K_z(i)$, $i = 0, 1, \cdots, n$, By substituting the transformation matrix T, the system matrix Φ, the autovariance function of $x(k)$, $K_x(k,k)$, and the variance of white observation noise, R, into the filtering equations, the filtering estimate of $W(k)$, $\hat{W}(k,k)$, is calculated. The filtering estimate of $x(k)$ is obtained by $\hat{x}(k,k) = T^{-1}W(k,k)$. The filtering estimate of $z(k)$, $\hat{z}(k,k)$, is obtained by $\hat{z}(k,k) = \hat{x}_1(k,k)$, which represents the first component of $\hat{x}(k,k)$.

Fig.3 illustrates the signal $z(k)$ and its filtering estimate $\hat{z}(k,k)$ vs. k for signal-to-noise ratio (SNR) 5 [dB] in the two-channel filter bank.

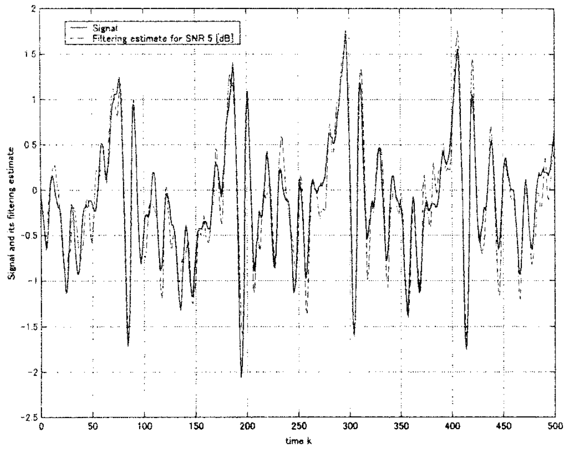

Fig.3 Signal $z(k)$ and its filtering estimate $\hat{z}(k,k)$ vs. k for signal-to-noise ratio (SNR) 5 [dB].

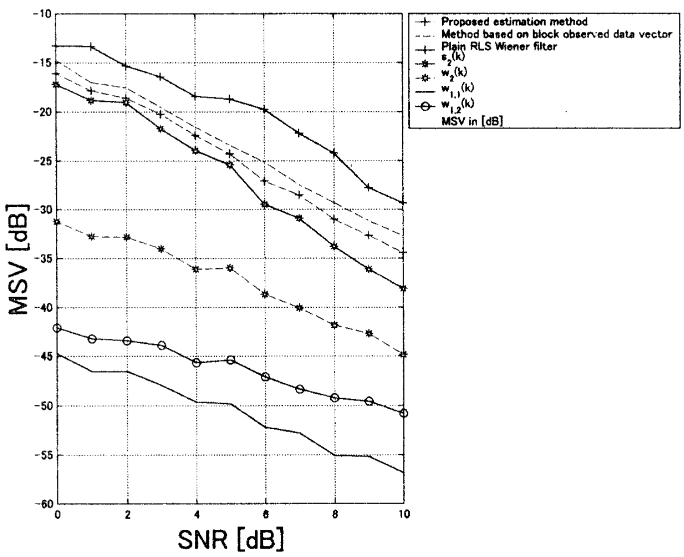

Fig.4 Mean-square values of filtering errors of the signal $z(k)$, $s_2(k)$, $w_2(k)$, $w_{1,1}(k)$ and $w_{1,2}(k)$ vs. SNR.

Table 1 shows the mean-square values (MSVs) of the filtering errors of the signal $z(k)$, the scaling coefficient $s_2(k)$, and the wavelet coefficients $w_2(k)$, $w_{1,1}(k)$ and $w_{1,2}(k)$ for SNRs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Fig.4 illustrates the MSVs of the filtering errors of $z(k)$, $s_2(k)$, $w_2(k)$, $w_{1,1}(k)$ and $w_{1,2}(k)$ vs. SNR. To the estimation of $z(k)$, the proposed filtering method, the estimation method in the case of using the transformation matrix $T=I$ (4-by-4 identity matrix) instead of T in (20) and the standard RLS Wiener filter [2] for $n=4$ are applied. Here, the MSV is calculated by $10\log_{10}(\sum_{k=1}^{1000}(z(k)-\hat{z}(k,k))^2/\sum_{k=1}^{1000}z^2(k))$ [dB]. The estimation accuacy is good in the order of $w_{1,1}(k)$, $w_{1,2}(k)$, $w_2(k)$, $s_2(k)$ and $z(k)$. In the estimation of the signal $z(k)$, the proposed estimation method is more accurate than that with the identity transformation matrx and the standard RLS Wierner filter. It is also noted that the filtering method with the identity transformation matrix is more accurate than the standard RLS Wiener filter. As the SNR becomes large, the MSV tends to be small as expected.

Table 1 Mean-square values of filtering errors of the signal $z(k)$, $s_2(k)$, $w_2(k)$, $w_{1,1}(k)$ and $w_{1,2}(k)$ for SNRs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in the two-channel filter bank.

