Design of RLS-FIR Filter Using Covariance Information in Linear Continuous-Time Stochastic Systems

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Abstract

This paper addresses a new design method of recursive least-squares (RLS) and finite impulse response (FIR) filter, using covariance information, in linear continuous-time stochastic systems. The signal process is observed with additive white noise. It is assumed that the white observation noise is independent of the signal process. The auto-covariance function of the signal is expressed in the semi-degenerate kernel form. The RLS-FIR filter uses the following information:

1. The auto-covariance function of the signal expressed in the semi-degenerate kernel form.
2. The variance of the white observation noise process.
3. The observed values.

Keywords: Continuous-time stochastic system; FIR filter; RLS filter; Signal estimation; Filtering algorithm

1 Introduction

In the filtering problem, the Kalman filter is recursively calculated based on the state-space model of the signal process, starting with initial values at time $t = 0$. Hence, the filtering estimate at time $t \geqslant 0$ uses the observed values $y(s), \quad 0 \leqslant s \leqslant t$. In [1], the finite impulse response (FIR) filter and smootherare shown for continuous time-invariant state-space models. The FIR estimators are calculated by solving a Riccati-type differential equations on a finite interval. Compared with growing-memory filtering, the FIR filter is useful for improving filter divergence due to modeling errors and for detecting signals in systems under sudden changes [2], [3]. Jazwinski [1] and Schweppe [4] introduce the FIR filter for discrete-time state-space models with no driving noises. Bruckstein and Kailath [5] derive recursive FIR filter for the case of general state-space models with driving noise based on the scattering description for both continuous-time and discrete-time stochastic systems. In [6], [7], [8], receding horizon Kalman FIR filter is shown for continuous-time and discrete-time stochastic systems. The horizon FIR filter is derived based on the information form of the Kalman filter and the horizon initial state is assumed to be unknown. Also, the H_2 smoother [9] and the H_∞ smoother [10], with the FIR structure, for discrete-time state-space signal models, are proposed.

As alternatives to the Kalman estimators based on the state-space models, the filter, the fixed-point smoother [11] and the fixed-lag smoother [12], using the covariance information of the signal and the observation noise, are devised. In [13], the extended recursive Wiener fixed-point smoother and filter are presented in discrete-time wide-sense stationary stochastic systems. It is assumed that the signal is observed with the nonlinear mechanism of the signal and with additional white observation noise. The extended recursive Wiener estimators are superior in estimation accuracy to the extended Kalman estimators based on the state-space models. In comparison with the FIR filter based on the state-space models, the estimators in [11]-[13] do not use the information of the input matrix and the variance of the input noise, etc. Hence, they can estimate the signal with less information.

This paper, based on the researches described above, newly designs the recursive least-squares (RLS) FIR filter, using the covariance information, in linear continuous-time stochastic systems. The following stochastic properties are assumed for the processes of the signal and the observation noise. (1) The signal is observed with additional white noise. (2) The white observation noise is independent of the signal process. (3) The auto-covariance function of the signal is expressed in the semi-degenerate kernel form. The RLS-FIR filter uses the following information:

1. The auto-covariance function of the signal expressed in the semi-degenerate kernel form.
2. The variance of the white observation noise process.
3. The observed values.

It is a characteristic that the proposed RLS-FIR filter uses the covariance information of the signal and do not require the state-space models for the signal.

The filtering error variance function, in section 4, of the proposed filter shows that, as the finite observation interval T becomes large, the estimation accuracy of the filter is improved. This is also assured by a numerical simulation example in section 5.

2 Least-squares FIR filtering problem

Let an m-dimensional observation equation be given by

$$y(t) = z(t) + v(t) \tag{1}$$

in linear continuous-time stochastic systems. Here, $z(t)$ is an $n-$dimensional signal with a mean of zero and $v(t)$ is zero-mean white observation noise. It is assumed that the signal process and the sequence of the observation noise are mutually independent. Let the auto-covariance function of $v(t)$ be given by

$$E[v(t)v^T(s)] = R\delta(t-s), \quad R > 0, \tag{2}$$

Here, $\delta(\cdot)$ denotes the Dirac δ function.

Let $K(t,s) = K(t-s)$ represent the auto-covariance function of the signal in wide-sense stationary stochastic systems [14], and let $K(t,s)$ be expressed in the semi-degenerate kernel form of

$$K(t,s) = \begin{cases} A(t)B^T(s), & 0 \leqslant s \leqslant t, \\ B(s)A^T(t), & 0 \leqslant t \leqslant s. \end{cases} \tag{3}$$

Let an FIR filtering estimate $\hat{z}(t,t+T)$ of $z(t+T)$ be expressed by

$$\hat{z}(t,t+T) = \int_t^{t+T} h(t+T,s)y(s)ds, \tag{4}$$

as a linear transformation of the observed value $y(s)$, $t \leqslant s \leqslant t+T$. In (4), $h(t+T,s)$ is a time-varying impulse response function.

