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The Current Volume and Change Estimations for Sampling with Partial Replacement on Two Occasions: Case of Swiss Continuous Forest Inventory

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Introduction

The Swiss continuous forest inventory (CFI) was commenced in 1983, and it continued for three years. The Swiss CFI was conducted with systematic permanent sampling on the ground. At this time the Swiss CFI is to be carried out every 10–15 years, but the next CFI method and the corresponding methods for checking predictions and success are still in the developing stage. For example, double samplings with aerial photographs instead of stratification on the basis of stands maps have been tested for the CFI method by Zobeiry[12]. The purpose of this paper is to estimate the current volume and change from Swiss CFIs' on two occasions, when double samplings with aerial photographs are applied in the next CFI procedure. For this estimation Sampling with Partial Replacement (SPR) on multiple occasions is used.

SPR was first introduced by Jessen[4] and was more fully developed by Patterson[6]. Ware and Cunia[11] introduced SPR to the forestry literature. Scott[9,10] gave a new look to SPR with estimators which are simpler and more logical to evaluate than those given in previous literature. The application of this SPR estimation to the stand-structure estimation based upon aerial photographs was presented by this author[13]. In this application we used the SPRs on two occasions, SPRs on multiple occasions are presented by Scott[9] too.

CFI method in Switzerland

In Switzerland the first CFI was conducted with systematic permanent sampling on the ground. This method gives sound and easily interpretable results. However, a combined inventory with photo plots and ground plots will be preferable to the sampling only on the ground. The general procedure for combined permanent sampling is presented by Schmid[7]. This consists of double sampling (DS) with aerial photographs for regression or for stratification on each occasion, and a combined inventory with permanent sample units based on aerial photographs (or other media of remote sensing) and on the ground. In this study of the Swiss CFI procedure, DS with aerial photographs for regression on each occasion, and SPR on succesive occasions are used, as this method is more general than the Swiss method. The Swiss method (combined inventory with permanent sample units on aerial photographs and on the ground) will be discussed in the next paper. Therefore, ground plots, photo plots and combined plots were established on both occasions, and some of them are permanent plots. A graphical description of the three plot-types is given in Table 1.

Table 1. Plot types of Swiss CFI

plots	occasions 1	2	permanent
ground	*	*	*
photo	*	*	*
combined	*	*	*

There are nine types of plots. It is desirable to use all types of plots for the estimation, but it is very difficult. If all the types of plots were used, the estimation method would become too much complicated. In this study ground plots established on all three occasions and photo plots established only on the second occasion were used for the estimation of current volume. By using this method, SPR on three occasions could be applied. The relationship between SPR's on three occasions and this case is given in Table 2.

Table 2. Relationship between SRR on three occasions and this study

method	SPR	this study
occasions	first	ground measurement at first occasion
	second	photointerpretation at second occasion
	third	ground measurement at second occasion

In the case of the Swiss CFI, SPR on three occasions were applied to the current volume and change-estimation.

General theory

In details readers should make references to the previous literature of Scott[9].

Notation

one occasion

n_1=number of plots established on the first occasion

second and third occasions

A description of the plot-types is noted in Table 3.

The number of different plot types is often reduced by practical considerations. There are two conventions used here, the first convention concerns the way in which different types of sample plots are denoted. The plot-numbers of each type are as follows. The first character in each subscript denotes the occasion at which the plot was established. The subscript p denotes the permanent plots which were established on the first occasion and remeasured during the second occasion. Each character following it denotes the type of plot.

The second convention is used to denote the occasion at which the mean is taken.