Signal to noise ratio [dB]	MSV of filtering error of signal by the proposed technique [dB] (MSV by identity transform matrix technique [dB])	MSV of filtering error of signal by plain recursive Wiener filter [dB]	MSV of filtering error of scaling coefficient $s_2(k)$ [dB]	MSV of filtering error of wavelet coefficient $w_2(k)$ [dB]	MSV of filtering error of wavelet coefficient $w_{1,1}(k)$ [dB]	MSV of filtering error of wavelet coefficient $w_{1,2}(k)$ [dB]
0	-16.0757 (-14.7103)	-13.2669	-17.1937	-31.2159	-44.7064	-42.1033
1	-17.8466 (-17.0137)	-13.3331	-18.8333	-32.7586	-46.5160	-43.2266
2	-18.5973 (-17.5585)	-15.3390	-19.0610	-32.8197	-46.4828	-43.4044
3	-20.2552 (-19.5147)	-16.4202	-21.7322	-34.0114	-47.9267	-43.9046
4	-22.4130 (-21.5560)	-18.4256	-23.9533	-36.1330	-49.6340	-45.6333
5	-24.2970 (-23.4130)	-18.6970	-25.4254	-35.9822	-49.7951	-45.3541
6	-25.2072 (-27.0868)	-19.7758	-29.4833	-38.6663	-52.2210	-47.0788
7	-28.5391 (-27.4660)	-22.2103	-30.9150	-40.0916	-52.7767	-48.3206
8	-30.9997 (-29.3006)	-24.2465	-33.7727	-41.8467	-55.0604	-49.2193
9	-32.6600 (-31.1365)	-27.7665	-36.1103	-42.7008	-55.1917	-49.5570
10	-34.4120 (-32.6696)	-29.3315	-38.0872	-44.8374	-56.8464	-50.7763

Table 2 shows the MSVs of filtering errors of the signal $z(k)$, $s_3(k)$, $w_{2,1}(k)$, $w_{2,2}(k)$, $w_{1,1}(k)$ $w_{1,2}(k)$ $w_{1,3}(k)$ and $w_{1,4}(k)$ for SNRs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in the four-channel filter bank. From Table 1 and Table 2, it is seen that the estimation accuracy of $z(k)$ in the wavelet RLS Wiener filter for four-channel filter bank is

Table 2 Mean-square values of filtering errors of the signal $z(k)$, $s_3(k)$, $w_{2,1}(k)$, $w_{2,2}(k)$, $w_{1,1}(k)$, $w_{1,2}(k)$, $w_{1,3}(k)$, and $w_{1,4}(k)$ for SNRs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in the four-channel filter bank.

Signal to noise ratio [dB]	MSV of filtering error of signal by the proposed technique [dB] (MSV by identity transform matrix technique [dB])	MSV of filtering error of signal by plain recursive Wiener filter [dB]	MSV of filtering error of scaling coefficient $s_3(k)$ ($w_3(k)$) [dB]	MSV of filtering error of wavelet coefficient $w_{2,1}(k)$ ($w_{2,2}(k)$) [dB]	MSV of filtering error of wavelet coefficient $w_{1,1}(k)$ ($w_{1,2}(k)$) [dB]	MSV of filtering error of wavelet coefficient $w_{1,3}(k)$ ($w_{1,4}(k)$) [dB]
0	-15.2509 (-15.0813)	-10.8011	-21.7993 (-25.2401)	-35.1828 (-29.4981)	-46.4430 (-46.3823)	-43.9198 (-39.5464)
1	-17.0109 (-16.4668)	-12.9089	-22.5226 (-27.0582)	-37.5282 (-30.8945)	-48.3307 (-48.3705)	-45.5565 (-40.6934)
2	-20.2374 (-19.0216)	-13.2187	-26.5697 (-29.2899)	-39.4125 (-32.8832)	-49.7219 (-50.0144)	-47.4813 (-42.4270)
3	-22.0092 (-21.4879)	-14.4161	-29.0793 (-30.8108)	-40.3256 (-33.3689)	-50.3176 (-50.5473)	-48.0393 (-42.3432)
4	-23.9084 (-22.3715)	-16.5499	-30.8757 (-33.2594)	-42.5213 (-35.0412)	-52.0092 (-52.2596)	-49.9950 (-43.2255)
5	-25.3750 (-24.1764)	-18.3945	-33.4695 (-34.0813)	-42.6665 (-35.6916)	-52.0683 (-52.4569)	-50.5659 (-43.7125)
6	-27.7475 (-26.3495)	-20.3667	-34.9570 (-37.3161)	-45.3790 (-37.9998)	-53.8656 (-54.1947)	-52.5806 (-45.2230)
7	-29.5266 (-28.3843)	-22.6263	-37.3536 (-39.5856)	-46.5526 (-39.3967)	-54.1442 (-54.6688)	-53.6858 (-46.0086)
8	-31.6181 (-30.7002)	-24.8163	-40.6265 (-42.0660)	-48.0828 (-41.9643)	-54.2998 (-55.2624)	-55.0136 (-48.0454)
9	-33.6890 (-32.1464)	-25.4656	-42.0206 (-44.6165)	-50.2056 (-43.6141)	-55.0750 (-56.0454)	-55.9999 (-48.7258)
10	-34.4689 (-33.7089)	-28.0822	-44.4124 (-44.6986)	-49.8928 (-43.4534)	-54.6439 (-55.4985)	-55.4907 (-48.3950)

superior to that for the two-channel filter bank when the SNR is larger than 1. Also, as a whole, the estimation accuracy of $z(k)$ by the estimation method with the identity transformation matrix for four-channel filter bank is superior to that for the two-channel filter bank when the SNR is larger than 1.

5. Conclusions

In this paper, in the relation with the wavelet transformation, the RLS filtering technique using the covariance information is introduced. The estimation accuracy of the proposed filter is better than the filtering method with the identity transformation matrix and also the standard RLS Wiener filter. It might be interesting that the filtering method with the identity transformation matrix is better in estimation accuracy than the standard RLS Wiener filter. The estimation accuracy of z(k) in the wavelet RLS Wiener fiiter for four-channel filter bank is superior to that for the two-channel filter bank when the SNR is larger than 1.

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