Let us consider the estimation problem, which minimizes the mean-square value (MSV)

$$J = E[||z(t+T) - \hat{z}(t,t+T)||^2] \tag{5}$$

of the FIR filtering error $z(t+T) - \hat{z}(t,t+T)$. From an orthogonal projection lemma [14],

$$z(t+T) - \int_{t}^{t+T} h(t+T,\tau)y(\tau)d\tau \perp y(s), \quad t \leqslant s \leqslant t+T, \tag{6}$$

the impulse response function satisfies the Wiener-Hopf integral equation

$$E[z(t+T)y^T(s)] = \int_{t}^{t+T} h(t+T,\tau)K_y(\tau,s)d\tau. \tag{7}$$

Here '$\perp$' denotes the notation of the orthogonality and $K_y(\tau,s)$ represents the auto-covariance function of the observed value.

Substituting (1) and (2) into (7), we obtain

$$h(t+T,s)R = K(t+T,s) - \int_{t}^{t+T} h(t+T,\tau)K(\tau,s)d\tau. \tag{8}$$

3 RLS-FIR algorithm for filtering estimate

Under the linear least-squares estimation problem of the signal $z(k)$ in section 2, [Theorem 1] shows the RLS-FIR filtering algorithm, which uses the covariance information of the signal and the observation noise.

[Theorem 1]

Let the auto-covariance function $K(t,s)$ of $z(t)$ be expressed by (3), and let the variance of the white observation noise be R. Then, the RLS-FIR algorithm for the filtering estimate consists of (9)-(19) in linear continuous-time stochastic systems.

RLS-FIR filtering estimate: $\hat{z}(t,t+T)$

$$\hat{z}(t,t+T) = A(t+T)e(t,t+T) \tag{9}$$

$$J(t+T,t+T) = (B^T(t+T) - r(t,t+T)A^T(t+T))R^{-1} \tag{10}$$

$$J(t+T,t) = (B^T(t) - \Gamma(t,t+T)B^T(t))R^{-1} \tag{11}$$

$$L(t+T,t+T) = (A^T(t+T) - q(t,t+T)A^T(t+T))R^{-1} \tag{12}$$

$$L(t+T,t) = (A^T(t) - p(t,t+T)B^T(t))R^{-1} \tag{13}$$

$$\begin{aligned}\frac{de(t,t+T)}{dt} &= J(t+T,t+T)(y(t+T) - \hat{z}(t,t+T)) \\ &\quad - J(t+T,t)(y(t) - B(t)e(t,t+T))\end{aligned} \tag{14}$$

Initial condition of $e(t,t+T)$ at $t=0$: $e(0,T)$

$$\begin{aligned}\frac{df(t,t+T)}{dt} &= L(t+T,t+T)(y(t+T) - \hat{z}(t,t+T)) - \\ &\quad L(t+T,t)(y(t) - B(t)f(t,t+T))\end{aligned} \tag{15}$$

Initial condition of $f(t,t+T)$ at $t=0$: $f(0,T)$

$$\begin{aligned}\frac{dr(t,t+T)}{dt} &= J(t+T,t+T)(B(t+T) - A(t+T)r(t,t+T)) - \\ &\quad J(t+T,t)(B(t) - B(t)q(t,t+T))\end{aligned} \tag{16}$$

Initial condition of $r(t,t+T)$ at $t=0$: $r(0,T)$

$$\begin{aligned}\frac{d\Gamma(t,t+T)}{dt} &= J(t+T,t+T)(A(t+T) - A(t+T)\Gamma(t,t+T)) \\ &\quad - J(t+T,t)(A(t) - B(t)p(t,t+T))\end{aligned} \tag{17}$$

Initial condition of $\Gamma(t,t+T)$ at $t=0$: $\Gamma(0,T)$

$$\begin{aligned}\frac{dq(t,t+T)}{dt} &= L(t+T,t+T)(B(t+T) - A(t+T)r(t,t+T)) \\ &\quad - L(t+T,t)(B(t) - B(t)q(t,t+T))\end{aligned} \tag{18}$$

Initial condition of $q(t,t+T)$ at $t=0$: $q(0,T)$

$$\begin{aligned}\frac{dp(t,t+T)}{dt} &= L(t+T,t+T)(A(t+T) - A(t+T)\Gamma(t,t+T)) \\ &\quad - L(t+T,t)(A(t) - B(t)p(t,t+T))\end{aligned} \tag{19}$$

Initial condition of $p(t,t+T)$ at $t=0$: $p(0,T)$

The initial conditions on the differential equations (14)-(19) are calculated by (20)-(27) recursively.

$$J(T,T) = (B^T(T) - r(0,T)A^T(T))R^{-1} \tag{20}$$

$$L(T,T) = (A^T(T) - q(0,T)A^T(T))R^{-1} \tag{21}$$

$$\frac{de(0,T)}{dT} = J(T,T)(y(T) - A(T)e(0,T)) \tag{22}$$

Initial condition of $e(0,T)$ at $T = 0$: $e(0,0) = 0$

$$\frac{df(0,T)}{dT} = L(T,T)(y(T) - A(T)e(0,T)) \tag{23}$$

Initial condition of $f(0,T)$ at $T = 0$: $f(0,0) = 0$

$$\frac{dr(0,T)}{dT} = J(T,T)(B(T) - A(T)r(0,T)) \tag{24}$$

Initial condition of $r(0,T)$ at $T = 0$: $r(0,0) = 0$

$$\frac{d\Gamma(0,T)}{dT} = J(T,T)(A(T) - A(T)\Gamma(0,T)) \tag{25}$$

Initial condition of $\Gamma(0,T)$ at $T = 0$: $\Gamma(0,0) = 0$

$$\frac{dq(0,T)}{dT} = L(T,T)(B(T) - A(T)r(0,T)) \tag{26}$$

Initial condition of $q(0,T)$ at $T = 0$: $q(0,0) = 0$

$$\frac{dp(0,T)}{dT} = L(T,T(A(T) - A(T)\Gamma(0,T)) \tag{27}$$

Initial condition of $p(0,T)$ at $T = 0$: $p(0,0) = 0$

Proof.