W: observation on the first occasion by aerial photograph

X: observation on the first occasion on the ground

Y: observation on the second occasion by aerial photograph

Z: observation on the second occasion on the ground

Table 3. The number and description of samples observed at each of the two measurement occasions

number of plots	occasions					
	1			2		
	type of plots			type of plots		
	ground	photo	combined	ground	photo	combined
n_{1g}	**			—		
n_{1a}		**			—	
n_{1c}			**			—
n_{pg}	**			*		
n_{pa}		**			*	
n_{pc}			**			*
n_{2g}				**		
n_{2a}					**	
n_{2c}						**

**: established *: remeasured —: not remeasured

σ_{WW}, σ_{XX}, σ_{YY} and σ_{ZZ}: true variance of W, X, Y and Z, respectively

σ_{WX}, σ_{XY}, σ_{YZ}, σ_{ZW}, σ_{WY} and σ_{WZ}: true covariance of W and X, X and Y, Y and Z, Z and W, W and Y, and W and Z, respectively

For example, the mean of the n_{2g} plots on the second occasion is denoted as $\bar{Z}_{2g}$ and its variance is:

$V(\bar{Z}_{2g})$: variance of $\bar{Z}_{2g}\,(=\sigma_{ZZ}/n_{2g})$

$v(\bar{Z}_{2g})$: sample based estimation of $V(\bar{Z}_{2g})$

R_{YW} and R_{ZX}: true correlation between the second and the first occasions on the aerial photo plot and ground plot, respectively

β_{ZY} and β_{XW}: true slope coefficient in the simple linear regression of Z on Y, and X on W, respectively

Two measurement occasions

Current volume

As was stated earlier, Scott[9,10] presented unbiased estimators for the third of three occasions and for the change between the second and the third occasions.

On the second of two occasions in this study, three independent estimators of the current mean which are the same as those presented by Scott[9] were found and are listed below.

$\bar{Z}_3$: mean estimator based on the newly established ground plots and sampled on the second occasion, n_{2g}, only

$\bar{Z}_2$: mean estimator based on the combined plots of the second occasion $(=\bar{Z}_{2c}+\beta_{ZY}(\bar{Y}_{2A}-\bar{Y}_{2c}))$ where: β_{ZY} is a slope coefficient in the simple linear regression of Z on Y (Z_{2c} on Y_{2c}).

$\bar{Z}_1$: mean estimator based on the matched ground plots on the first and the second occasion $(=\bar{Z}_{pg}+\beta_{ZX}(\bar{X}_{1G}-\bar{X}_{pg}))$ where: β_{ZX} is a slope coefficient in the simple linear regression of Z on X (Z_{pg} on X_{pg}).

The inverse-weighted procedure[1-3,5] is applied to combine these three independent estimators into a single estimator of the mean of the second occasion. The combined estimator of the mean for the second occasion is:

$$\bar{Z} = \sum_{t=1}^{3} w_{\bar{Z}t}\bar{Z}_t / w_{\bar{Z}}$$

where:

$w_{\bar{Z}t}$=reciprocal of the variance of t-th estimator $\bar{Z}_t (=1/V(\bar{Z}_t))$

$w_{\bar{Z}}$ =summation of the individual weights $(=\sum_{t=1}^{3} w_{\bar{Z}t})$

with variance: $V(\bar{Z})=1/w_{\bar{Z}}$

In most of the practical applications, estimates of mean, variances, covariances, slope coefficients and correlations must be obtained from samples using the common approach of substitution of the sample-based estimations for each of the components of the estimator.

When the variance of each independent estimator is estimated from samples, the sample-based variance of a single estimator of mean for the second occasion is taken from Meier[5] as:

$$v(\hat{\bar{Z}}) = \frac{1}{\hat{w}_{\bar{Z}}}\left\{1 + \frac{4}{\hat{w}^2_{\bar{Z}}}\sum_{t=1}^{3} \hat{w}_{\bar{Z}t}(\hat{w}_{\bar{Z}} - \hat{w}_{\bar{Z}t})/m_t\right\}$$

where: m_t is degrees of freedom at occasion t

Change

Four alternative unbiased estimators of change between the second and the third occasions are presented for the case by Scott[9] for the three occasions. In this study the estimator $\bar{D}_g$ recommended by Scott[9] is used.