From (3) and (8), we have

$$h(t+T,s)R = A(t+T)B^T(s) - \int\limits_t^{t+T} h(t+T,\tau)K(\tau,s)d\tau. \tag{28}$$

Introducing an auxiliary function $J(t+T,s)$, which satisfies

$$J(t+T,s)R = B^T(s) - \int\limits_t^{t+T} J(t+T,\tau)K(\tau,s)d\tau, \quad t \leqslant s \leqslant t+T, \tag{29}$$

we obtain

$$h(t+T,s) = A(t+T)J(t+T,s). \tag{30}$$

Let us introduce an auxiliary function $L(t+T,s)$, which satisfies

$$L(t+T,s)R = A^T(s) - \int\limits_t^{t+T} L(t+T,\tau)K(\tau,s)d\tau, \quad t \leqslant s \leqslant t+T. \tag{31}$$

Differentiating (29) with respect to t, we have

$$\begin{aligned} \frac{\partial J(t+T,s)}{\partial t}R = & -J(t+T,t+T)K(t+T,s) + J(t+T,t)K(t,s) \\ & - \int\limits_t^{t+T} \frac{\partial J(t+T,\tau)}{\partial t}K(\tau,s)d\tau. \end{aligned} \tag{32}$$

From (3), (29) and (31), we obtain

$$\begin{aligned} \frac{\partial J(t+T,s)}{\partial t} = & -J(t+T,t+T)A(t+T)J(t+T,s) \\ & + J(t+T,t)B(t)L(t+T,s). \end{aligned} \tag{33}$$

The function $J(t+T,t+T)$ satisfies, by putting $s = t+T$ in (29),

$$J(t+T,t+T)R = B^T(t+T) - \int\limits_t^{t+T} J(t+T,\tau)B(\tau)d\tau A^T(t+T). \tag{34}$$

Introducing

$$r(t,t+T) = \int\limits_t^{t+T} J(t+T,\tau)B(\tau)d\tau, \tag{35}$$

we obtain

$$J(t+T,t+T) = (B^T(t+T) - r(t,t+T)A^T(t+T))R^{-1}. \tag{36}$$

Similarly, by putting $s = t$ in (29), the function $J(t+T,t)$ satisfies

$$J(t+T,t)R = B^T(t) - \int\limits_t^{t+T} J(t+T,\tau)A(\tau)d\tau B^T(t). \tag{37}$$

Introducing

$$\Gamma(t,t+T) = \int\limits_t^{t+T} J(t+T,\tau)A(\tau)d\tau, \tag{38}$$

we obtain

$$J(t+T,t) = (B^T(t+T) - \Gamma(t,t+T)B^T(t))R^{-1}. \tag{39}$$

Differentiating (35) with respect to t, we have

$$\begin{aligned}\frac{dr(t,t+T)}{dt} &= J(t+T,t+T)B(t+T) - J(t+T,t)B(t) \\ &+ \int_t^{t+T} \frac{\partial J(t+T,\tau)}{\partial t} B(\tau)d\tau.\end{aligned} \tag{40}$$

Substituting (33) into (40) and introducing

$$q(t,t+T) = \int_t^{t+T} L(t+T,\tau)B(\tau)d\tau, \tag{41}$$

we have

$$\begin{aligned}\frac{dr(t,t+T)}{dt} &= J(t+T,t+T)B(t+T) - J(t+T,t)B(t) \\ &- J(t+T,t+T)A(t+T)\int_t^{t+T} J(t+T,\tau)B(\tau)d\tau \\ &+ J(t+T,t)B(t)\int_t^{t+T} L(t+T,\tau)B(\tau)d\tau \\ &= J(t+T,t+T)(B(t+T) - A(t+T)r(t,t+T)) \\ &- J(t+T,t)(B(t) - B(t)q(t,t+T)).\end{aligned} \tag{42}$$

Differentiating (38) with respect to t, we have

$$\begin{aligned}\frac{d\Gamma(t,t+T)}{dt} &= J(t+T,t+T)A(t+T) - J(t+T,t)A(t) \\ &+ \int_t^{t+T} \frac{\partial J(t+T,\tau)}{\partial t} A(\tau)d\tau.\end{aligned} \tag{43}$$

Substituting (33) into (43) and introducing

$$p(t,t+T) = \int_t^{t+T} L(t+T,\tau)A(\tau)d\tau, \tag{44}$$

we have

$$\begin{aligned}\frac{d\Gamma(t,t+T)}{dt} &= J(t+T,t+T)A(t+T) - J(t+T,t)A(t) \\ &- J(t+T,t+T)A(t+T)\int_t^{t+T} J(t+T,\tau)A(\tau)d\tau \\ &+ J(t+T,t)B(t)\int_t^{t+T} L(t+T,\tau)A(\tau)d\tau \\ &= J(t+T,t+T)(A(t+T) - A(t+T)\Gamma(t,t+T)) \\ &- J(t+T,t)(A(t) - B(t)p(t,t+T)). \end{aligned} \tag{45}$$

Differentiating (31) with respect to t, we have

$$\begin{aligned}\frac{\partial L(t+T,s)}{\partial t}R &= -L(t+T,t+T)K(t+T,s) + L(t+T,t)K(t,s) \\ &- \int_t^{t+T} \frac{\partial L(t+T,\tau)}{\partial t}K(\tau,s)d\tau. \end{aligned} \tag{46}$$

From (3), (29) and (31), we obtain

$$\begin{aligned}\frac{\partial L(t+T,s)}{\partial t} &= -L(t+T,t+T)A(t+T)J(t+T,s) \\ &+ L(t+T,t)B(t)L(t+T,s). \end{aligned} \tag{47}$$

The function $L(t+T,t+T)$ satisfies, by putting $s=t+T$ in (31),

$$L(t+T,t+T)R = A^T(t+T) - \int_t^{t+T} L(t+T,\tau)B(\tau)d\tau A^T(t+T). \tag{48}$$

From (41), we obtain

$$L(t+T,t+T) = (A^T(t+T) - q(t,t+T)A^T(t+T))R^{-1}. \tag{49}$$

Similarly, by putting $s=t$ in (31), the function $L(t+T,t)$ satisfies,

$$L(t+T,t)R = A^T(t) - \int_t^{t+T} L(t+T,\tau)A(\tau)d\tau B^T(t). \tag{50}$$

From (44), we obtain

$$L(t+T,t) = (A^T(t) - p(t,t+T)B^T(t))R^{-1}. \tag{51}$$

Differentiating (41) with respect to t, we have

$$\frac{dq(t,t+T)}{dt} = L(t+T,t+T)B(t+T) - L(t+T,t)B(t) + \int\limits_t^{t+T} \frac{\partial L(t+T,\tau)}{\partial t} B(\tau)d\tau. \quad (52)$$

Substituting (47) into (52) and using (35), we obtain

$$\begin{aligned}\frac{dq(t,t+T)}{dt} &= L(t+T,t+T)B(t+T) - L(t+T,t)B(t) \\ &- L(t+T,t+T)A(t+T)\int\limits_t^{t+T} J(t+T,\tau)B(\tau)d\tau \\ &+ L(t+T,t)B(t)\int\limits_t^{t+T} L(t+T,\tau)B(\tau)d\tau \\ &= L(t+T,t+T)(B(t+T) - A(t+T)r(t,t+T)) \\ &- L(t+T,t)(B(t) - B(t)q(t,t+T)).\end{aligned} \quad (53)$$

Differentiating (44) with respect to t, we have

$$\frac{dp(t,t+T)}{dt} = L(t+T,t+T)A(t+T) - L(t+T,t)A(t) + \int\limits_t^{t+T} \frac{\partial L(t+T,\tau)}{\partial t} A(\tau)d\tau. \quad (54)$$

Substituting (47) into (54) and using (38), we obtain

$$\begin{aligned}\frac{dp(t,t+T)}{dt} &= L(t+T,t+T)A(t+T) - L(t+T,t)A(t) \\ &- L(t+T,t+T)A(t+T)\int\limits_t^{t+T} J(t+T,\tau)A(\tau)d\tau \\ &+ L(t+T,t)B(t)\int\limits_t^{t+T} L(t+T,\tau)A(\tau)d\tau \\ &= L(t+T,t+T)(A(t+T) - A(t+T)\Gamma(t,t+T)) \\ &- L(t+T,t)(A(t) - B(t)p(t,t+T)).\end{aligned} \quad (55)$$

Substituting (30) into (4), we have

$$\hat{z}(t,t+T) = A(t+T)\int\limits_t^{t+T} J(t+T,s)y(s)ds. \quad (56)$$

Introducing the function

$$e(t,t+T) = \int\limits_{t}^{t+T} J(t+T,s)y(s)ds, \tag{57}$$

we obtain

$$\hat{z}(t,t+T) = A(t+T)e(t,t+T). \tag{58}$$

Differentiating (57) with respect to t, using (33) and introducing

$$f(t,t+T) = \int\limits_{t}^{t+T} L(t+T,s)y(s)ds, \tag{59}$$

we obtain

$$\begin{aligned}\frac{de(t,t+T)}{dt} &= J(t+T,t+T)y(t+T) - J(t+T,t)y(t)\\ &+ \int\limits_{t}^{t+T} \frac{\partial J(t+T,s)}{\partial t} y(s)ds\\ &= J(t+T,t+T)y(t+T) - J(t+T,t)y(t)\\ &- J(t+T,t+T)A(t+T)\int\limits_{t}^{t+T} J(t+T,s)y(s)ds\\ &+J(t+T,t)B(t)\int\limits_{t}^{t+T} L(t+T,s)y(s)ds\\ &= J(t+T,t+T)(y(t+T) - A(t+T)e(t,t+T))\\ &- J(t+T,t)(y(t) - B(t)f(t,t+T)).\end{aligned} \tag{60}$$