Estimator $\bar{D}_g$ consists of three independent estimators. They are:

$\bar{D}_{g3}$: the change estimator based on unmatched ground plots, n_{2g} and n_{1g}, on the first and the second occasions $(=\bar{Z}_{2g}-\bar{X}_{1g})$
with variance: $V(\bar{D}_{g3})=\sigma_{ZZ}/n_{2g}+\sigma_{XX}/n_{1g}$
where: σ_{ZZ} and σ_{XX} are true variance of Z and X

$\bar{D}_{g2}$: the change estimator based on matched ground plots, n_{pg} on the first and the second occasions $(=\bar{Z}_{pg}-\bar{X}_{pg})$
with variance: $V(\bar{D}_{g2})=(\sigma_{ZZ}+\sigma_{XX})/n_{pg}-2\sigma_{ZX}/n_{pg}$

$\bar{D}_{g1}$: the change estimator based on the aerial photo plots of each occasion can be developed as the difference between the mean updated to the second occasion minus the mean updated to the first occasions $(=\bar{Z}_1-\bar{X}_1)$
with variance: $V(\bar{D}_{g1})=V(\bar{Z}_1)+V(\bar{X}_1)$
where: $\bar{Z}_1=\bar{Z}_{2c}+\beta_{ZY}(\bar{Y}_{2A}-\bar{Y}_{2c})$ and $\bar{X}_1=\bar{X}_{1c}+\beta_{XW}(\bar{W}_{1A}-\bar{W}_{1c})$
with variance: $V(\bar{Z}_1)=\left|1-(n_{2a})/n_{2A}{}^{*}\,R^2_{ZY}\right|\sigma_{ZZ}/n_{2c}$ and
$V(\bar{X}_1)=\left|1-(n_{1a})/n_{1A}{}^{*}\,R^2_{XW}\right|\sigma_{XX}/n_{1c}$

The Subscript A denotes all photo-plots (photo plots only plus combined plots). As estimators, $\bar{Z}_1$ and $\bar{X}_1$ are independent of each other, we can get the variance of $\bar{D}_{g1}$ easily. That is:

$$V(\bar{D}_{g1})=V(\bar{Z}_1)+V(\bar{X}_1)$$

The combined difference estimator $\bar{G}$ of the change between the first and the second

occasions is found, using the inverse weighting formulas in the same way presented in the estimation of current volume.
That is:

$$\bar{G} = \sum_{s=1}^{3} w_{\bar{D}gt} \bar{D}_{gt} / w_{\bar{D}g}$$

where:
$w_{\bar{D}gt}$=reciprocal of the variance of t-th difference estimator $\bar{D}_{gt}$ $(=1/V(\bar{D}_{gt}))$

$w_{\bar{D}g}$ =summation of the individual weights $(= \sum_{t=1}^{3} w_{\bar{D}gt})$
with variance: $V(\bar{G})=1/w_{\bar{D}g}$

In most of the practical applications, the above estimators must be obtained from samples in the same way as in case of the current volume estimation.

Discussion

Estimators of the current volume and change on the second occasion of double sampling for regression with aerial photographs and SPR are presented by the application of SPR on three occasions. There are many alternative estimators which were not presented here. Those estimators become very complex, but the extension of change estimation becomes especially complex. The estimator presented here can, however, be used with a minimum of lost information, besides being simpler. When the third or more CFI are conducted, this estimator will be useful for the last two occasions. The amount of information ignored is small, and moreover, due to the fact that the Swiss CFI is now planned to be conducted every 12 years, and the change to be estimated is only between the two most recent occasions, the correlation after three or more measurement occasions is also usually small in forest sampling applications.

Summary

The estimators of current volume on the second occasion and the change between the first and the second occasion are presented, using double sampling with aerial phorographs for regression on each occasion, and the sampling partial replacement (SPR) on succesive occasions is applied as a continuous forest inventory (CFI) method. For this estimation SPR on mutiple occasions is used. The CFI method makes the usage of more information (informations from ground and aerial photographs) possible, and its estimator has a very simple form. The CFI method is useful for the next CFI's not only in case of Switzerland, but also in other countries.

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