Differentiating (59) with respect to t and using (47) and (57), we obtain

$$
\begin{aligned}
\frac{df(t,t+T)}{dt} &= L(t+T,t+T)y(t+T) - L(t+T,t)y(t) \\
&+ \int_t^{t+T} \frac{\partial L(t+T,s)}{\partial t} y(s)ds \\
&= L(t+T,t+T)y(t+T) - L(t+T,t)y(t) \\
&- L(t+T,t+T)A(t+T) \int_t^{t+T} J(t+T,s)y(s)ds \\
&+L(t+T,t)B(t) \int_t^{t+T} L(t+T,s)y(s)ds \\
&= L(t+T,t+T)(y(t+T) - A(t+T)e(t,t+T)) \\
&- L(t+T,t)(y(t) - B(t)f(t,t+T)). \qquad (61)
\end{aligned}
$$

The initial condition on the differential equation (42) for $r(t,t+T)$, at $t=0$, is $r(0,T)$, which is expressed, from (35), by

$$
r(0,T) = \int_0^T J(T,\tau)B(\tau)d\tau. \qquad (62)
$$

From (29), $J(T,s)$ satisfies

$$
J(T,s)R = B^T(s) - \int_0^T J(T,\tau)K(\tau,s)d\tau. \qquad (63)
$$

Differentiating (63) with respect to T, we have

$$
\frac{\partial J(T,s)}{\partial T}R = -J(T,T)K(T,s) - \int_0^T \frac{\partial J(T,\tau)}{\partial T}K(\tau,s)d\tau. \qquad (64)
$$

From (3) and (63), we obtain

$$
\frac{\partial J(T,s)}{\partial T} = -J(T,T)A(T)J(T,s). \qquad (65)
$$

Differentiating (62) with respect to T and using (65), we obtain

$$
\begin{aligned}
\frac{dr(0,T)}{dT} &= J(T,T)B(T) + \int_0^T \frac{\partial J(T,\tau)}{\partial T}B(\tau)d\tau \\
&= J(T,T)B(T) - J(T,T)A(T) \int_0^T J(T,\tau)B(\tau)d\tau \\
&= J(T,T)(B(T) - A(T)r(0,T)). \qquad (66)
\end{aligned}
$$

The initial condition on the differential equation (66) at $T = 0$ is $r(0,0) = 0$ from (62).

The initial condition on the differential equation (45) for $\Gamma(t, t+T)$, at $t = 0$, is $\Gamma(0,T)$, which is expressed, from (38), by

$$\Gamma(0,T) = \int_0^T J(T,\tau)A(\tau)d\tau. \tag{67}$$

Differentiating (67) with respect to T and using (65), we obtain

$$\begin{aligned}\frac{d\Gamma(0,T)}{dT} &= J(T,T)A(T) + \int_0^T \frac{\partial J(T,\tau)}{\partial T}A(\tau)d\tau \\ &= J(T,T)A(T) - J(T,T)A(T)\int_0^T J(T,\tau)A(\tau)d\tau \\ &= J(T,T)(A(T) - A(T)\Gamma(0,T)).\end{aligned} \tag{68}$$

The initial condition on the differential equation (68) at $T = 0$ is $\Gamma(0,0) = 0$ from (67).

The initial condition on the differential equation (53) for $q(t, t+T)$, at $t = 0$, is $q(0,T)$, which is expressed, from (41), by

$$q(0,T) = \int_0^T L(T,\tau)B(\tau)d\tau. \tag{69}$$

From (31), $L(T,s)$ satisfies

$$L(T,s)R = A^T(s) - \int_0^T L(T,\tau)K(\tau,s)d\tau. \tag{70}$$

Differentiating (70) with respect to T, we have

$$\frac{\partial L(T,s)}{\partial T}R = -L(T,T)K(T,s) - \int_0^T \frac{\partial L(T,\tau)}{\partial T}K(\tau,s)d\tau. \tag{71}$$

From (3) and (63), we obtain

$$\frac{\partial L(T,s)}{\partial T} = -L(T,T)A(T)J(T,s). \tag{72}$$

Differentiating (69) with respect to T and using (62) and (72), we obtain

$$\begin{aligned}\frac{dq(0,T)}{dT} &= L(T,T)B(T) + \int_0^T \frac{\partial L(T,\tau)}{\partial T}B(\tau)d\tau \\ &= L(T,T)B(T) - L(T,T)A(T)\int_0^T J(T,\tau)A(\tau)d\tau \\ &= L(T,T)(B(T) - A(T)r(0,T)). \end{aligned} \tag{73}$$

The initial condition on the differential equation (73) at $T = 0$ is $q(0,0) = 0$ from (69).

The initial condition on the differential equation (55) for $p(t, t+T)$, at $t = 0$, is $p(0,T)$, which is expressed, from (44), by

$$p(0,T) = \int_0^T L(T,\tau)A(\tau)d\tau. \tag{74}$$

Differentiating (74) with respect to T and using (67) and (72), we obtain

$$\begin{aligned}\frac{dp(0,T)}{dT} &= L(T,T)A(T) + \int_0^T \frac{\partial L(T,\tau)}{\partial T}A(\tau)d\tau \\ &= L(T,T)A(T) - L(T,T)A(T)\int_0^T J(T,\tau)A(\tau)d\tau \\ &= L(T,T)(A(T) - A(T)\Gamma(0,T)). \end{aligned} \tag{75}$$

The initial condition on the differential equation (75) at $T = 0$ is $p(0,0) = 0$ from (74).

The initial condition on the differential equation (60) for $e(t, t+T)$, at $t = 0$, is $e(0,T)$, which is expressed, from (57), by

$$e(0,T) = \int_0^T J(T,\tau)y(\tau)d\tau. \tag{76}$$

Differentiating (76) with respect to T and using (65), we obtain

$$\begin{aligned}\frac{de(0,T)}{dT} &= J(T,T)y(T) + \int_0^T \frac{\partial J(T,\tau)}{\partial T}y(\tau)d\tau \\ &= J(T,T)y(T) - J(T,T)A(T)\int_0^T J(T,\tau)y(\tau)d\tau \\ &= J(T,T)(y(T) - A(T)e(0,T)). \end{aligned} \tag{77}$$

The initial condition on the differential equation (77) at $T = 0$ is $e(0,0) = 0$ from (76).

The initial condition on the differential equation (61) for $f(t, t+T)$, at $t = 0$, is $f(0,T)$, which is expressed, from (59), by

$$f(0,T) = \int_0^T L(T,\tau)y(\tau)d\tau. \tag{78}$$

Differentiating (78) with respect to T and using (72) and (76), we obtain

$$\begin{aligned}\frac{df(0,T)}{dT} &= L(T,T)y(T) + \int_0^T \frac{\partial L(T,\tau)}{\partial T} y(\tau)d\tau \\ &= L(T,T)y(T) - L(T,T)A(T)\int_0^T J(T,\tau)y(\tau)d\tau \\ &= L(T,T)(y(T) - A(T)e(0,T)). \end{aligned} \tag{79}$$

The initial condition on the differential equation (79) at $T = 0$ is $f(0,0) = 0$ from (78).

The function $J(T,T)$ satisfies, by putting $s = T$ in (63) and using (3),

$$J(T,T)R = B^T(T) - \int_0^T J(T,\tau)B(\tau)d\tau A^T(T).$$

By using (62), $J(T,T)$ is expressed as

$$J(T,T) = (B^T(T) - r(0,T)A^T(T))R^{-1}. \tag{80}$$

The function $L(T,T)$ satisfies, by putting $s = T$ in (70) and using (3),

$$L(T,T)R = A^T(T) - \int_0^T L(T,\tau)B(\tau)d\tau A^T(T).$$

By using (69), $L(T,T)$ is expressed as

$$L(T,T) = (A^T(T) - q(0,T)A^T(T))R^{-1}. \tag{81}$$

(Q.E.D.)

Let $\hat{z}(0,t) = A(t)e(0,t)$ represent the filtering estimate of $z(t)$ and let $\hat{z}(0,t)$ be calculated recursively by using (20), (22) and (24). The algorithm for the filtering estimate $\hat{z}(0,t)$ uses the observed values $y(\tau)$, $0 \leqslant \tau \leqslant t$. It is noted that the filtering algorithm is same as that in [11].

4 RLS-FIR filtering error variance function

The RLS-FIR filtering error variance function is expressed by

$$\begin{aligned} E[\tilde{z}(t+T)\tilde{z}^T(t+T)] &= K(t+T,t+T) - P_{\hat{z}}(t,t+T), \\ \tilde{z}(t+T) &= z(t+T) - \hat{z}(t,t+T). \end{aligned} \tag{82}$$

Here, $P_{\hat{z}}(t,t+T)$ represents the auto-variance function of the RLS-FIR filtering estimate $\hat{z}(t,t+T)$ as $P_{\hat{z}}(t,t+T) = E[\hat{z}(t,t+T)\hat{z}^T(t,t+T)]$. From (8), it is seen that the RLS-FIR filtering error variance function is given by

$$h(t+T,t+T)R = K(t+T,t+T) - \int_t^{t+T} h(t+T,\tau)K(\tau,t+T)d\tau. \tag{83}$$

Substituting (30) into (83) and using (35), we obtain

$$\begin{aligned} h(t+T,t+T)R &= K(t+T,t+T) \\ &\quad - A(t+T)\int_t^{t+T} J(t+T,\tau)B(\tau)d\tau A^T(t+T) \\ &= K(t+T,t+T) - A(t+T)r(t,t+T)A^T(t+T) \geqslant 0, \\ K(t+T,t+T) &= E[z(t+T)z^T(t+T)] = K(0). \end{aligned} \tag{84}$$

The RLS-FIR filtering error variance function $h(t+T,t+T)R$, the variance function $K(t+T,t+T)$ of the signal $z(t+T)$ and the filtering variance function $P_{\hat{z}}(t,t+T) = A(t+T)r(t,t+T)A^T(t+T)$ are the positive semi-definite symmetric matrices. Hence, it is seen that, as the integral interval T becomes large, in (84), the estimation accuracy of the RLS-FIR filter is improved.

5 A numerical simulation example

Let a scalar observation equation be given by

$$y(t) = z(t) + v(t). \tag{85}$$

Let the observation noise $v(t)$ be a zero-mean white Gaussian process with the variance R, $N(0,R)$. Let the auto-covariance function of the signal $z(t)$ be given by

$$K(t,s) = \frac{3}{16}e^{-|t-s|} + \frac{5}{48}e^{-3|t-s|}. \tag{86}$$

From (86), the functions $A(t)$ and $B(s)$ in (3) are expressed as follows:

$$A(t) = [\ \tfrac{3}{16}e^{-t} \quad \tfrac{5}{48}e^{-3t}\ ], \quad B(s) = [\ e^{s} \quad e^{3s}\ ]. \tag{87}$$

If we substitute (87) into the RLS-FIR filtering algorithm of [Theorem 1], we can calculate the RLS-FIR filtering estimate recursively. Fig.1 illustrates the signal $z(t)$ and the RLS filtering estimate $\hat{z}(0,t) = A(t)e(0,t)$, $0 \leqslant t \leqslant 2.5$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the filter in [11]. Fig.2 illustrates the signal $z(t)$ and the filtering estimate $\hat{z}(t-T,t)$, $0.5 \leqslant t \leqslant 2.5$, $T = 500 \cdot \Delta = 0.5$, $\Delta = 0.001$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the RLS-FIR filtering algorithm in [Theorem1]. Here, Δ represents the step size of the numerical integration in terms of the fourth-order Runge-Kutta-Gill method. As time t advances, the RLS-FIR filtering estimate converges to the signal.

Table 1 compares the mean-square values (MSVs) of the filtering errors by the proposed RLS-FIR filter for $T = 500 \cdot \Delta$ with the RLS filter in [11] for the white Gaussian observation noises $N(0, 0.1^2)$, $N(0, 0.3^2)$, $N(0, 0.5^2)$ and $N(0, 0.7^2)$. As the variance of the observation noise becomes large, the MSV becomes large for the both filters. The MSV for the Case 2-1 is larger than those of the Case 1 and the Case 2-2 for each observation noise. This is based on the fact that the filtering estimate for the Case 2-1 starts with the filtering estimate $\hat{z}(0,t)|_{t=0} = 0$ and the absolute value of the filtering error $z(t) - \hat{z}(0,t)$ is relatively large around $t = 0$. In Case 1, the filtering estimate, by the RLS-FIR filter in [Theorem 1], is calculated recursively based on the 500 observed values at each time. The MSV of the proposed RLS-FIR filter is relatively larger than that for the Case 2-2. This might be based on the fact that the RLS filter for the Case 2-2 uses more observed values as time t advances for $0.5 < t \leqslant 2.5$ in comparison with the constant number of the 500 observed values used for the Case 1.

Table 2 compares the MSVs of the filtering errors by the proposed RLS-FIR filter for $T = 1000 \cdot \Delta$ with the RLS filter in [11] for the white Gaussian observation noises $N(0, 0.1^2)$, $N(0, 0.3^2)$, $N(0, 0.5^2)$ and $N(0, 0.7^2)$. In the Case 1, the filtering estimate is calculated recursively based on 1,000 observed values at each time. As the variance of the observation noise becomes large, the MSV becomes large for the both filters. The MSV for the Case 1 is smaller than that of the Case 2-1 and slightly smaller than that of the Case 2-2 for each observation noise.

Here, the MSVs of the RLS-FIR filtering errors for the Case 1 are evaluated

Table 1: Comparison of the MSVs of the proposed RLS-FIR filter with the RLS filter in [11], using covariance information, for $T = 500 \cdot \Delta$

White Gaussian observation noise	Proposed RLS-FIR filter Case 1: MSV of filtering errors for $0.5 < t \leqslant 2.5$.	RLS filter in [11] Case 2-1: MSV of filtering errors for $0 < t \leqslant 2.5$.	RLS filter in [11] Case 2-2: MSV of filtering errors for $0.5 < t \leqslant 2.5$.
$N(0, 0.1^2)$	0.00424440904600	0.06380784056532	0.00359374059285
$N(0, 0.3^2)$	0.09770149325886	0.38374146305639	0.06583993506543
$N(0, 0.5^2)$	0.26227092046015	0.65616674303527	0.18864273401713
$N(0, 0.7^2)$	0.37579151353174	0.82624196537842	0.29854112344278

Table 2: Comparison of the MSVs of the proposed RLS-FIR filter with the RLS filter in [11], using covariance information, for $T = 1000 \cdot \Delta$.

White Gaussian observation noise	Proposed RLS-FIR filter Case 1: MSV of filtering errors for $1 < t \leqslant 2.5$.	RLS filter in [11] Case 2-1: MSV of filtering errors for $0 < t \leqslant 2.5$.	RLS filter in [11] Case 2-2: MSV of filtering errors for $1 \leqslant t \leqslant 2.5$.
$N(0, 0.1^2)$	6.196625300693155e-004	0.05644795547704	6.954391413372565e-004
$N(0, 0.3^2)$	0.01645365817254	0.35940956729006	0.01664718252321
$N(0, 0.5^2)$	0.06373211503018	0.64216747801833	0.06546634938394
$N(0, 0.7^2)$	0.11196511664697	0.82632200623893	0.12245408078970

by $\sum_{i=1}^{2000} (z(500 \cdot \Delta + i \cdot \Delta) - \hat{z}(i \cdot \Delta, 500 \cdot \Delta + i \cdot \Delta))^2/2000$, $\Delta = 0.001$, in Table 1 and $\sum_{i=1}^{1500} (z(1000 \cdot \Delta + \Delta i) - \hat{z}(i \cdot \Delta, 1000 \cdot \Delta + i \cdot \Delta))^2/1500$ in Table 2.

For references, the state-space model, which generates the signal process, is given by

$$\begin{aligned} z(t) &= x_1(t), \\ \frac{dx_1(t)}{dt} &= x_2(t) + u(t), \\ \frac{dx_2(t)}{dt} &= -3x_1(t) - 4x_2(t) - 2u(t),\ E[u(t)u(s)] = \delta(t-s). \end{aligned} \tag{88}$$

6 Conclusions

In this paper, the new RLS-FIR filtering algorithm, using the information of the covariance function of the signal, in the semi-degenerate kernel form, and the variance of the white observation noise, has been devised in linear continous-time stochastic systems. From the simulation result in section 5, the proposed RLS-FIR filtering algorithm is feasible. As the observation interval T increases, the MSV of the filtering errors becomes small and the estimation accuracy of the RLS-FIR filter is improved.

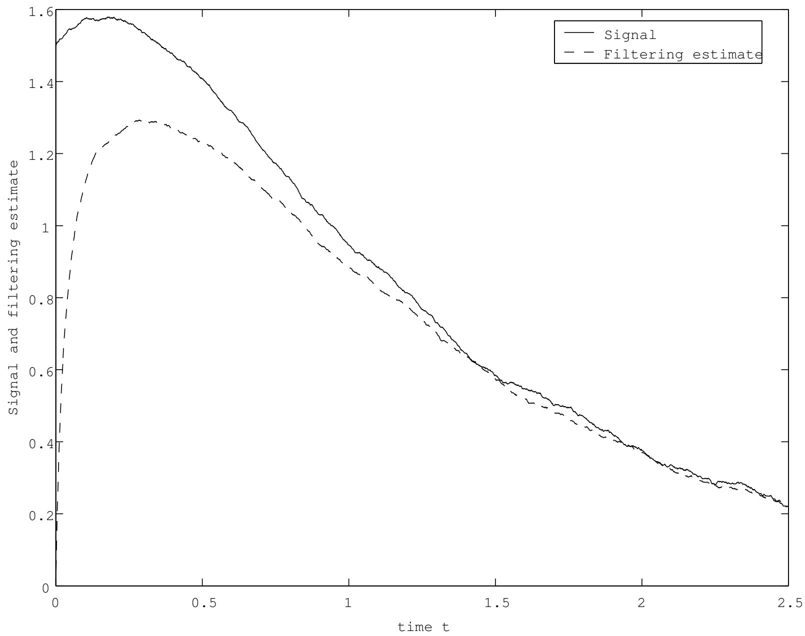

Figure 1: Signal $z(t)$ and filtering estimate $\hat{z}(0,t) = A(t)e(0,t)$, $0 \leqslant t \leqslant 2.5$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the RLS filter in [11].

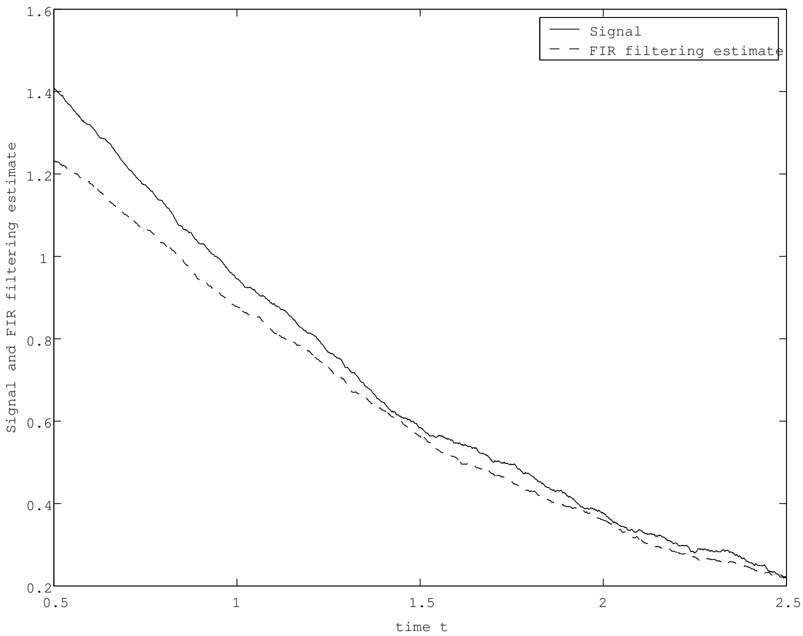

Figure 2: Signal $z(t)$ and filtering estimate $\hat{z}(t-T,t)$, $0.5 \leqslant t \leqslant 2.5$, $T = 500 \cdot \Delta = 0.5$, $\Delta = 0.001$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the RLS-FIR filtering algorithm in [Theorem1].

7 References

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8 Captions of figures and tables

Fig.1 Signal $z(t)$ and filtering estimate $\hat{z}(0,t) = A(t)e(0,t)$, $0 \leqslant t \leqslant 2.5$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the RLS filter in [11].

Fig.2 Signal $z(t)$ and filtering estimate $\hat{z}(t-T,t)$, $0.5 \leqslant t \leqslant 2.5$, $T = 500 \cdot \Delta = 0.5$, $\Delta = 0.001$, for the white Gaussian observation noise $N(0, 0.1^2)$, by the RLS-FIR filtering algorithm in [Theorem1].

Table 1 Comparison of the MSVs of the proposed RLS-FIR filter with the RLS filter in [11], using covariance information, for $T = 500 \cdot \Delta$

Table 2 Comparison of the MSVs of the proposed RLS-FIR filter with the RLS filter in [11], using covariance information, for $T = 1000 \cdot \Delta$.